The physics of the HI 21cm line
Observing the early Universe

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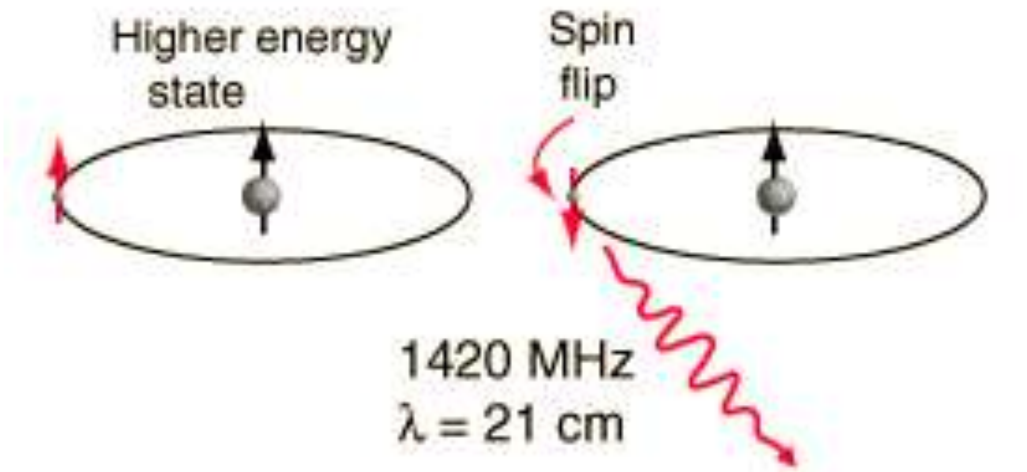
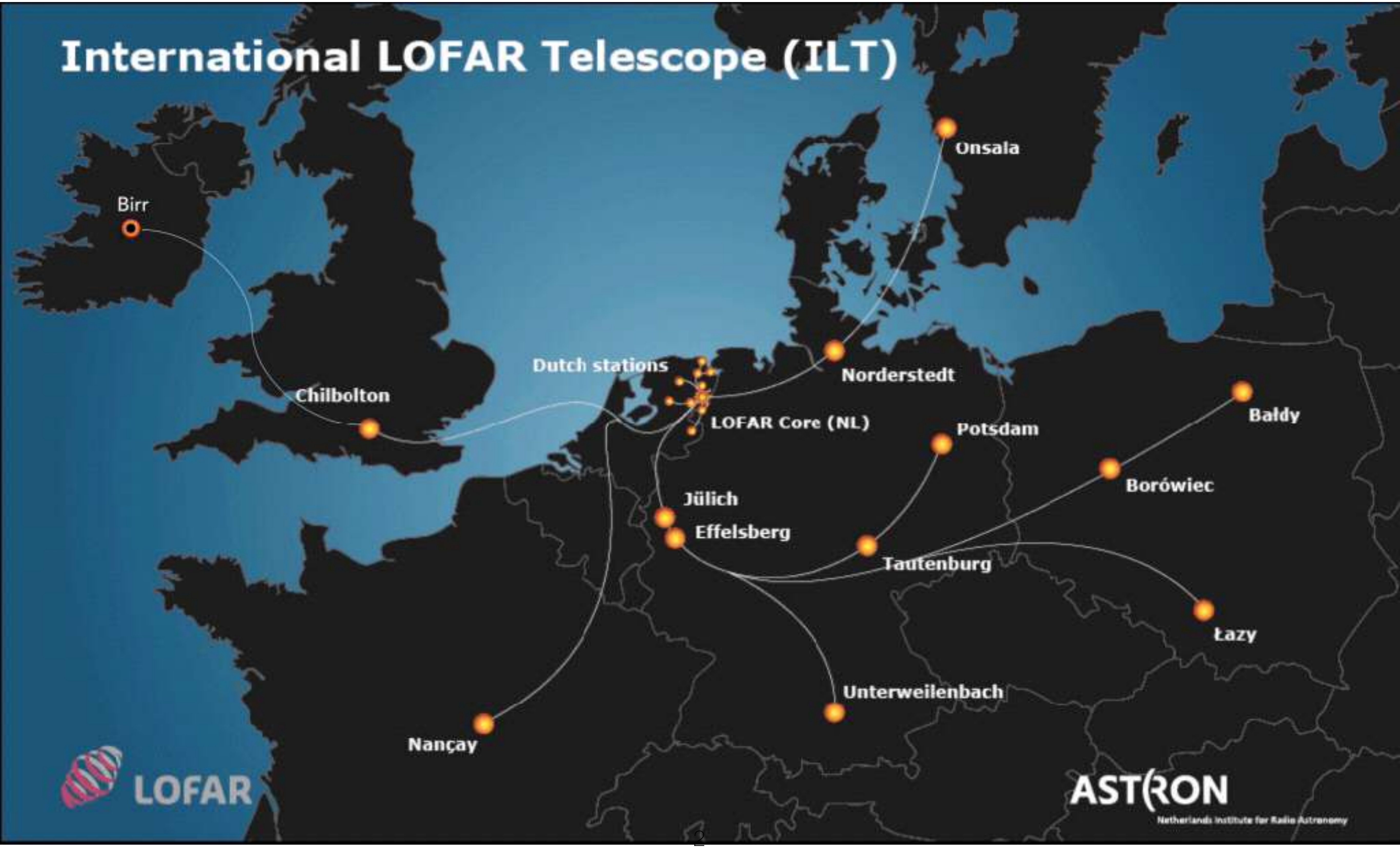
6ª Escola Avançada de Astrofísica do INPE
Cosmologia de 21 cm no século 21



http://hyperphysics.phy-astr.gsu.edu/hbase/quantum/h21.html

Low Frequency Array (LOFAR)

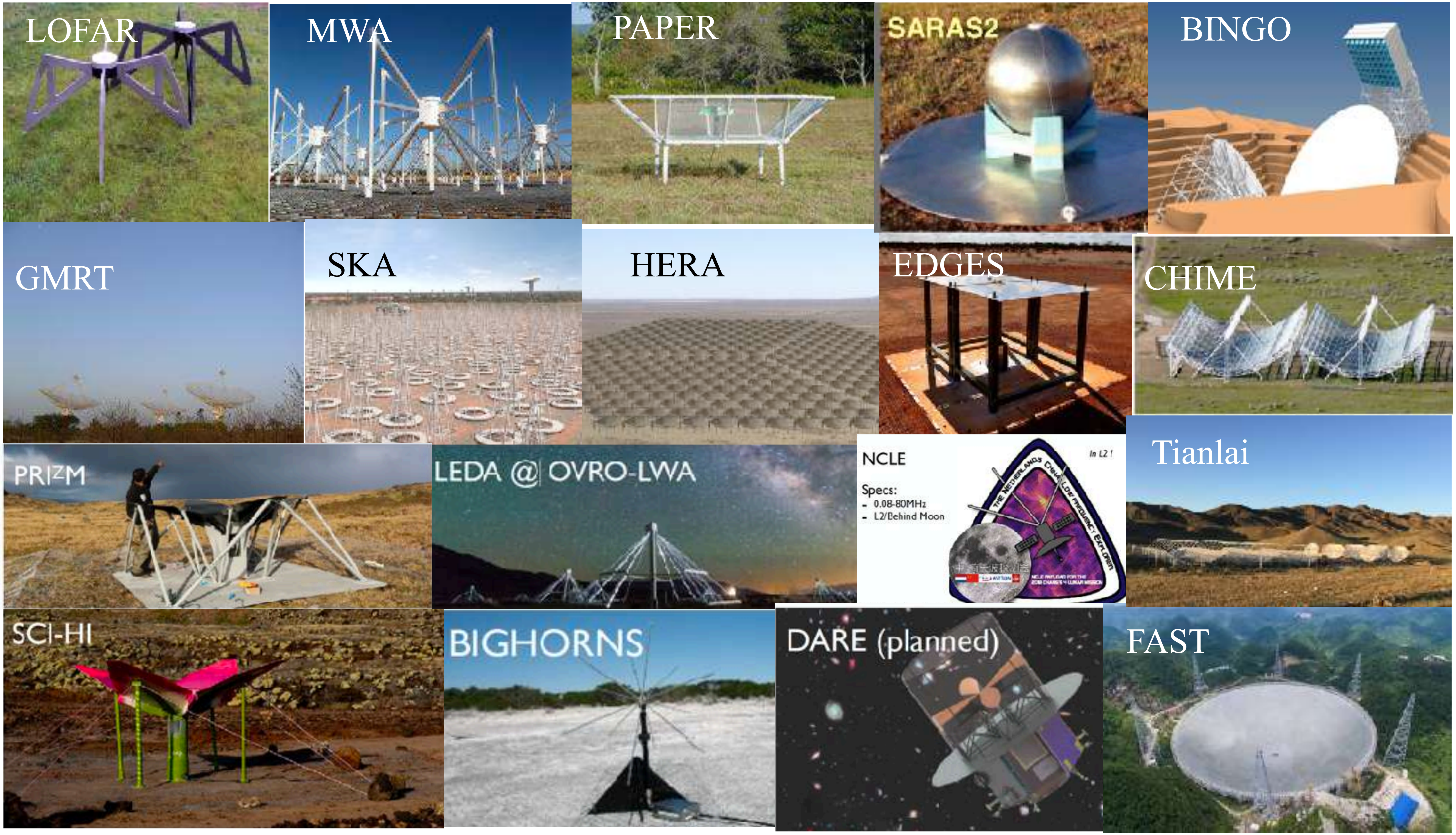


LOFAR

Low band: 30 - 80 MHz
High Band: 115-189 MHz

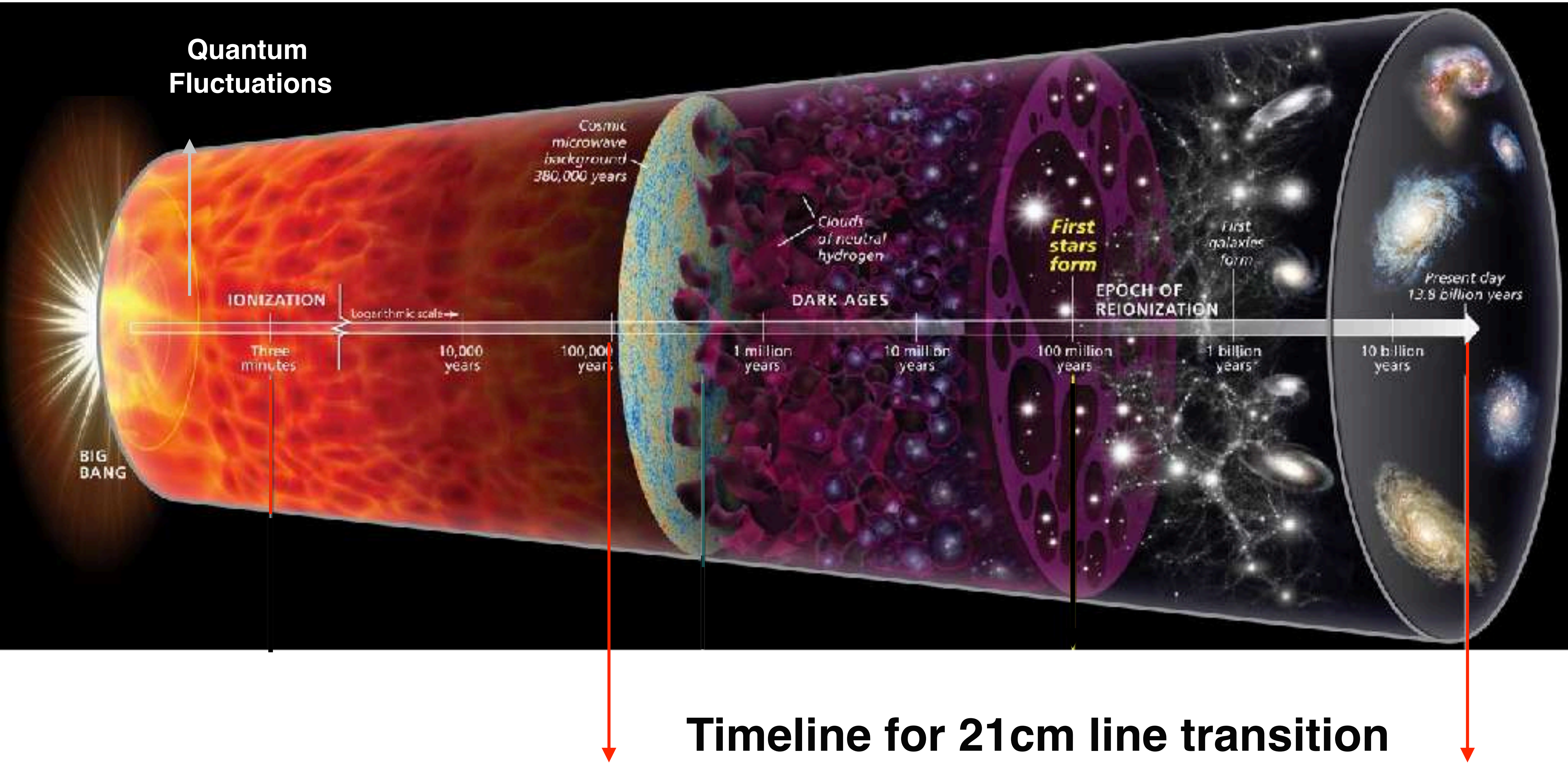
21cm line

A hot topic in today’s astronomy



Why is this line so important?

