Cosmological solutions from Induced Matter Model applied to 5D $f(R,T)$ gravity

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Introduction

- To match the cosmological observations as supernovae Ia, the universe in ΛCDM model needs to be filled by an exotic component, named “dark energy" (DE), which makes its expansion to accelerate.
- Physically, what causes the acceleration would be the existence of a quantum vacuum energy, with negative equation of state (EoS) $p \sim -\rho$.
- However, there is a huge discrepancy between the quantum vacuum energy values obtained from Cosmology and from Particle Physics.

- A proposal of changing standard gravity is through the consideration of extra dimensions. The Kaluza-Klein (KK) gravitational model proposes the universe is empty in five dimensions (5D).

- Cosmological models derived from KK theory are continuously presented in the literature.

- KK theories usually admit compactified extradimensions. In fact, compactification is the only mechanism able to explain the apparent 4D nature of the universe in KK gravity. However, it is common to see such a compactification as an imposed feature of KK cosmological models instead of a natural characteristic of the extra coordinate evolution.

- A innovative form of physically interpreting KK gravitational model was brought up by P.S. Wesson, for which the properties of matter of the usual 4D universe (i.e., density and pressure) are regarded as the extra parts - due to the extra dimension - of the 5D Einstein's field equations (FEs) for vacuum (remind that in KK theory, the 5D universe is empty: $G_{AB} = 0$), namely, the Wesson's Induced Matter Model (IMM).

- In fact, Wesson has showed that a 5D theory does not necessarily need an explicit energy-momentum tensor; the extra terms of the 5D Einstein tensor may work as an induced energy-momentum tensor.

- Modifications on the Einstein's FEs are also presented by assuming the gravitational part of the action is given by a generic function of the Ricci scalar R, contemplating the $f(R)$ gravity theories.

- Recently, it was proposed a more generic gravity model, for which the action depends still on a generic function of R, but also on a function of T, the trace of the energy-momentum tensor $T_{\mu\nu}$, namely, the $f(R,T)$ theory of gravity.

- The present work proposes a cosmological model which unifies KK and $f(R,T)$ theory.

$f(R,T)$ gravity

$$S=\frac{1}{16\pi}\int f(R,T)\sqrt{-g}d^4x+\int\mathcal{L}_m\sqrt{-g}d^4x \tag{1}$$

$$f(R,T)=R+2\lambda T\rightarrow G_{\mu\nu}=8\pi T_{\mu\nu}+\lambda T g_{\mu\nu}+2\lambda(T_{\mu\nu}+pg_{\mu\nu}). \tag{2}$$

$$\nabla^\mu T_{\mu\nu}=\frac{2\lambda}{2\lambda-8\pi}\nabla^\mu(2T_{\mu\nu}+pg_{\mu\nu}), \tag{3}$$

5D $f(R,T)$ field equations and their cosmological solutions from Induced Matter Model application

Let me consider a general KK metric of the form

$$ds^2 = e^{\alpha(t,l)}dt^2 - e^{\beta(t,l)}(dx^2 + dy^2 + dz^2) - e^{\gamma(t,l)}dl^2. \tag{4}$$

$$\frac{3}{2}\left[\frac{\dot{\beta}\dot{\gamma}}{2}e^{-\alpha}+\left(\frac{\beta'\gamma'}{2}-\beta'^2-\beta''\right)e^{-\gamma}\right]=(8\pi+3\lambda)\rho-\lambda p, \tag{5}$$

$$\alpha'\dot{\beta}+\dot{\gamma}\beta'=\dot{\beta}\beta'+2\dot{\beta}', \tag{6}$$

$$\begin{aligned}&\left(\dot{\beta}\dot{\gamma}-\frac{\dot{\alpha}\dot{\gamma}}{2}+\frac{\dot{\gamma}^2}{2}+\ddot{\gamma}\right)e^{-\alpha}+ \\ &\left(-\alpha'\beta'-\frac{3}{2}\beta'^2+\beta'\gamma'-2\beta''+\frac{\alpha'\gamma'}{2}-\frac{\alpha'^2}{2}-\alpha''\right)e^{-\gamma} \\ =\ & 2[\lambda\rho-(8\pi+3\lambda)p],\end{aligned} \tag{7}$$

