Two scalar field cosmology from coupled one-field models - TO APPEAR IN PHYS. REV. D

P.H.R.S. Moraes and J.R.L. Santos

Outline

Introduction

Why alternative models?

Cosmological constant problem

Quintessence models

$$S = \int d^4x \sqrt{-g} \left[-\frac{R}{4} + \mathcal{L}(\phi, \partial_\mu \phi) \right] \tag{1}$$

$$\mathcal{L} = \frac{1}{2} \partial_\mu \partial^\mu \phi - V(\phi) \tag{2}$$

$$\ddot{\phi} + 3\,H\,\dot{\phi} + V_\phi = 0 \tag{3}$$

$$\rho = \dot{\phi}^2 + V(\phi) \tag{4}$$

$$p = \dot{\phi}^2 - V(\phi) \tag{5}$$

$$H^2 = \frac{2}{3}\left[\frac{\dot{\phi}^{\,2}}{2} + V(\phi)\right] - \frac{k}{a^2} \tag{6}$$

$$\dot{H} = -\dot{\phi}^{\,2} + \frac{k}{a^2} \tag{7}$$

The two scalar field quintessence model

Fundaments

$$S = \int d^4 x \sqrt{-g} \left[-\frac{R}{4} + \mathcal{L}(\phi_i , \partial_\mu \phi_i) \right] \tag{8}$$

$$\mathcal{L} = \frac{\dot{\phi}^2}{2} + \frac{\dot{\chi}^2}{2} - V(\phi, \chi) \tag{9}$$

In our notations, $i = 1, 2$, $\phi_1 = \phi(t)$, $\phi_2 = \chi(t)$.

$$\ddot{\phi} + 3\,H\,\dot{\phi} + V_{\phi} = 0 \tag{10}$$

$$\ddot{\chi} + 3\,H\,\dot{\chi} + V_{\chi} = 0 \tag{11}$$

$$\rho = \frac{\dot{\phi}^2}{2} + \frac{\dot{\chi}^2}{2} + V(\phi, \chi) \tag{12}$$

$$p = \frac{\dot{\phi}^2}{2} + \frac{\dot{\chi}^2}{2} - V(\phi, \chi) \tag{13}$$

$$H^2 = \frac{\dot{\phi}^2}{3} + \frac{\dot{\chi}^2}{3} + \frac{2}{3}\,V(\phi, \chi) \tag{14}$$

$$\dot{H} = -\dot{\phi}^2 - \dot{\chi}^2 \tag{15}$$

Solutions

$$H(t) = B + Ae^{-4At} + \alpha \cosh\left[2arccoth\left(e^{\alpha\beta^2 t}\right)\right] \tag{16}$$

$$a(t) = a_0\left[2\left(1 - e^{2\alpha\beta^2 t}\right)\right]^{\beta^{-2}} \exp\left[-\frac{1}{4}e^{-4At} + (B - \alpha)\,t\right] \tag{17}$$

$$\omega(t) = \frac{8A^2\cosh(4At) - 3\left[B + Ae^{-4At} + \alpha\coth\left(\alpha\beta^2 t\right)\right]^2 + 2\alpha^2\beta^2 csch^2\left(\alpha\beta^2 t\right) - 8A^2\sinh(4At)}{3\left[B + Ae^{-4At} + \alpha\coth\left(\alpha\beta^2 t\right)\right]^2} \tag{18}$$

Dimensional Analysis

- we see from Eq.(17) that the non-dimensional property of the scale factor is respected, since it is given by the product of an exponential with an arbitrary non-dimensional constant.

- The dimension of the Hubble parameter $H(t)$ in Eq.(16) is directly connected to the dimension of the constants A, B and α. From (16) we see that it would be interesting if those constants had the dimension inverse time, which is in fact the Hubble parameter dimension. Eq. (17) only strengthens this assumption. One can see that for the argument of the first exponential to be dimensionless, $[\alpha] = [\beta] = [t]^{-1}$, as also required in the second exponential.

- One can check that ω is dimensionless in Eq. (18).

Cosmological Interpretations

- In this section we show that our model presents physical and cosmological consistence for some given values of A, B, α and β.