The first light in the Universe

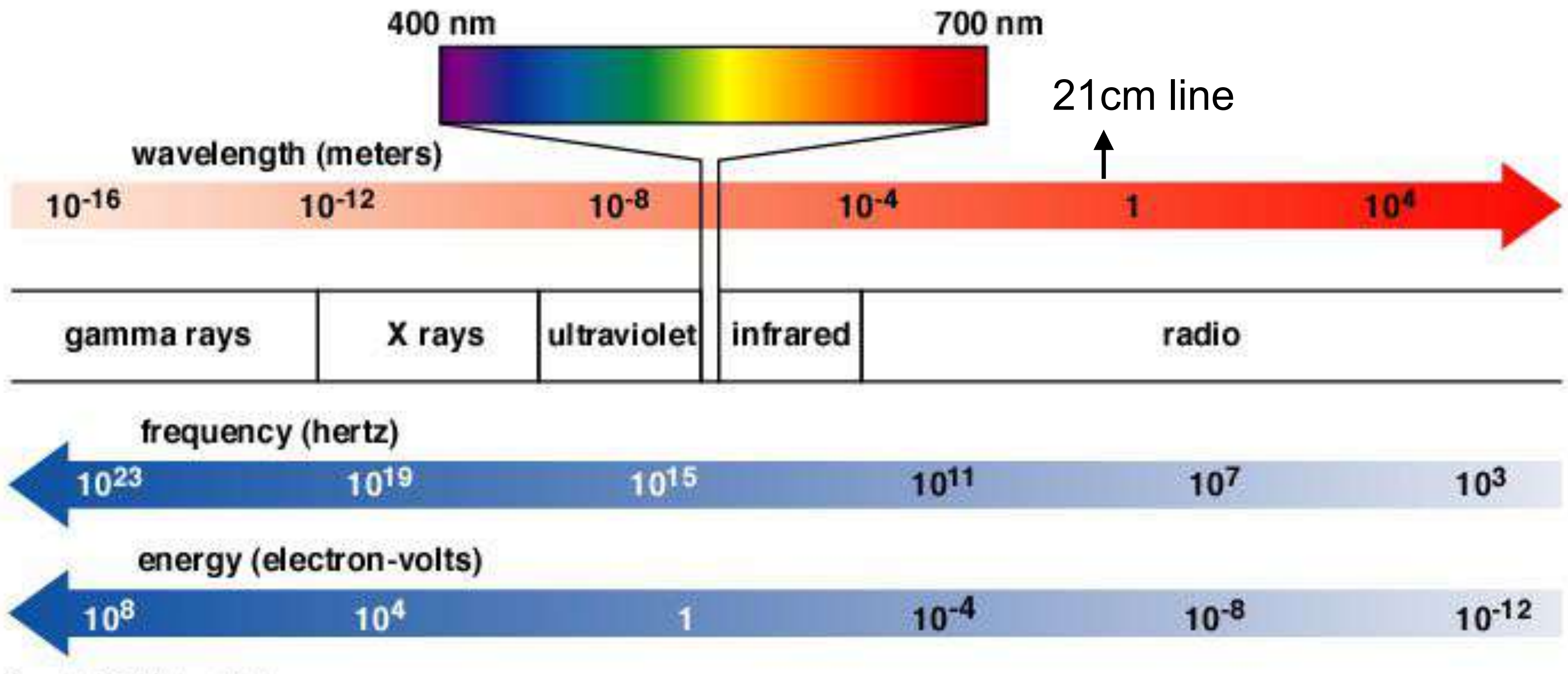


Quantum Fluctuations

Timeline for 21cm line transition

Credit - http://discovermagazine.com/~/media/Images/Issues/2014/April/21-centimeter-cosmology.jpg

The electromagnetic spectrum



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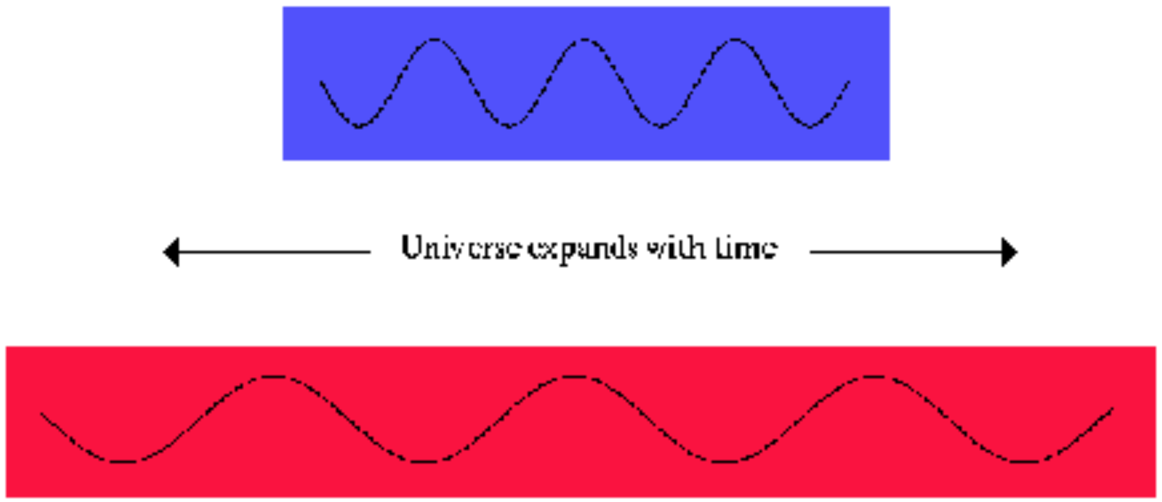


$$\lambda_0 = 21 \text{ cm}$$
$$\lambda_{obs} = \lambda_0 (1+z)$$

The HI 21cm line
The first light in the Universe

Lecture 1

i) The 21 cm line

ii) CMB Formation

iii) The mean 21cm line signal

Lecture 2

iV) Cosmic Dawn/ The First Stars

V) Reionization

Vi) Observing the EoR

Lecture 3

Vii) Impact of the EoR on the CMB

Viii) Other probes of the EoR

ViV) 21cm line in the post EoR

Lecture 1

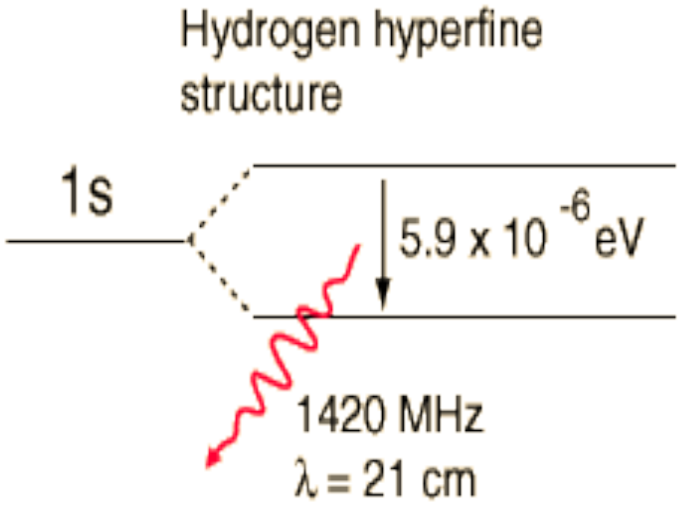
i) The 21 cm line

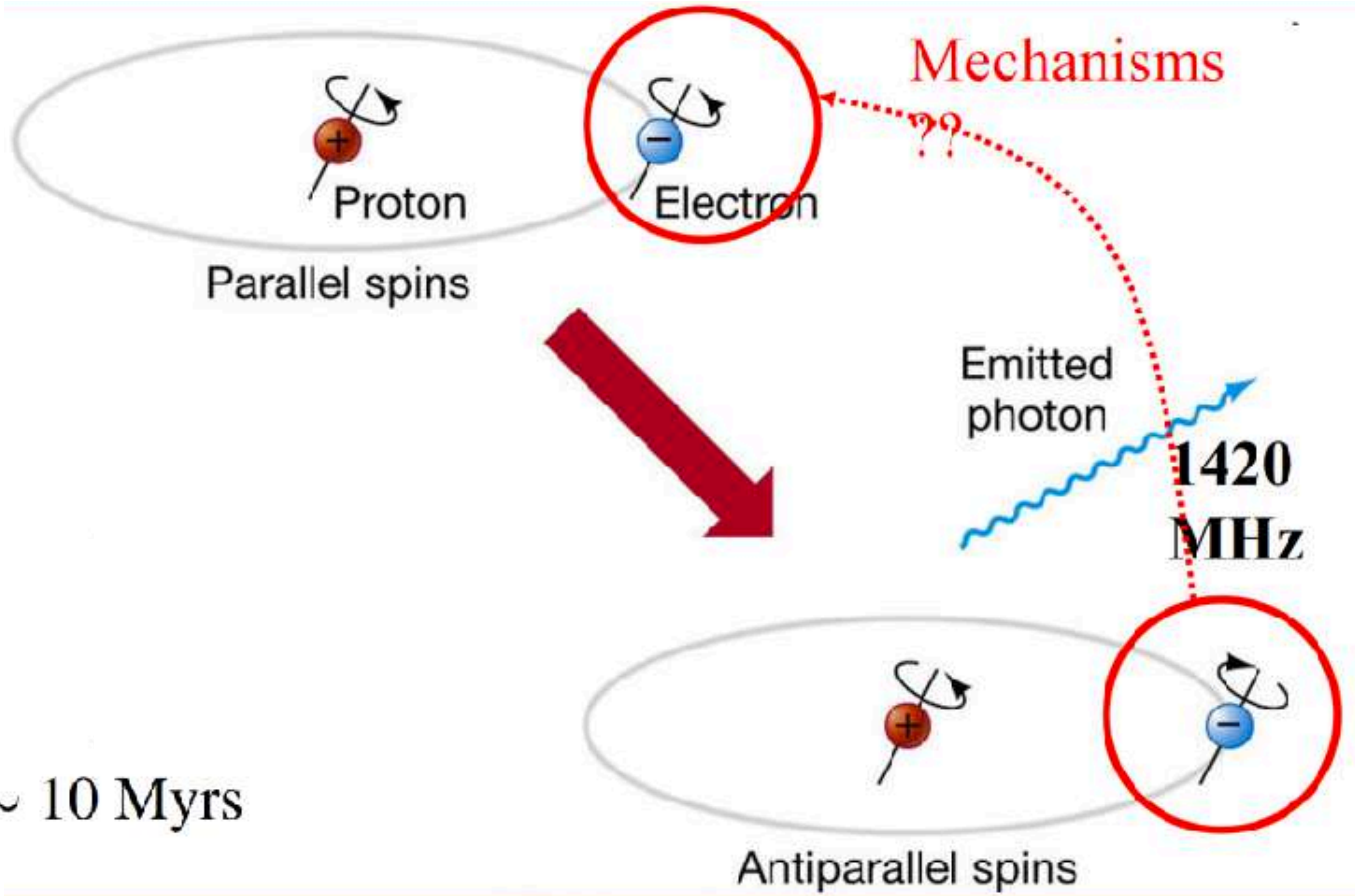
ii) CMB Formation

iii) The mean 21cm line signal

iV) Structure formation

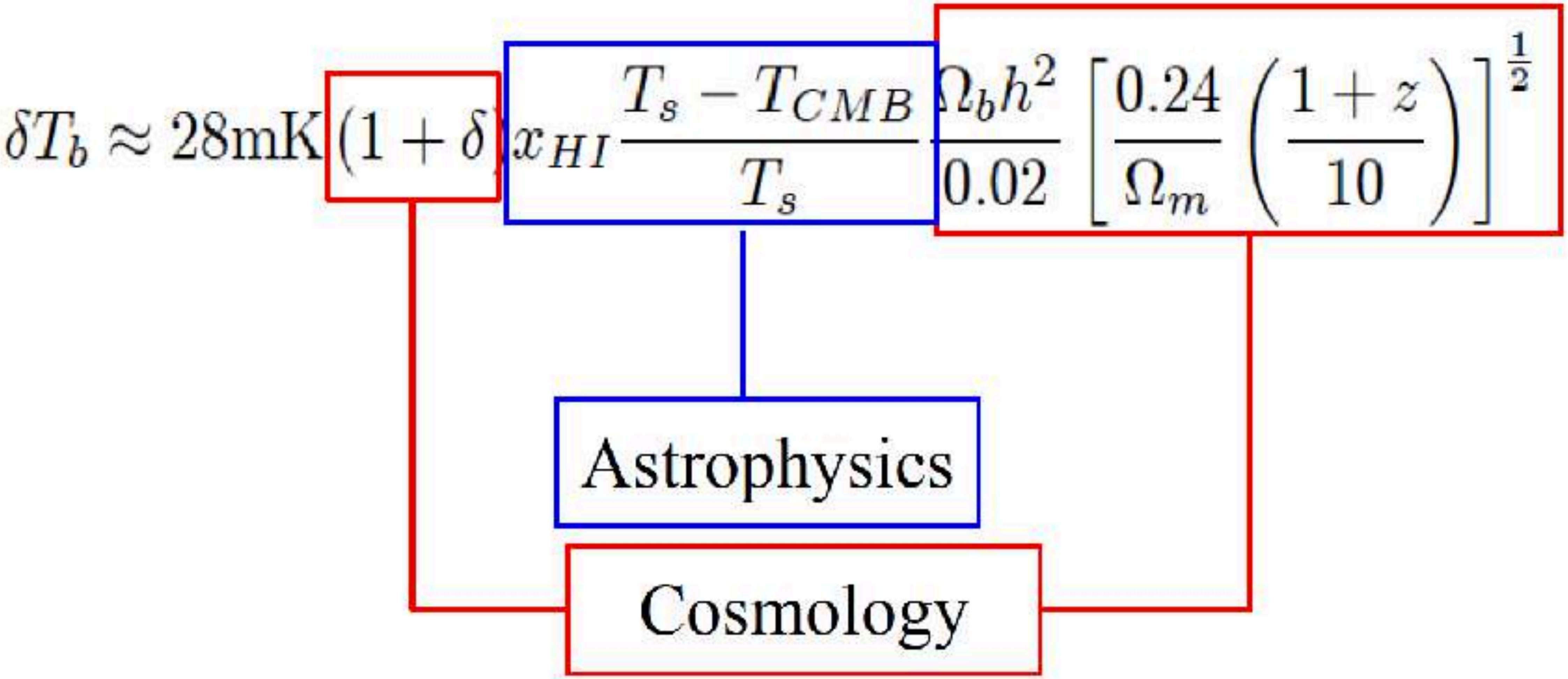
The 21-cm line from neutral hydrogen





Mean Lifetime $\sim$ 10 Myrs

21 cm line Brightness Temperature



$$\delta T_b \approx 28\mathrm{mK}\boxed{(1+\delta)}\boxed{x_{HI}\frac{T_s - T_{CMB}}{T_s}}\boxed{\frac{\Omega_b h^2}{0.02}\left[\frac{0.24}{\Omega_m}\left(\frac{1+z}{10}\right)\right]^{\frac{1}{2}}}$$