$$-\frac{3}{4}(\beta'^2+\alpha'\beta')e^{-\gamma}=\lambda(\rho-p), \tag{8}$$

$$\frac{3}{4}\dot{\beta}^2 e^{-\alpha} + (8\pi + 3\lambda)\rho - \lambda p = 0, \tag{9}$$

$$\left(-\frac{\dot{\alpha}\dot{\beta}}{2} + \frac{3}{4}\dot{\beta}^2 + \ddot{\beta}\right) e^{-\alpha} - (8\pi + 3\lambda)p + \lambda\rho = 0, \tag{10}$$

$$\frac{3}{2}\left(-\frac{\dot{\alpha}\dot{\beta}}{2} + \dot{\beta}^2 + \ddot{\beta}\right) e^{-\alpha} + \lambda(\rho - p) = 0. \tag{11}$$

General solutions

- In order to find a solution for β, let me use the separation of variables method, i.e., let me take $\beta = T_\beta L_\beta$ with T_β and L_β respectively representing functions of t and l only. By integrating (6), one obtains

$$\gamma = \beta + 2ln(T_\beta). \tag{12}$$

- Moreover, from (8) and (11), one is able to write

$$e^{-\gamma} = 2\frac{(\dot{\beta}^2 + \ddot{\beta})}{\beta'^2}. \tag{13}$$

- Note that Eqs.(12)-(13) allow us to find a differential equation for T_β and a differential equation for L_β if ρ and p are eliminated of the FEs. By plausibly assuming $T_\beta \neq const$, those equations are

$$\ddot{T}_\beta = 0, \tag{14}$$

$$L_\beta'^2 - L_\beta L_\beta'' = 0. \tag{15}$$

- Therefore, the solution for β is

$$\beta = C_1 t e^{C_2 l}. \tag{16}$$

- By using solution (16), the model FEs yield, for the density of the universe,

$$\rho = \frac{3C_1 e^{C_2 l}}{8(4\pi + \lambda)t}(C_1 t e^{C_2 l} - 2). \tag{17}$$

- The substitution of Eqs.(13) and (17) in (8) yields, for the pressure of the universe:

$$p = \frac{3C_1 e^{C_2 l}}{8\lambda}\left[2C_1 C_2^2 e^{h(t,l)} + \frac{C_1 t e^{C_2 l} - 2}{(4\pi + \lambda)t}\lambda\right], \tag{18}$$

with $h(t,l) \equiv -C_1 t e^{C_2 l} + C_2 l$.

Non-conservation of the energy-momentum tensor and the time evolution of the extra coordinate

- The non-conservation of the energy-momentum tensor in $f(R,T)$ theory will have a valuable application in the 5D case, as it will be demonstrated below.

- Note that by taking $A = B = 0$ in the 5D version of Eq.(3), one has

$$\frac{8\pi - \lambda}{2\lambda}\dot{\rho} + \dot{p} = 0, \tag{19}$$

while $A = B = 4$ yields

$$\frac{p'}{p}e^{\gamma} + (e^{\gamma})' = 0. \tag{20}$$

- Now let me substitute the general solutions (17)-(18) in (19)-(20). Such a procedure yields the following equations

$$C_1C_2t\sqrt{e^{h(t,l)+C_2l}} = \sqrt{\frac{8\pi+\lambda}{2(4\pi+\lambda)}}, \tag{21}$$

$$e^{-h(t,l)+C_2l} = 4C_1C_2^2\frac{(4\pi+\lambda)}{\lambda}\frac{te^{C_2l}}{(2-C_1^2t^2e^{C_2l})}. \tag{22}$$

- By substituting (22) in (21) and solving for l, one obtains

$$l_i = \frac{1}{C_2}\{ln[\xi_i(t)] - ln(t)\} \tag{23}$$

as solutions for l as functions of t, with $i = 1$ or 2 and

$$\xi_1 = \frac{C_1\lambda - \sqrt{-16C_1^3\pi\lambda t + C_1^2\lambda^2 - 2C_1^3\lambda^2 t}}{C_1^3\lambda^2 t}. \tag{24}$$