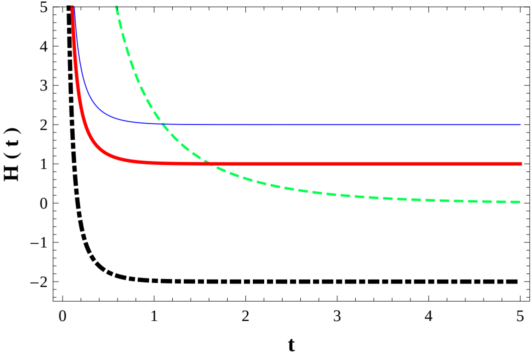

Figure 1: Plots of parameter $H(t)$, with $A = 5$, $\alpha = -1$, $\beta = 3/2$ and $B = -3$ for the black curve, $B = 0$ for the red curve, $B = 1$ for the blue curve, and $A = 5$, $\alpha = -2$, $\beta = 1/2$, $B = -2$ for the green curve.

- Since $H \sim t^{-1}$, with t being the Hubble time, $H(t)$ must decrease with time, as observed in Fig.(1). Also, we discard the black (dot-dashed) curve once it allows negative values of $H(t)$, which is a physical inconsistency in an expanding universe.

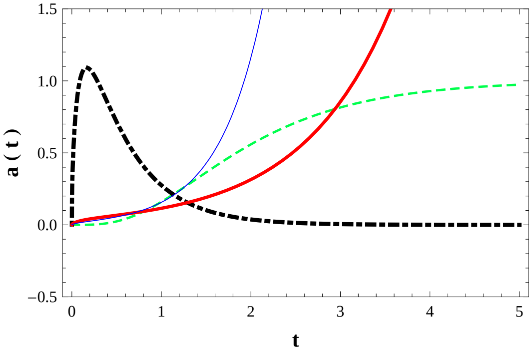

Figure 2: Plots of parameter $a(t)$, with $A = 5$, $\alpha = -1$, $\beta = 3/2$ and $B = -3$ for the black curve, $B = 0$ for the red curve, $B = 1$ for the blue curve, and $A = 5$, $\alpha = -2$, $\beta = 1/2$, $B = -2$ for the green curve.

- An interesting feature about the black curve for $a(t)$ in Fig.(2) is the bump for small values of t. In the inflationary phase, when the energy density of the universe is dominated by a (cosmological) constant, the Friedmann equation solution is a scale factor that grows exponentially with time as $a(t) \propto e^{H_\iota t}$, with H_ι being the value of the Hubble parameter during inflation. This bump might thus represent the inflationary phase. Nevertheless, in the present case, the black (dot-dashed) curve for $H(t)$ has been discarded, so for cosmological purposes, all the curves with $A = 5$, $\alpha = -1$, $\beta = 1.5$, $a_0 = 3/2$ and $B = -3$ must also be discarded.

- In Fig.(2), one can see that the green curve for $a(t)$ represents $\dot{a} \to 0$ for large values of t, which implies a null Hubble parameter.

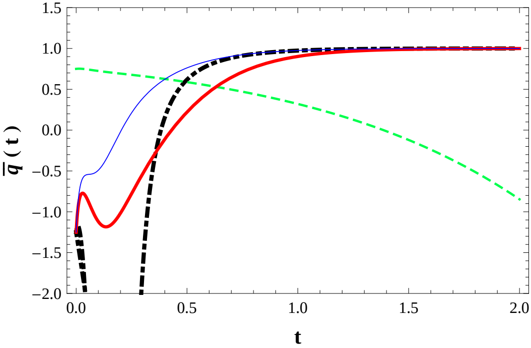

Figure 3: Here we show the different forms of the acceleration parameter $\bar{q}(t)$, in which we considered $A = 5$, $\alpha = -1$, $\beta = 3/2$ with $B = -3$ for the black curve, $B = 0$ for the red curve, $B = 1$ for the blue curve, and $A = 5$, $\alpha = -2$, $\beta = 1/2$, $B = -2$ for the green curve.

- The information on the green curve for $a(t)$ combined with the anomalous behavior of the acceleration parameter of the green curve (see Fig.(2)) configures an unpleasant cosmological scenario. Therefore, we focus our attention to the blue and red curves.