Astrophysics

Cosmology

Brightness Temperature

Rayleigh-Jeans approximation

$$h\nu \ll kT$$

$$T_b(\nu) \approx I_\nu \, c^2 / 2k_B \nu^2$$

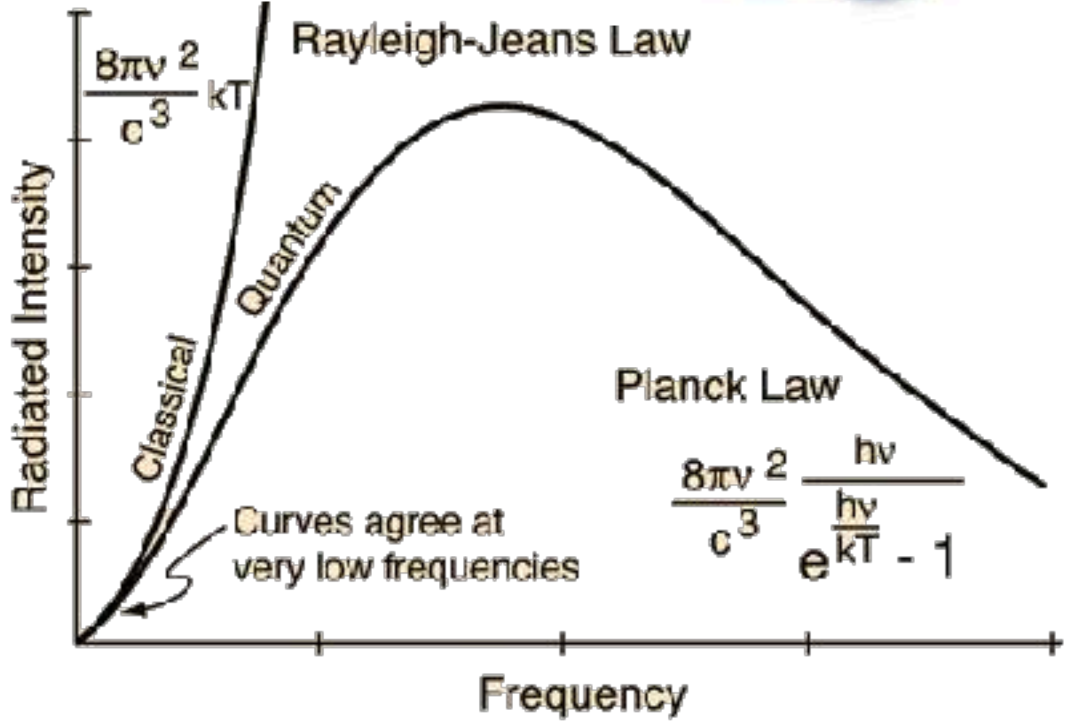
Apparent brightness

$$T_b(\nu) = T_b'(\nu_0)/(1+z)$$

$$\nu_0 = 1420.4057 \text{ MHz}$$

$$\nu = \nu_0/(1+z)$$

$$E = h_p\nu = h_p\nu_0/(1+z)$$



Credit: http://hyperphysics.phy-astr.gsu.edu/hbase/mod6.html

$$\lambda = c/\nu$$

Brightness Temperature

Apparent brightness

$$T_b(\nu) = T_b'(\nu_0)/(1+z)$$

$$\nu_0 = 1420.4057 \text{ MHz}$$

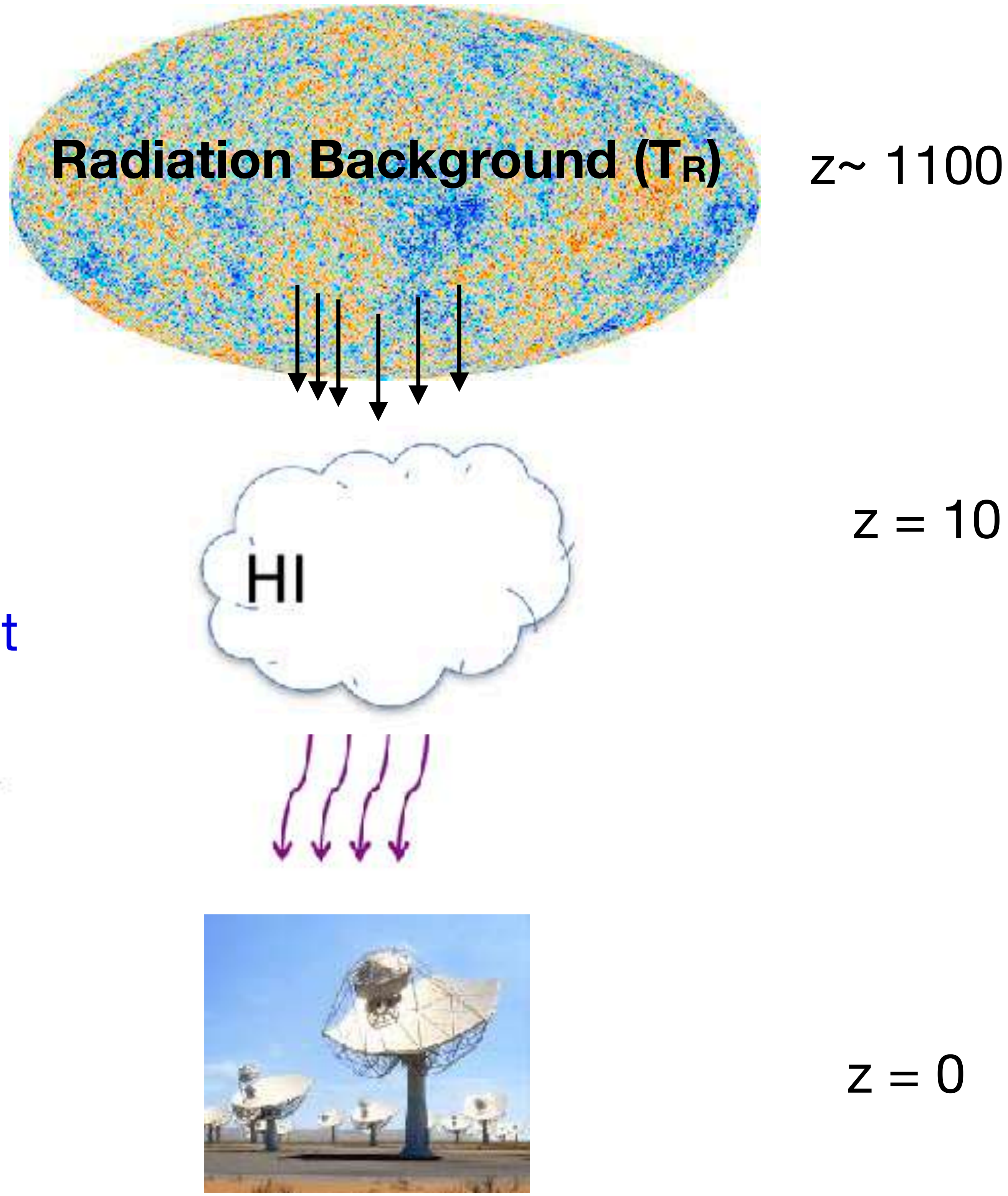
$$\nu = \nu_0/(1+z)$$

Radiative transfer along a line of sight

$$T_b'(\nu) = T_{\rm ex}(1 - e^{-\tau_\nu}) + T_R'(\nu)e^{-\tau_\nu}$$

$\tau_\nu \equiv \int \mathrm{d}s\, \alpha_\nu$ (optical depth)

Integral along the path of
the absorption coefficient



Radiation Background (T_R)

z~ 1100

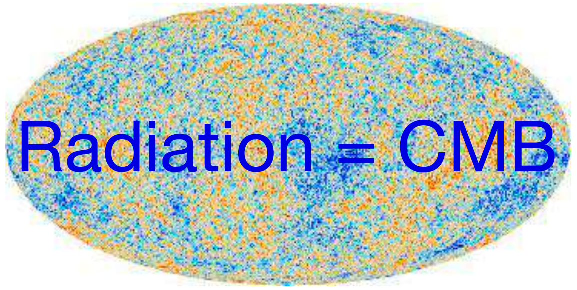
HI

z = 10

z = 0

Brightness Temperature

$$T_b'(\nu) = T_{\rm ex}(1 - e^{-\tau_\nu}) + T_R'(\nu)e^{-\tau_\nu}$$



Radiation = CMB

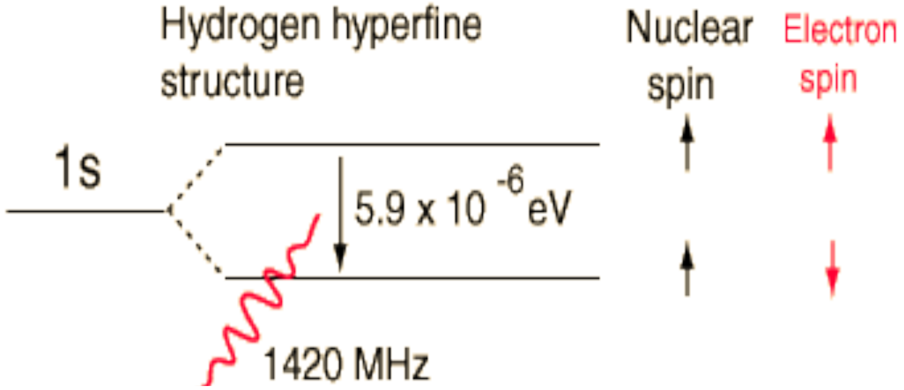
$$T_{\rm R} = T_\gamma$$

$$T_\gamma(z) = T_\gamma(0)(1+z)$$

$$T_\gamma(0) = 2.73\,{\rm K}$$

Spin Temperature

$$T_{\rm exc} = T_S$$



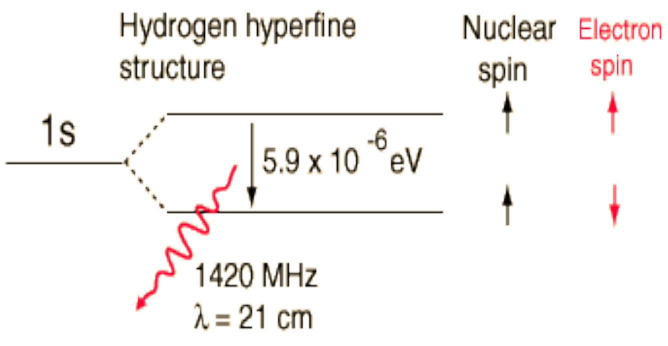
$$\frac{n_1}{n_0} = 3\exp\left(-T_*/T_S\right)$$

$$T_\star \equiv E_{10}/k_B = 0.068\ {\rm K}$$

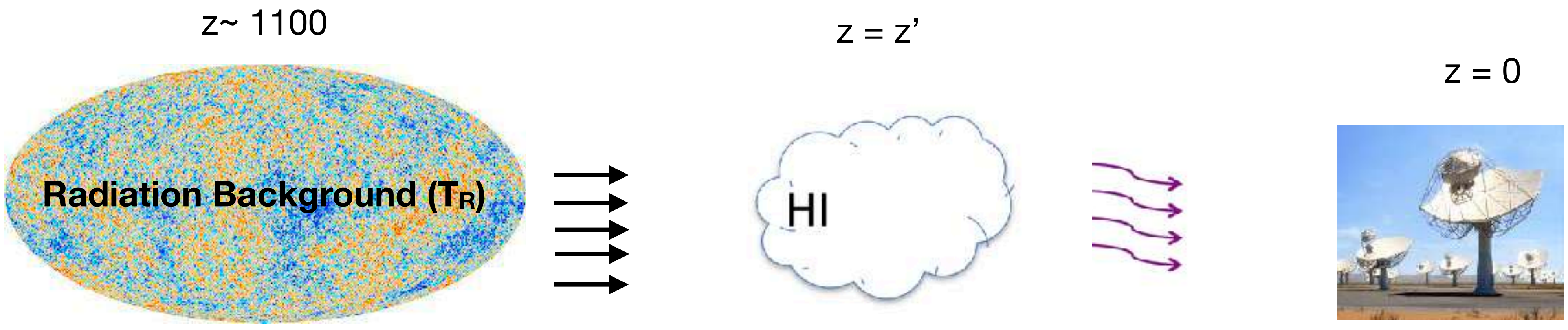
Tb: Optical depth

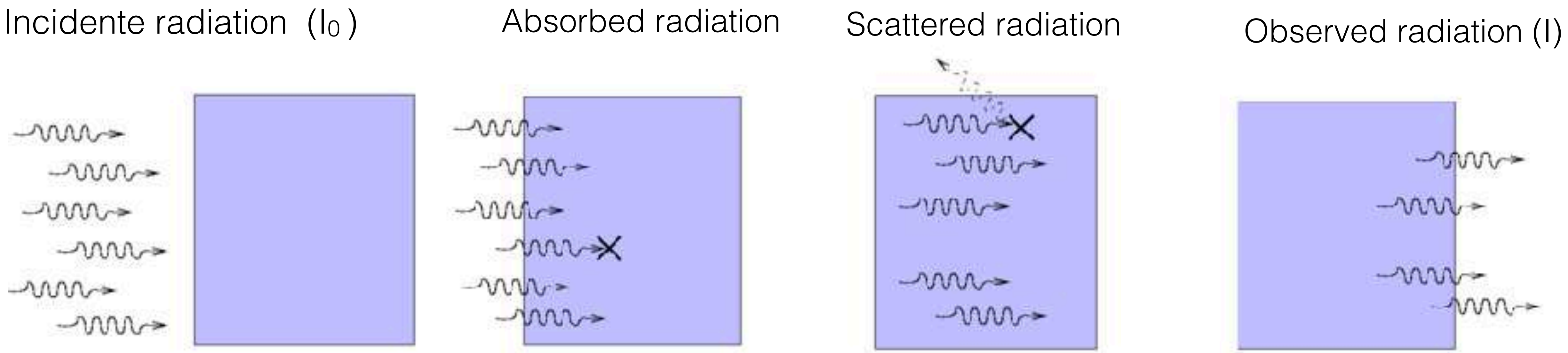
$$T'_b(\nu) = T_{\mathrm{ex}}(1 - e^{-\tau_\nu}) + T'_R(\nu)e^{-\tau_\nu}$$