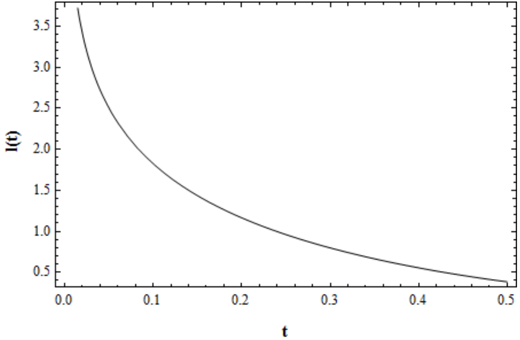

Figure 1: Time evolution of l from Eqs.(23)-(24) with $C_1 = -1$, $C_2 = 1$ and $\lambda = -5\pi$.

Cosmological parameters

- In this subsection, I will present the time evolution of some cosmological parameters of the present model. Those will be derived from the scale factors (12) and (16). However, for the dependence on l, I will use Eqs.(23)-(24). Such a procedure allow us to verify the effects that the evolution of the extra coordinate causes in our 4D observable universe.

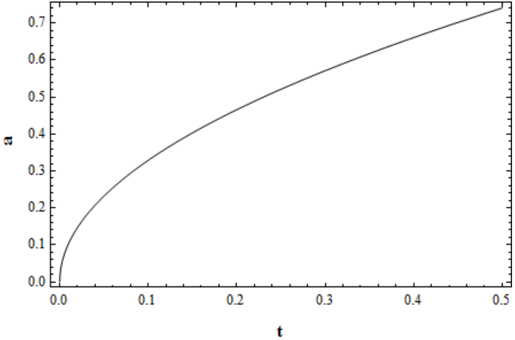

Figure 2: Time evolution of the mean scale factor a with $C_1 = -1$ and $\lambda = -5\pi$.

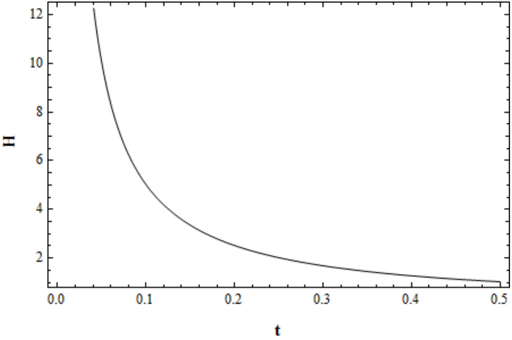

Figure 3: Time evolution of the Hubble parameter H with $C_1 = -1$ and $\lambda = -5\pi$.

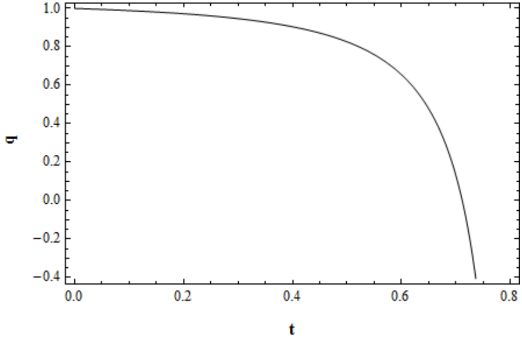

Figure 4: Time evolution of the deceleration parameter q with $C_1 = -1$ and $\lambda = -5\pi$.

Discussion

In this work I presented IMM solutions to 5D $f(R,T)$ theory from a KK metric in which the coefficients depend on both time and extra coordinate. Taking into account the covariant divergence of the energy-momentum tensor, I have obtained an equation for the evolution of the extra coordinate through time. Such an equation has revealed the compactification of the fifth dimension.

When the relation $l(t)$ is substituted in the cosmological parameters of the model, one obtains a projection of the fifth coordinate evolution in our observable 4D universe. Such a substitution is in accordance with recent observations of anisotropies in the cosmic microwave background temperature, which indicates a negative deceleration parameter for the present universe dynamics.

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