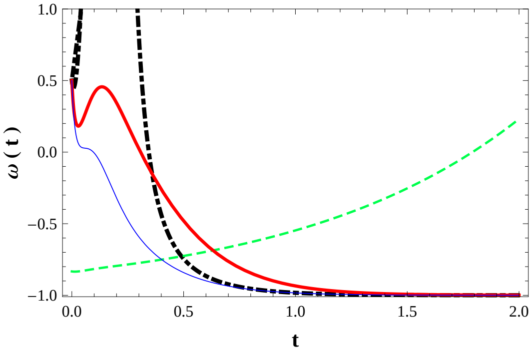

Figure 4: Plots of $\omega(t)$, with $A = 5$, $\alpha = -1$, $\beta = 3/2$, $B = -3$ in the black curve, $B = 0$ in the red curve, $B = 1$ in the blue curve, and $A = 5$, $\alpha = -2$, $\beta = 1/2$, $B = -2$ in the green curve.

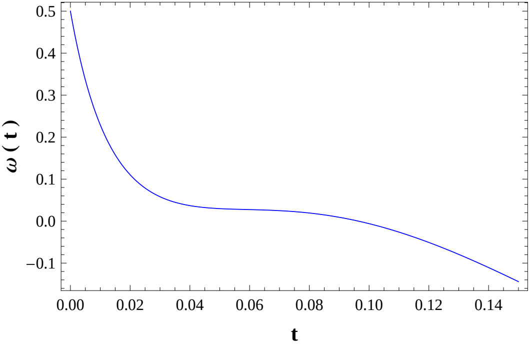

Figure 5: The figure shows in more detail the plateau-like behavior of $\omega(t)$, which occurs in the blue curve.

- In Figs.(4-5) we plot the EoS parameter ω. We zoom in on the blue curve in Fig.(4), and in order to clarify its features, let us briefly review some aspects concerning the density of the universe and the EoS parameter.

- The conservation of the energy-momentum tensor ($\nabla_\mu T^{\mu\nu} = 0$) in standard Einstein's field equations results in

$$\rho = \rho_0 a^{-3(1+\omega)} \tag{19}$$

for the density of the universe if we consider ρ_0 a constant and $a_0 = 1$ the present value of the scale factor.

- From FRW cosmology, we can separate the universe dynamics in three regimes: the relativistic matter scenario, related to $\omega = 1/3$ (which implies $\rho_r \propto a^{-4}$); the non-relativistic matter scenario, related to $\omega = 0$ ($\rho_m \propto a^{-3}$); and the quantum vacuum scenario, corresponding to $\omega = -1$ ($\rho_\Lambda = \rho_0$).
- From the blue curve in Figs.(4-5), for early times, ω assumes the value $1/3$ and values near to it, which shall represent the radiation dominated era. As the universe expands and cools down, the matter-radiation decoupling makes the universe propitious to form the stars, galaxies and clusters of galaxies. This era is dominated, then, by matter, with $p = 0$ ($\omega = 0$), which in Fig.(5) is presented as a plateau-like behavior of the blue curve for a non-negligible period of time. Note also that for high values of time, $\omega \to -1$, in agreement with recent observations of Planck satellite, which have constrained the EoS parameter to $\omega = -1.073^{+0.090}_{-0.089}$.

Discussion and perspective

- Some of the plotted results, as the blue curves, have showed very interesting features which we revisit in the following.
- In Fig.(4) there is a plateau-like behavior around $\omega = 0$ (consequently $p = 0$) which could represent the matter-dominating era of the universe. The derivatives of ω with respect to time are near zero in the interval $t \in [0.04 - 0.10]$.
- Also, for $t < 0.04$, one can see an abrupt variation of ω in a small interval of time. Note that this variation is continuous and constrained to values around $1/3$, which is the value of ω for a radiation-like EoS.
- Furthermore, the model predicts the late acceleration of the universe expansion since $\omega \to -1$ for high values of time.

- In conclusion, the previous results support the two coupled scalar models description since new nontrivial behavior from the coupling between the fields was unveiled.

- Possible interest related to the paper arXiv:1403.5009. Possible collaboration with Bibhas Ranjan Majhi, The Hebrew University of Jerusalem, extending such approach to $f(R)$ theories of gravity.

Thanks!