$$\tau_\nu \equiv \int \mathrm{d}s\, \alpha_\nu$$



$$\tau_\nu = \tau_{21\,cm}$$



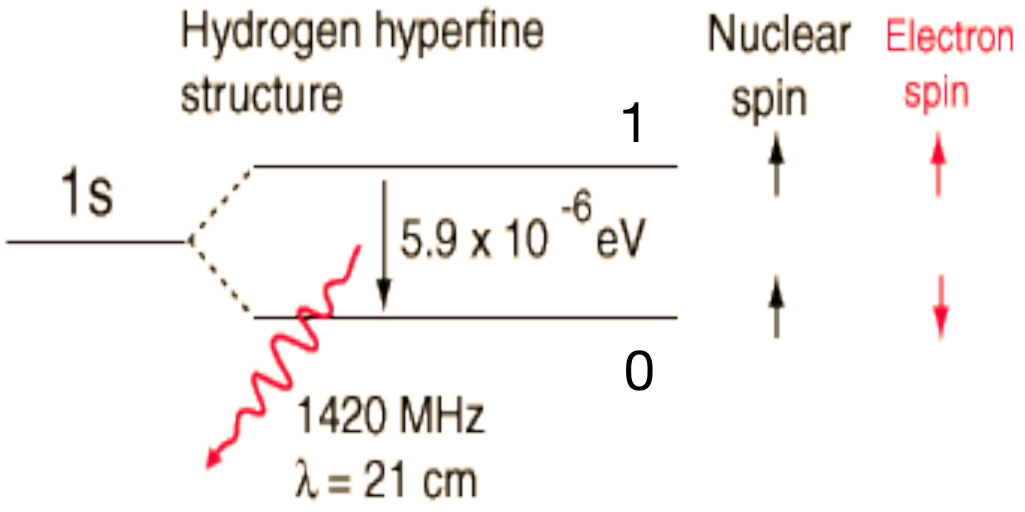


credit: http://spiff.rit.edu/classes/phys440/lectures/optd/optd.html

Tb: Optical depth

$$T_b'(\nu) = T_{\rm ex}(1-e^{-\tau_\nu}) + T_R'(\nu)e^{-\tau_\nu}$$

$$\tau_\nu = \tau_{21\ cm}$$



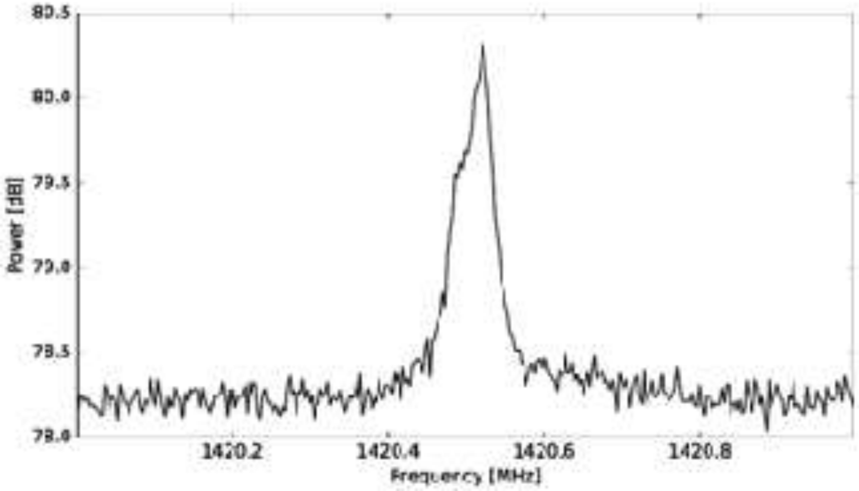
$$\tau_\nu \equiv \int \mathrm{d}s\, \alpha_\nu$$

$$\tau_\nu = \int \mathrm{d}s\, \sigma_{01}\left(1-e^{-E_{10}/k_BT_S}\right)\phi(\nu)\, n_0$$
$$\approx \sigma_{01}\left(\frac{h\nu}{k_BT_S}\right)\left(\frac{N_{\rm HI}}{4}\right)\phi(\nu),$$

$$\tau_{\nu_0} = \frac{3}{32\pi}\,\frac{hc^3A_{10}}{k_BT_S\nu_0^2}\,\frac{x_{\rm HI}n_H}{(1+z)\,(\mathrm{d}v_\|/\mathrm{d}r_\|)}$$
$$\approx 0.0092\,(1+\delta)\,(1+z)^{3/2}\,\frac{x_{\rm HI}}{T_S}\left[\frac{H(z)/(1+z)}{\mathrm{d}v_\|/\mathrm{d}r_\|}\right]$$

$$\sigma_{01} \equiv \frac{3c^2A_{10}}{8\pi\nu^2}$$

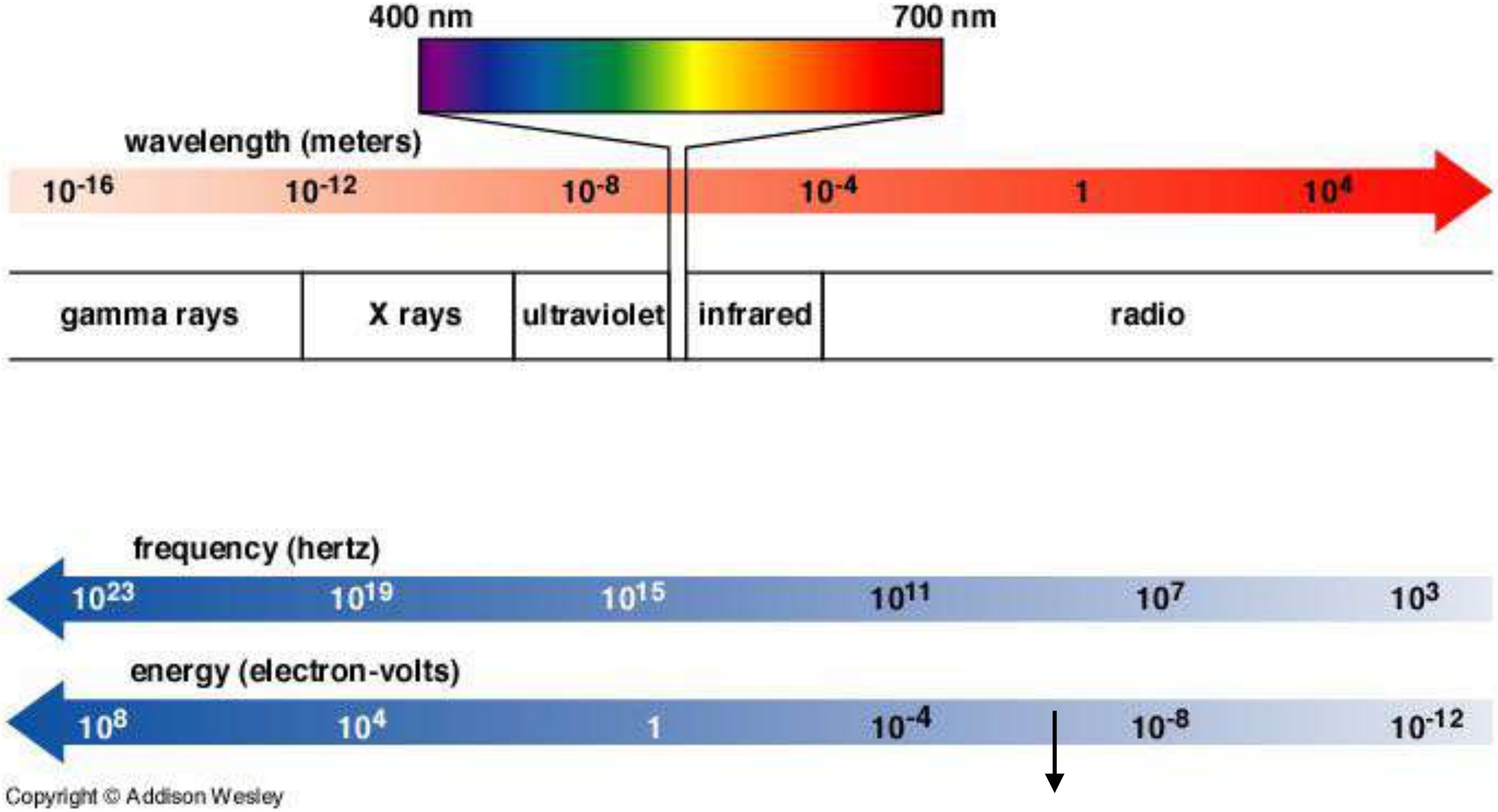
(Cross section)



credit: DC Price 2016

$$\phi(\nu) \sim c/[sH(z)\nu]$$

The energy of the 21cm radiation



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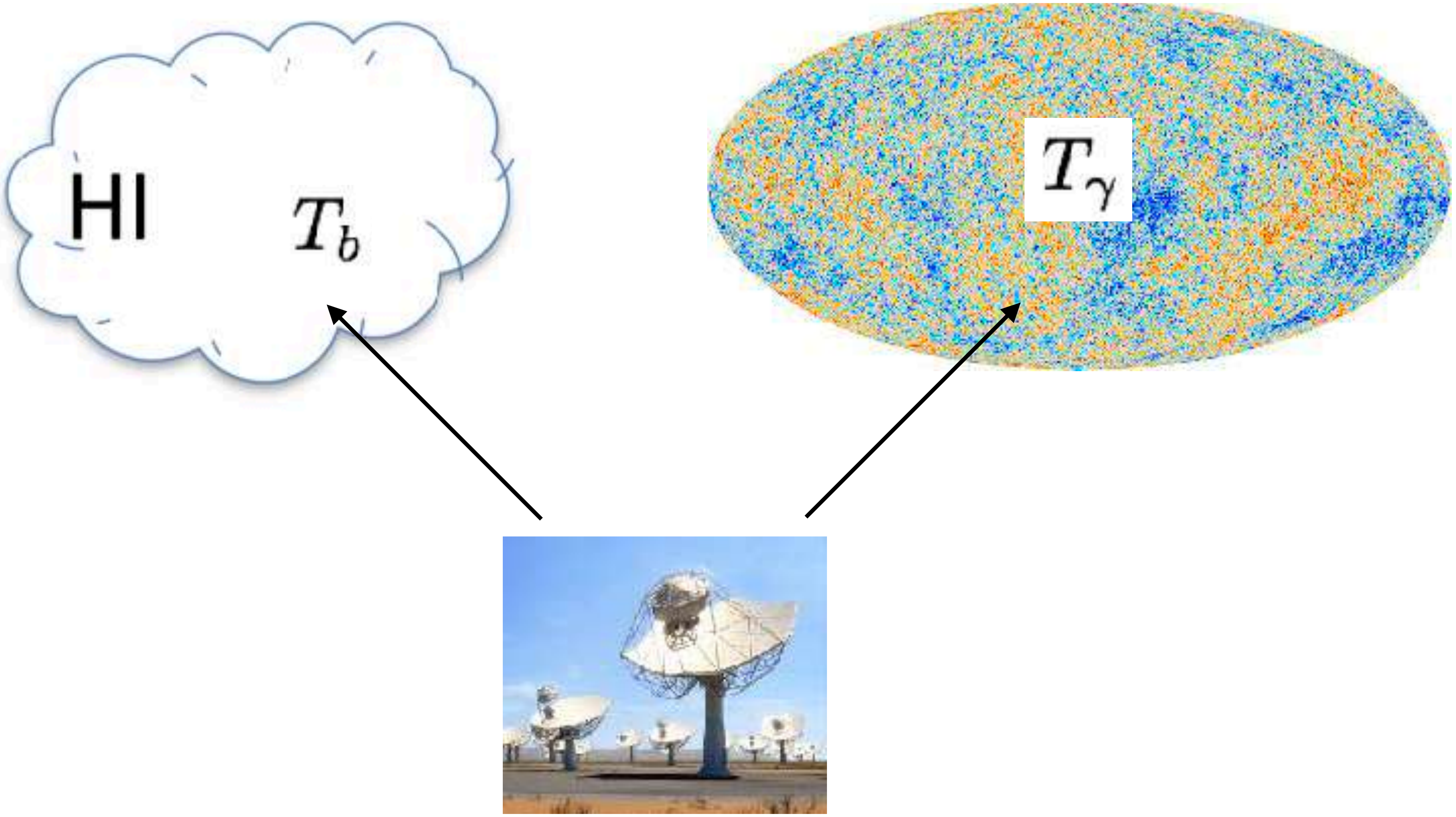
$E_{21cm} = 5.9 \times 10^{-6}$ eV

δTb: The Brightness Temperature

$$T_b(\nu) \;=\; T_S\left(1-e^{-\tau_\nu}\right)+T_\gamma(\nu)e^{-\tau_\nu}$$

$$\delta T_b(\nu) \;\equiv\; T_b-T_\gamma=(T_S-T_\gamma)\left(1-e^{-\tau_\nu}\right)$$



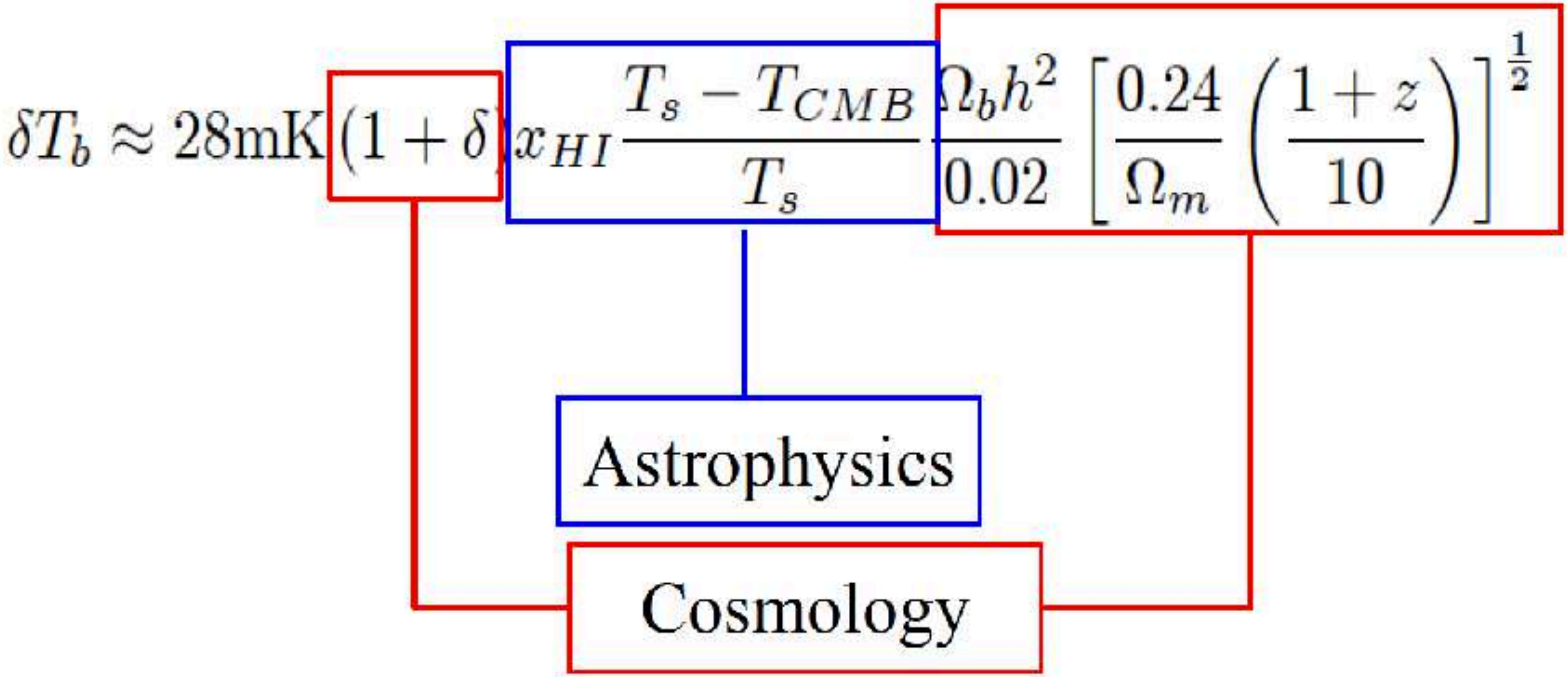
21 cm line Brightness Temperature

$$\delta T_b(\nu) = \frac{T_S - T_\gamma(z)}{1+z}\left(1 - e^{-\tau_{\nu_0}}\right) \approx \frac{T_S - T_\gamma(z)}{1+z}\,\tau_{\nu_0}$$

$$\approx 9\, x_{\mathrm{HI}}(1+\delta)\,(1+z)^{1/2}\left[1 - \frac{T_\gamma(z)}{T_S}\right]\left[\frac{H(z)/(1+z)}{\mathrm{d}v_\parallel/\mathrm{d}r_\parallel}\right]\ \mathrm{mK}$$

If we assume this is 0
(Except for the Hubble flow the gas is not moving in the direction of the line of sight)

21 cm line Brightness Temperature



$$\delta T_b \approx 28\text{mK}\boxed{(1+\delta)}\boxed{x_{HI}\frac{T_s - T_{CMB}}{T_s}}\boxed{\frac{\Omega_b h^2}{0.02}\left[\frac{0.24}{\Omega_m}\left(\frac{1+z}{10}\right)\right]^{\frac{1}{2}}}$$

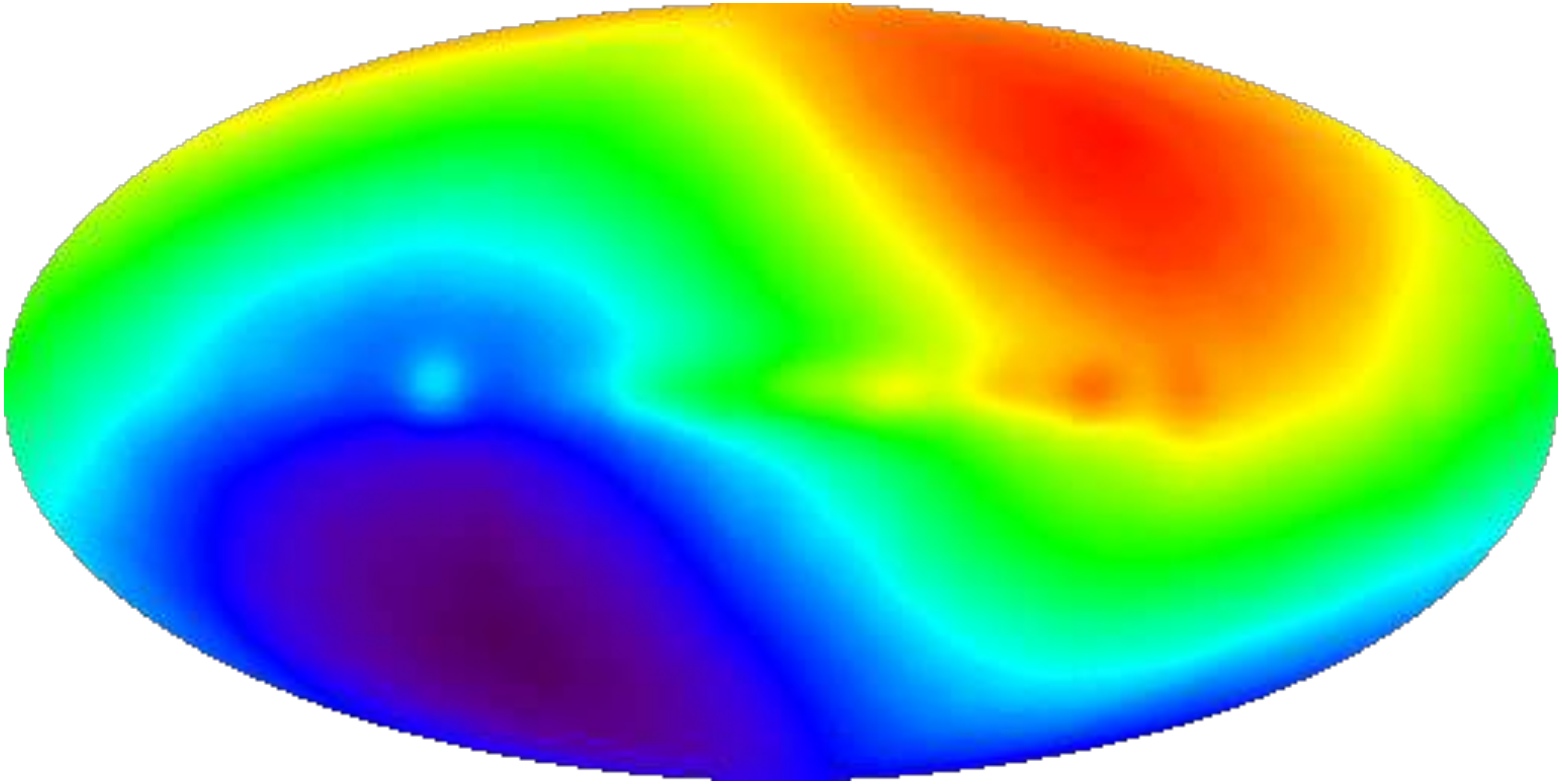
Astrophysics

Cosmology

How do we constrain cosmology in the early Universe?

The Cosmic Microwave Background (CMB)

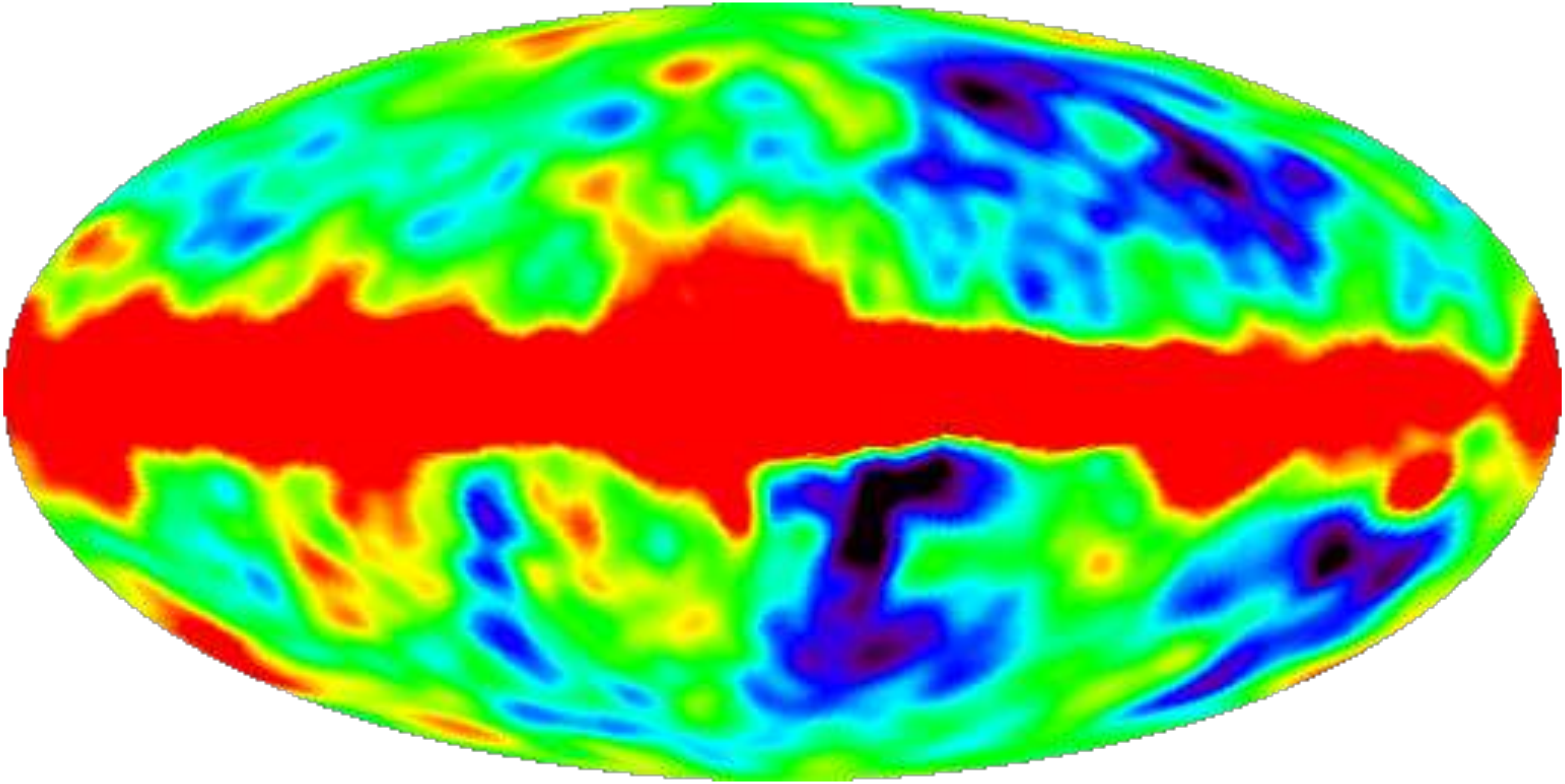
Raw image:
Typical dipole appearance because our Galaxy is moving in a particular direction



credit: http://abyss.uoregon.edu/~js/ast123/lectures/lec23.html

The Cosmic Microwave Background (CMB)

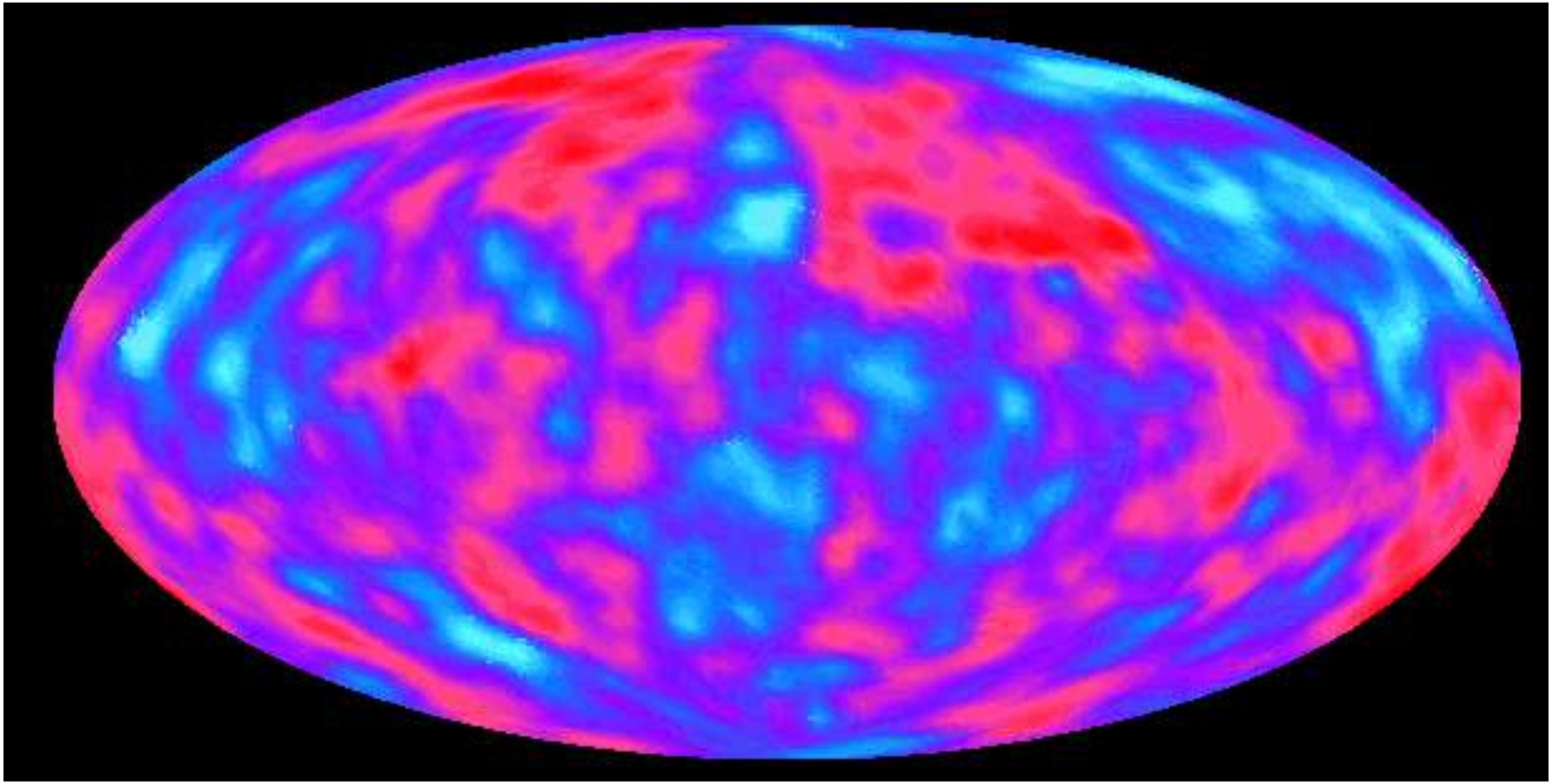
After removing the Galaxy’s motion:
Map dominated by the far-infrared emission from gas in our own Galaxy



credit: http://abyss.uoregon.edu/~js/ast123/lectures/lec23.html

The Cosmic Microwave Background (CMB)

After removing our galaxy emission:



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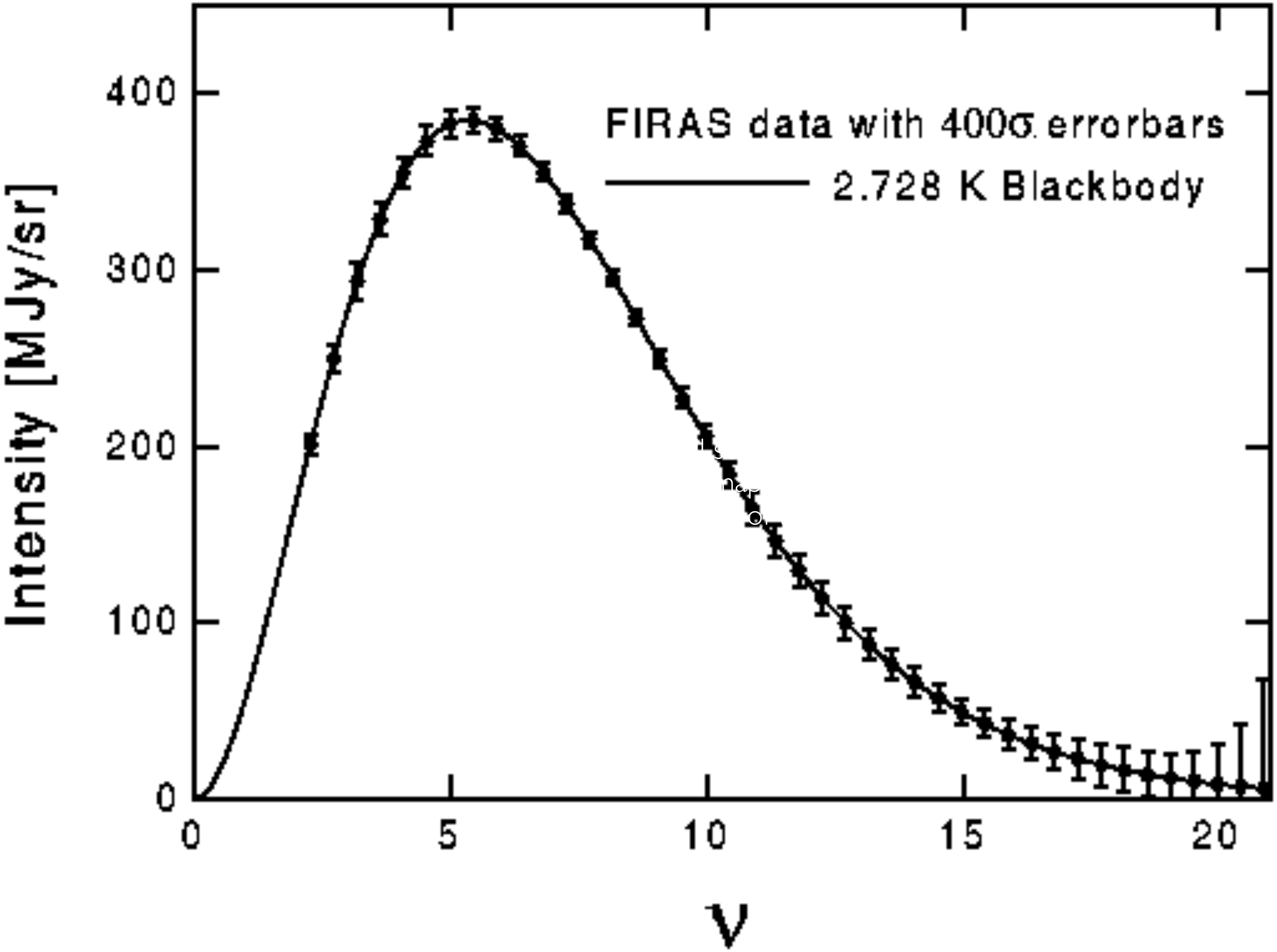
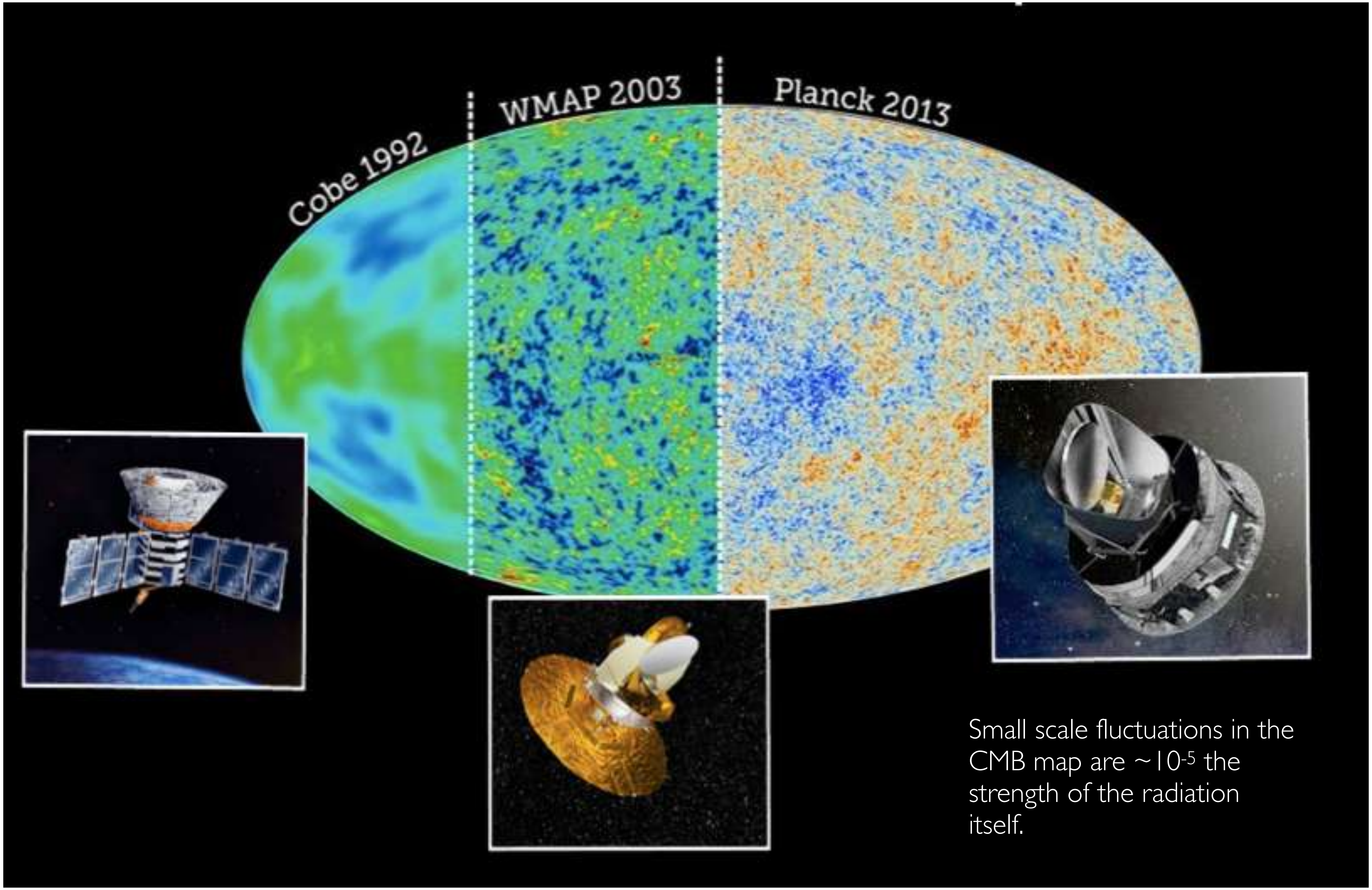
The Cosmic Microwave Background (CMB)



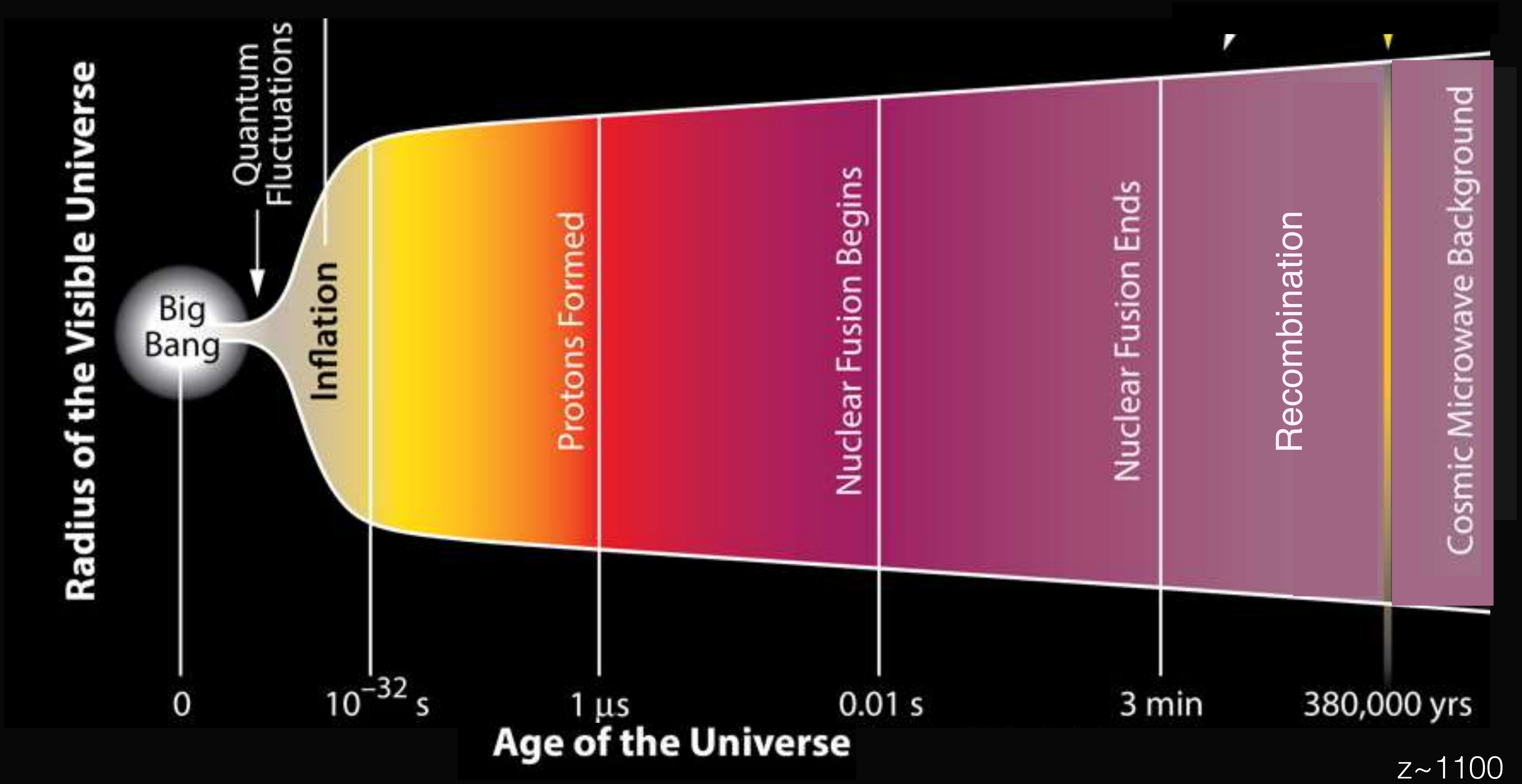
The Cosmic Microwave Background (CMB)



Small scale fluctuations in the CMB map are ~10^{-5} the strength of the radiation itself.

Where does this radiation come from?

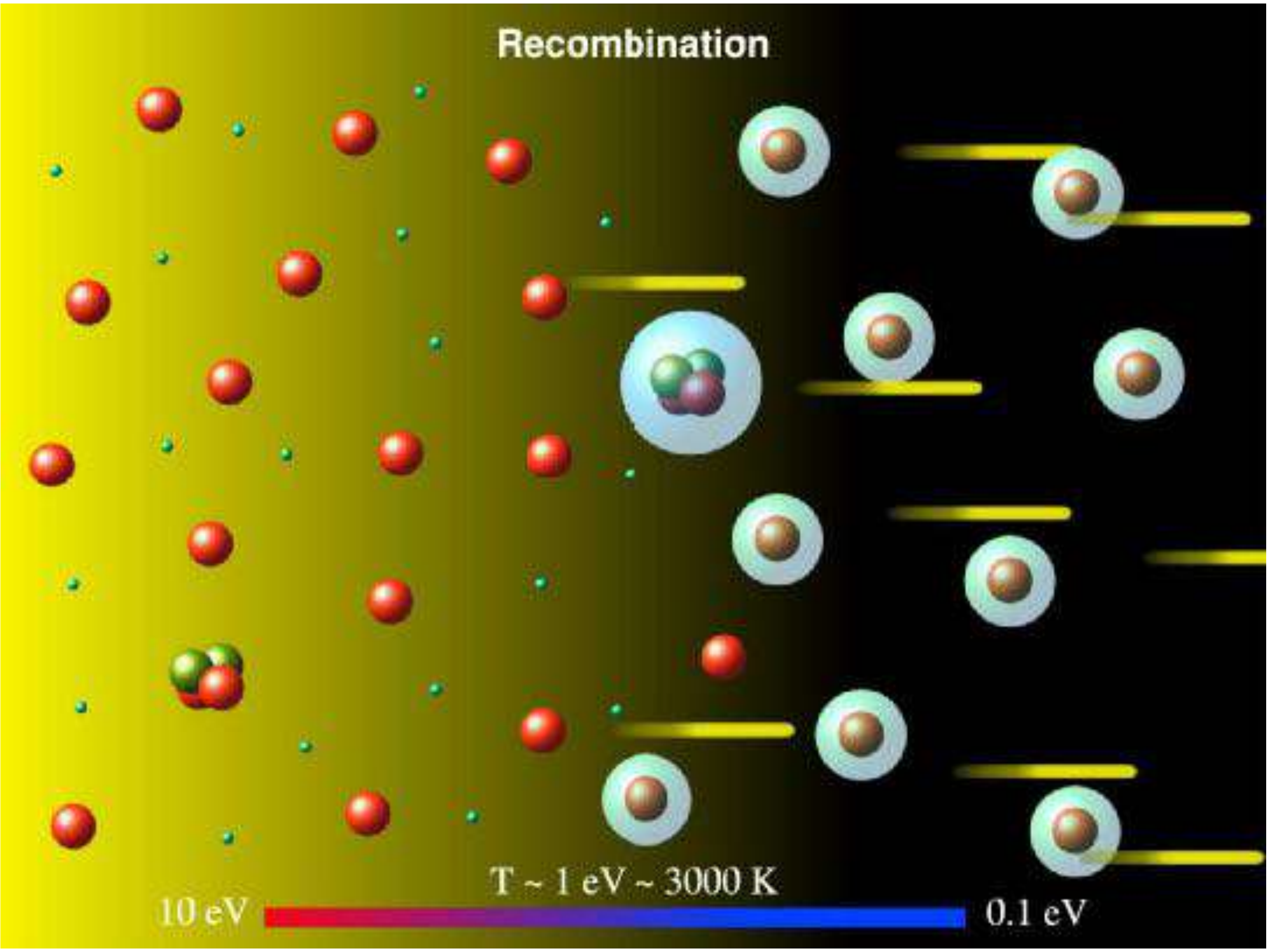
From the Big Bang to the Epoch of Recombination



z~1100

Decoupling

Recombination and Photon decoupling



Recombination to an excited state followed by photon emission

↓

When the photon mean free path became bigger than the rate of expansion of the Universe

$\lambda_{mfp} > H^{-1}$

↓

Photon Decoupling
(CMB formation)

The Cosmic Microwave Background (CMB)

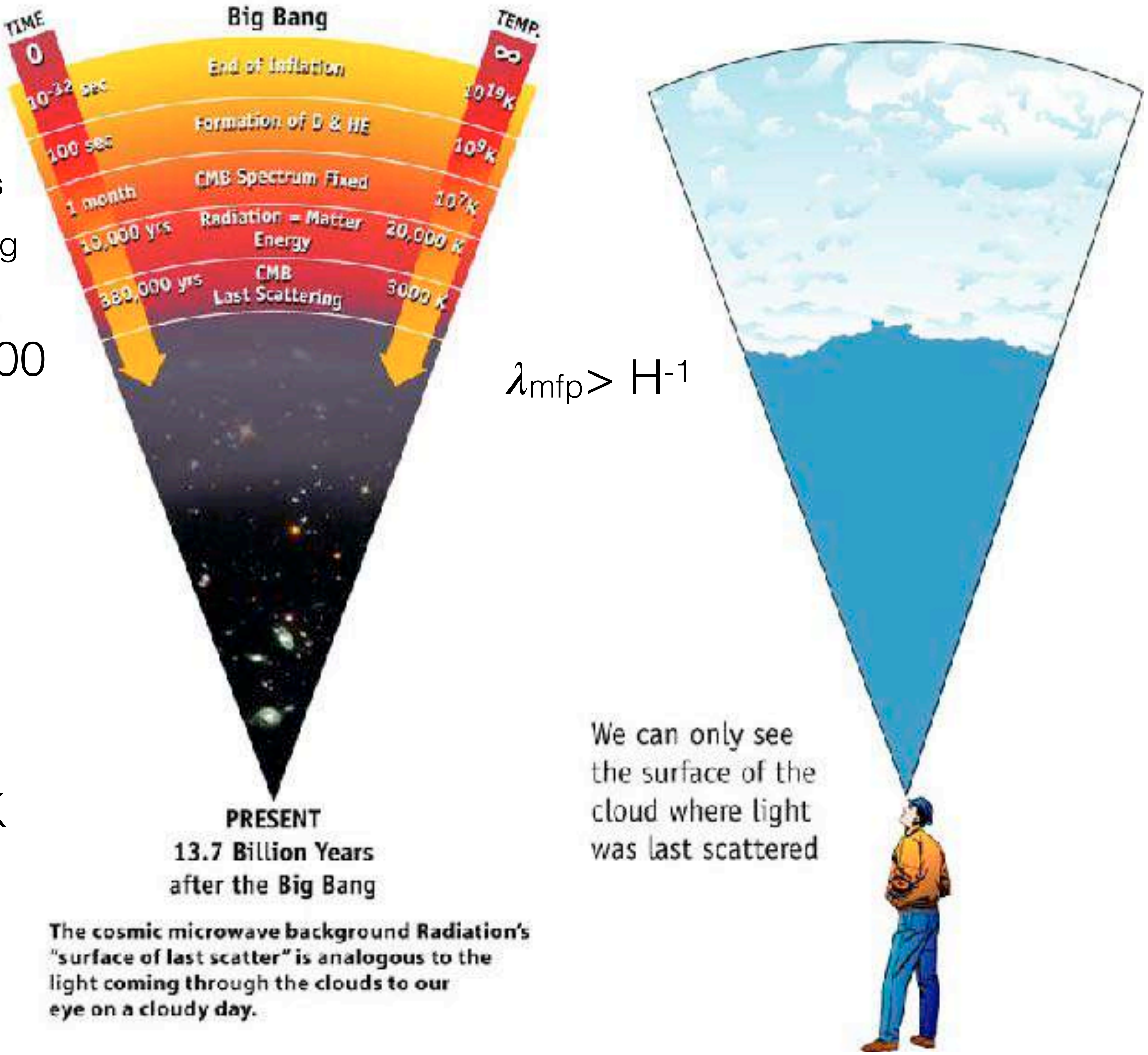
Planck Epoch

Nucleosynthesis

Decoupling

$z \sim 1100$

$\lambda_{\mathrm{mfp}} > H^{-1}$



The cosmic microwave background Radiation's "surface of last scatter" is analogous to the light coming through the clouds to our eye on a cloudy day.

$$T_{\mathrm{CMB}} \propto (1+z)$$

$$T_{\mathrm{CMB},z} = 3000(1+\Delta z)^{-1}\ \mathrm{K}$$

$$T_{\mathrm{CMB},0} = 3000(1+1100)^{-1}\ \mathrm{K}$$
$$= 2.725\ \mathrm{K}$$

credit: http://abyss.uoregon.edu/~js/ast123/lectures/lec23.html

The 21 cm line Brightness Temperature

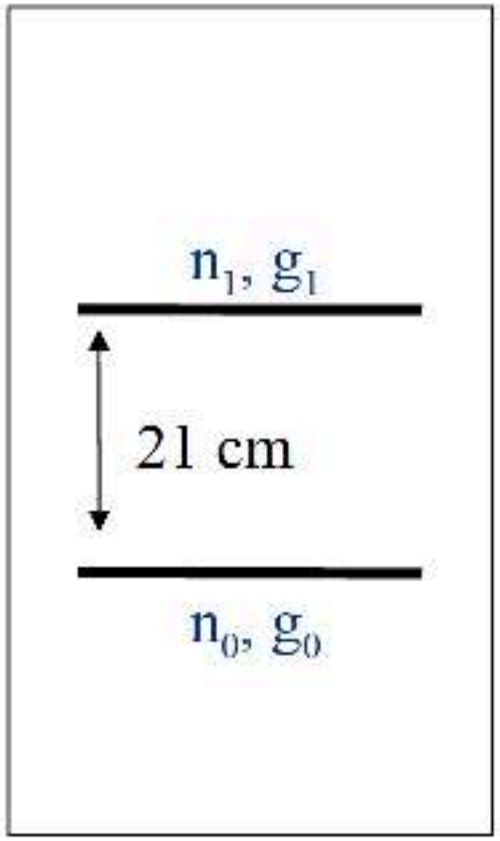
$$\delta T_b \approx 28\text{mK}\,(1+\delta)\,x_{HI}\,\boxed{\frac{T_s - T_{CMB}}{T_s}}\,\frac{\Omega_b h^2}{0.02}\left[\frac{0.24}{\Omega_m}\left(\frac{1+z}{10}\right)\right]^{\frac{1}{2}}$$

$$\delta T_{\mathrm{b}} = T_b - T_{\mathrm{CMB}}$$

$T_{\mathrm{s}} = T_{CMB} \Rightarrow \delta T_b = 0$	No signal
$T_{\mathrm{s}} > T_{CMB} \Rightarrow \delta T_b > 0$	Signal in emission
$T_{\mathrm{s}} < T_{CMB} \Rightarrow \delta T_b < 0$	Signal in absorption

The Spin Temperature

$$\delta T_b \approx 28\mathrm{mK}\,(1+\delta)x_{HI}\boxed{\frac{T_s - T_{CMB}}{T_s}}\frac{\Omega_b h^2}{0.02}\left[\frac{0.24}{\Omega_m}\left(\frac{1+z}{10}\right)\right]^{\frac{1}{2}}$$



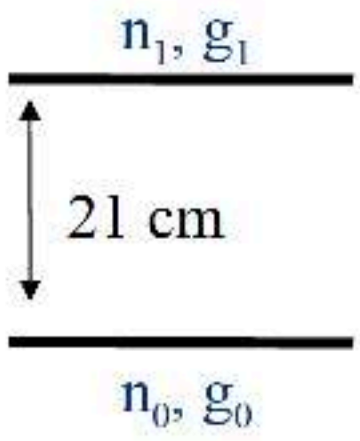
- The value of the T_s is given by:

$$T_s^{-1} = \frac{T_{CMB}^{-1} + x_c T_k^{-1} + x_\alpha T_k^{-1}}{1 + x_c + x_\alpha}$$

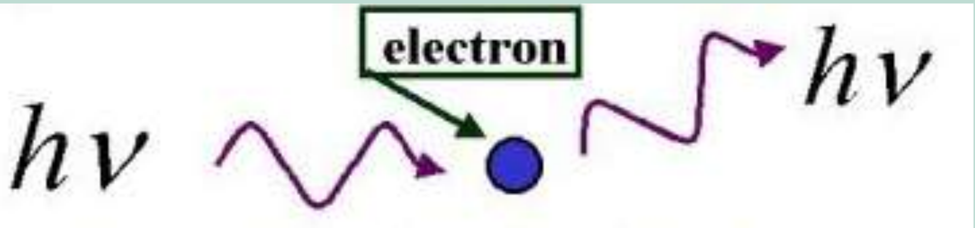
Field 1958
Madau et al 98
Ciardi&Madau 2003

The Spin Temperature

$$T_s^{-1} = \frac{T_{CMB}^{-1} + x_c T_k^{-1} + x_\alpha T_k^{-1}}{1 + x_c + x_\alpha}$$

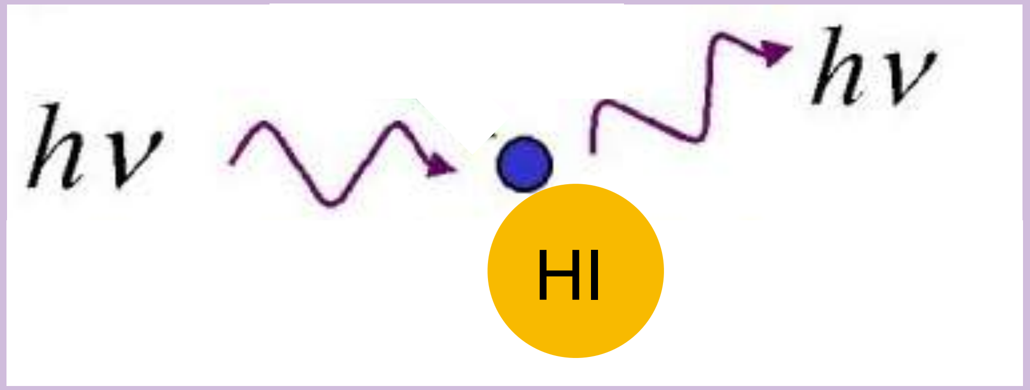


Coupling to the CMB

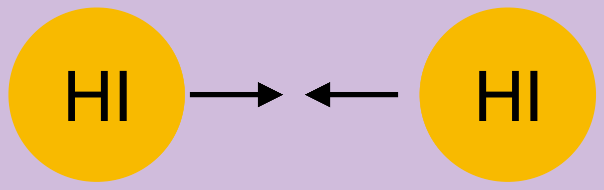


Thomson scattering of CMB photons (low energy photons) by free electrons

Coupling to the gas



Scattering of Lyα photons



Collision with other HI atoms, electrons and protons

Temperature Evolution

In a Universe expanding adiabatically

$$T_{\mathrm{CMB}} \propto (1+z)$$

$$T_{\mathrm{CMB,z}} = T_{\mathrm{CMB,z=1100}}(1+\Delta z)^{-1}\,\mathrm{K}$$

$$T_{\mathrm{CMB,z}} = 3000(1+1100-z)^{-1}\,\mathrm{K}$$

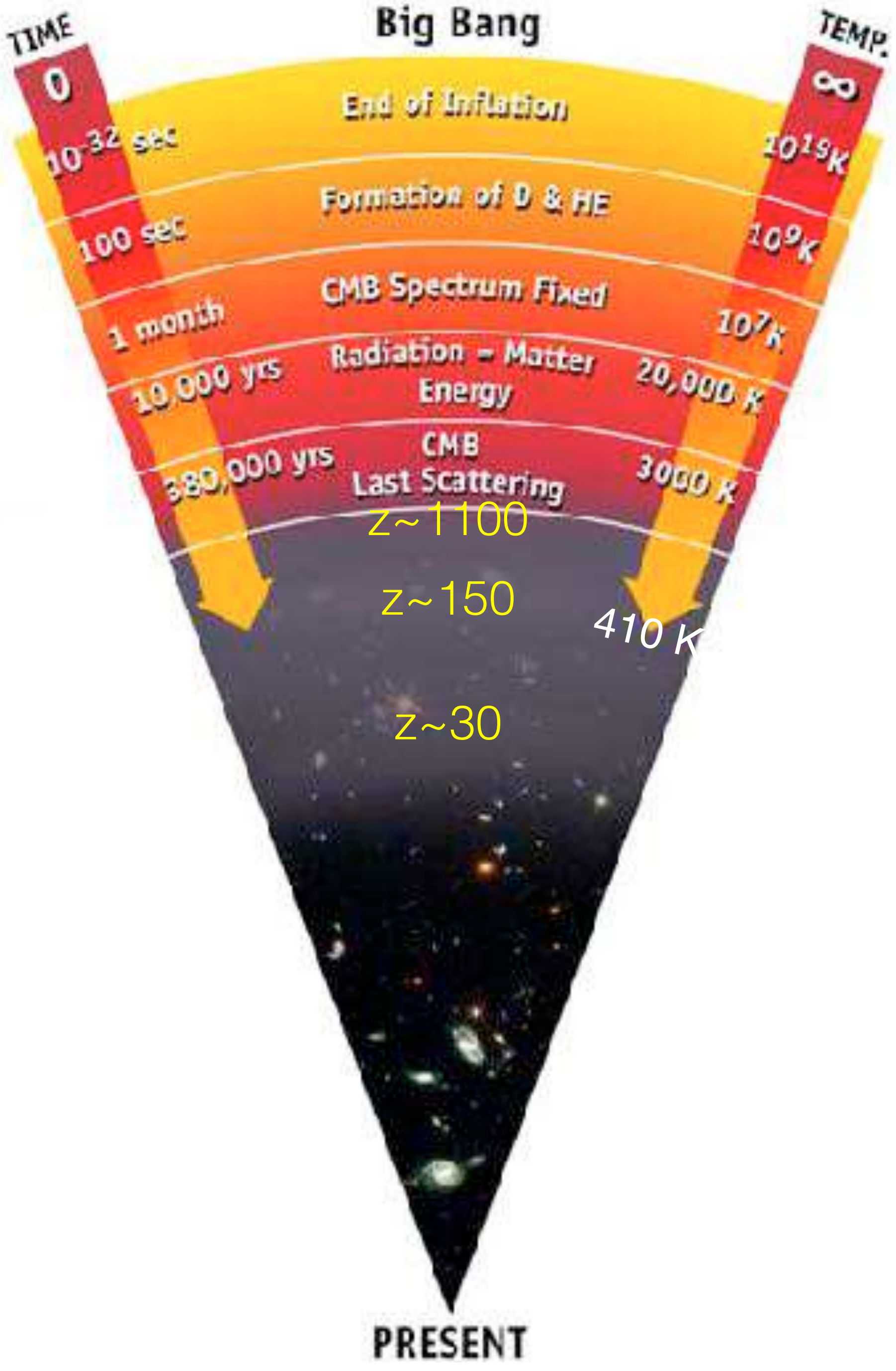
$$T_{\mathrm{CMB}}(0) = 2.73\,\mathrm{K}$$

T_{K} coupled to T_{CMB} through Thomson scattering of residual free electrons

$$T_{\mathrm{K}} \approx T_{\mathrm{CMB}} \text{ down to } z \sim 200-150$$

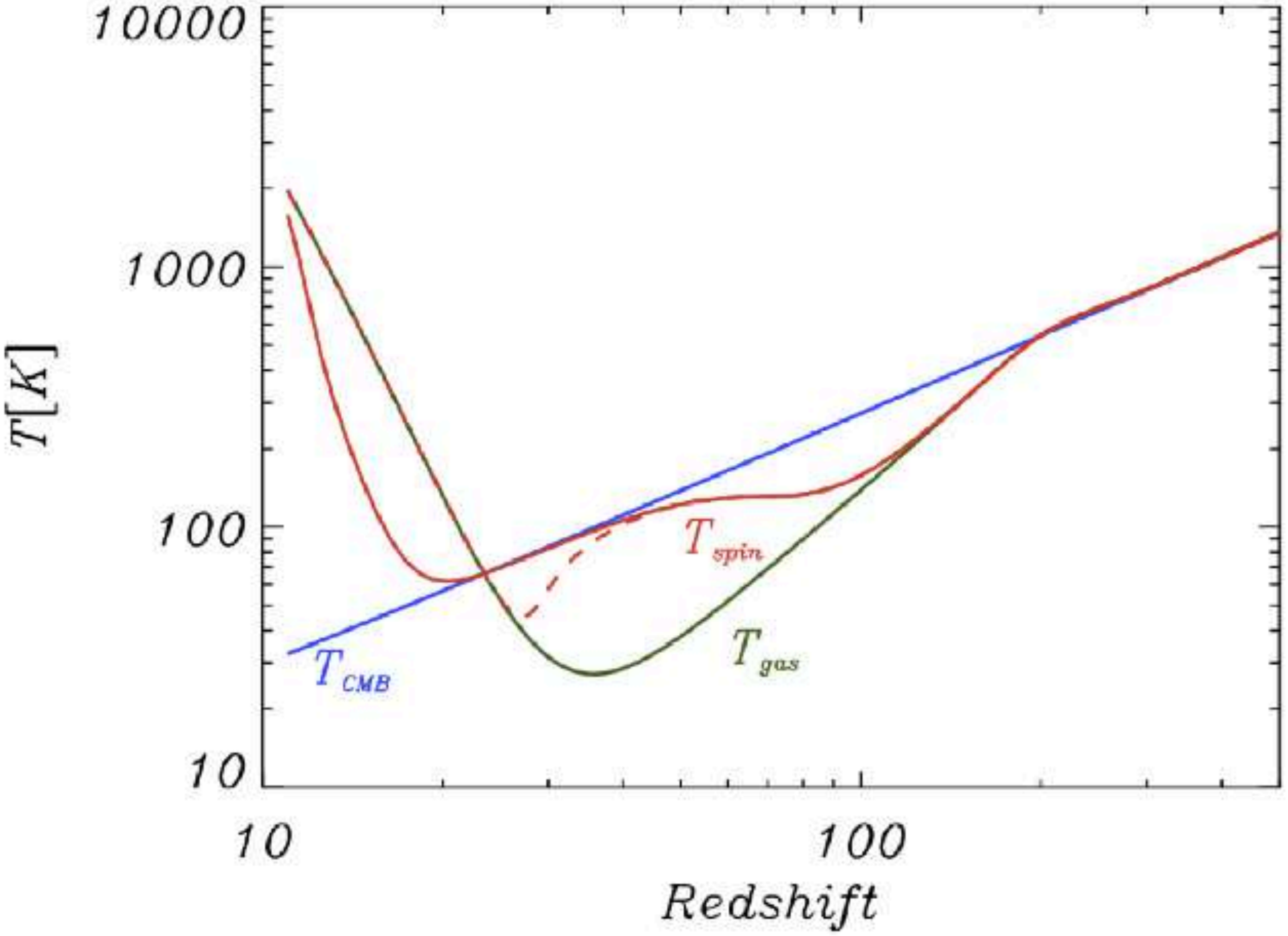
$$T_{\mathrm{K}} \propto (1+z)^2$$

$$T_{\mathrm{K}} = T_{\mathrm{K,z\approx150}}(1+\Delta z)^{-2}\,\mathrm{K}$$



T_{K} will increase with the onset of Star Formation

The global evolution of TS



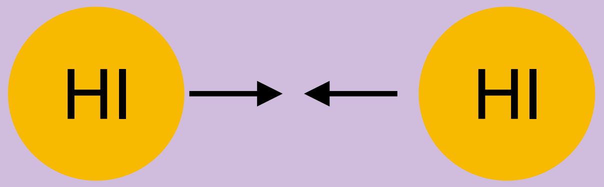
$$T_s^{-1} = \frac{T_{CMB}^{-1} + x_c T_k^{-1} + x_\alpha T_k^{-1}}{1 + x_c + x_\alpha}$$

Loeb & Zaldarriaga 2004,Pritchard & Loeb 2008,
Baek et al. 2010, Thomas & Zaroubi 2010

The global evolution of T_S:
Collisional and radiative coupling to T_K

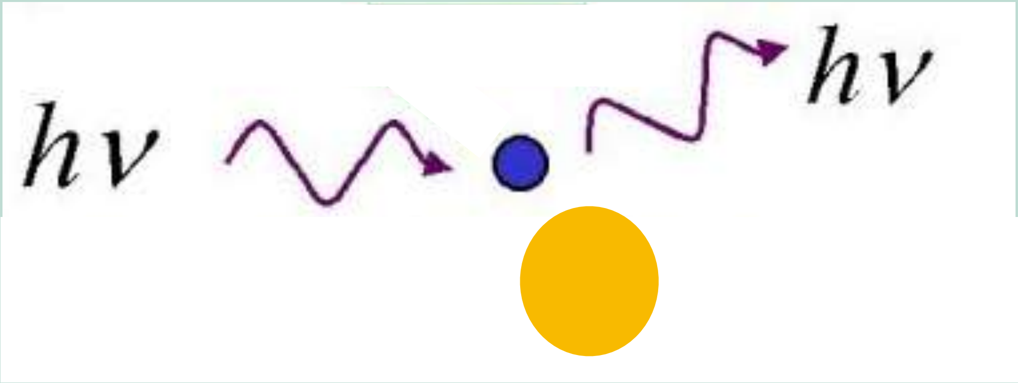
$$T_s^{-1} = \frac{T_{CMB}^{-1} + x_c T_k^{-1} + x_\alpha T_k^{-1}}{1 + x_c + x_\alpha}$$

Collision with other HI atoms, electrons and protons

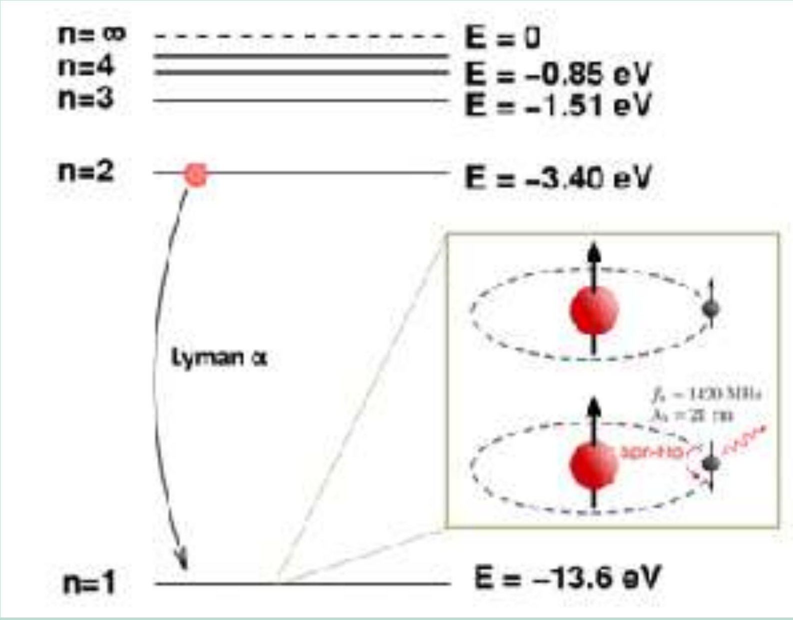


Important when the densities are high

Scattering Lyα photons

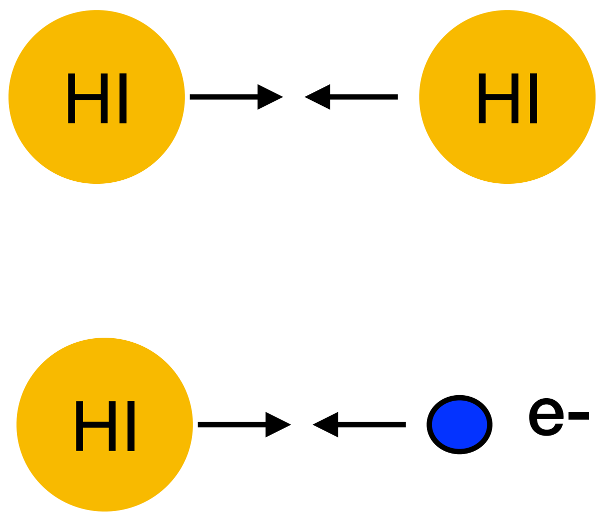


Lyα photons are emitted by stars



Collisional coupling: x_c

Collision with other HI atoms, electrons and protons



$$y_c = \frac{4T_\star}{3A_{10}T_\gamma}\left[\kappa^{HH}_{1-0}(T_k)n_H + \kappa^{eH}_{1-0}(T_k)n_e\right]$$

Spontaneous emission coefficient

tabulated coefficients for collisions

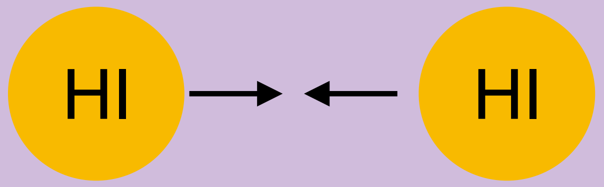
$$T_\star \equiv hc/k\lambda_{21\mathrm{cm}} = 0.0628\,\mathrm{K}$$

Important coupling mechanism when the densities are high

The global evolution of TS: Collisional and radiative coupling

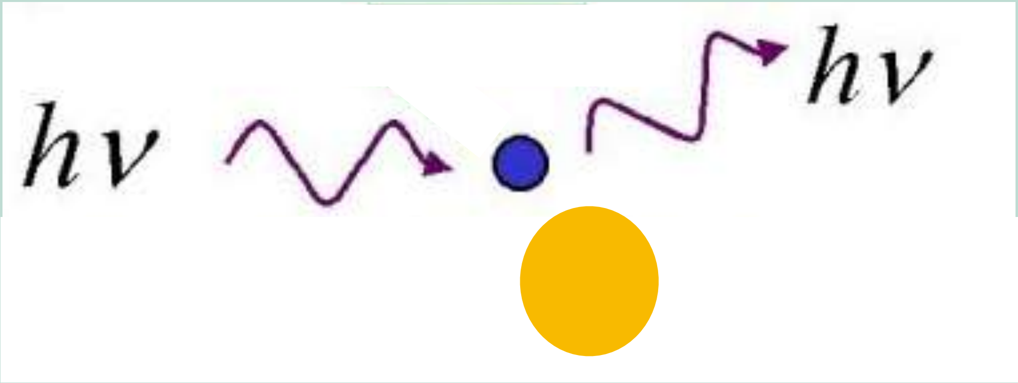
$$T_s^{-1} = \frac{T_{CMB}^{-1} + x_c T_k^{-1} + x_\alpha T_k^{-1}}{1 + x_c + x_\alpha}$$

Collision with other HI atoms, electrons and protons

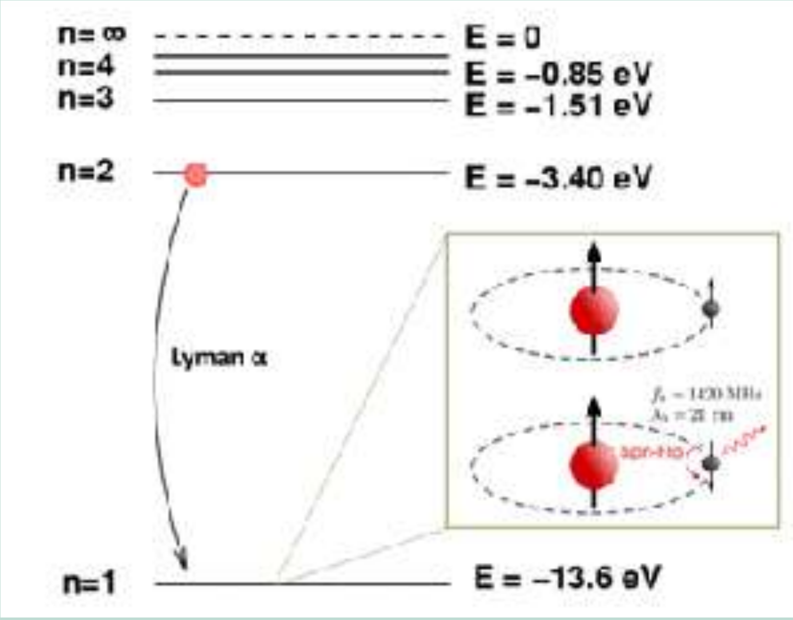


Important when the densities are high

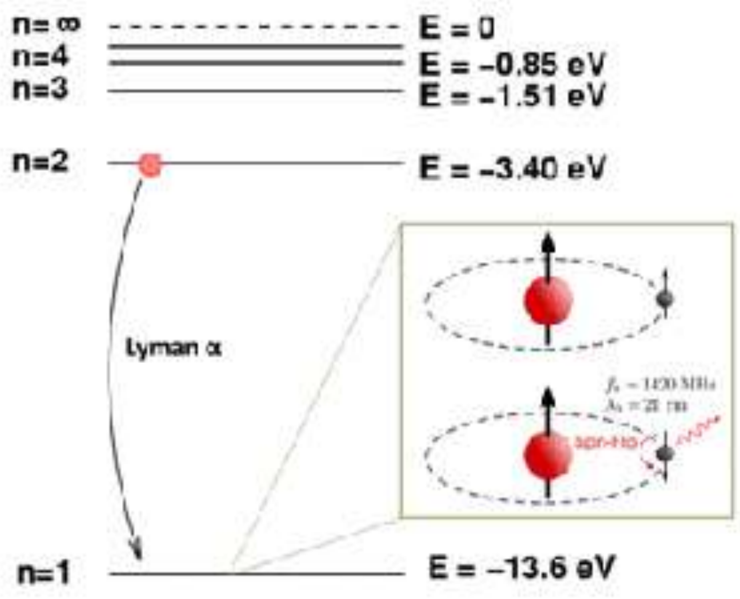
Scattering Lyα photons



Lyα photons are emitted by stars



The Lyman-alpha line



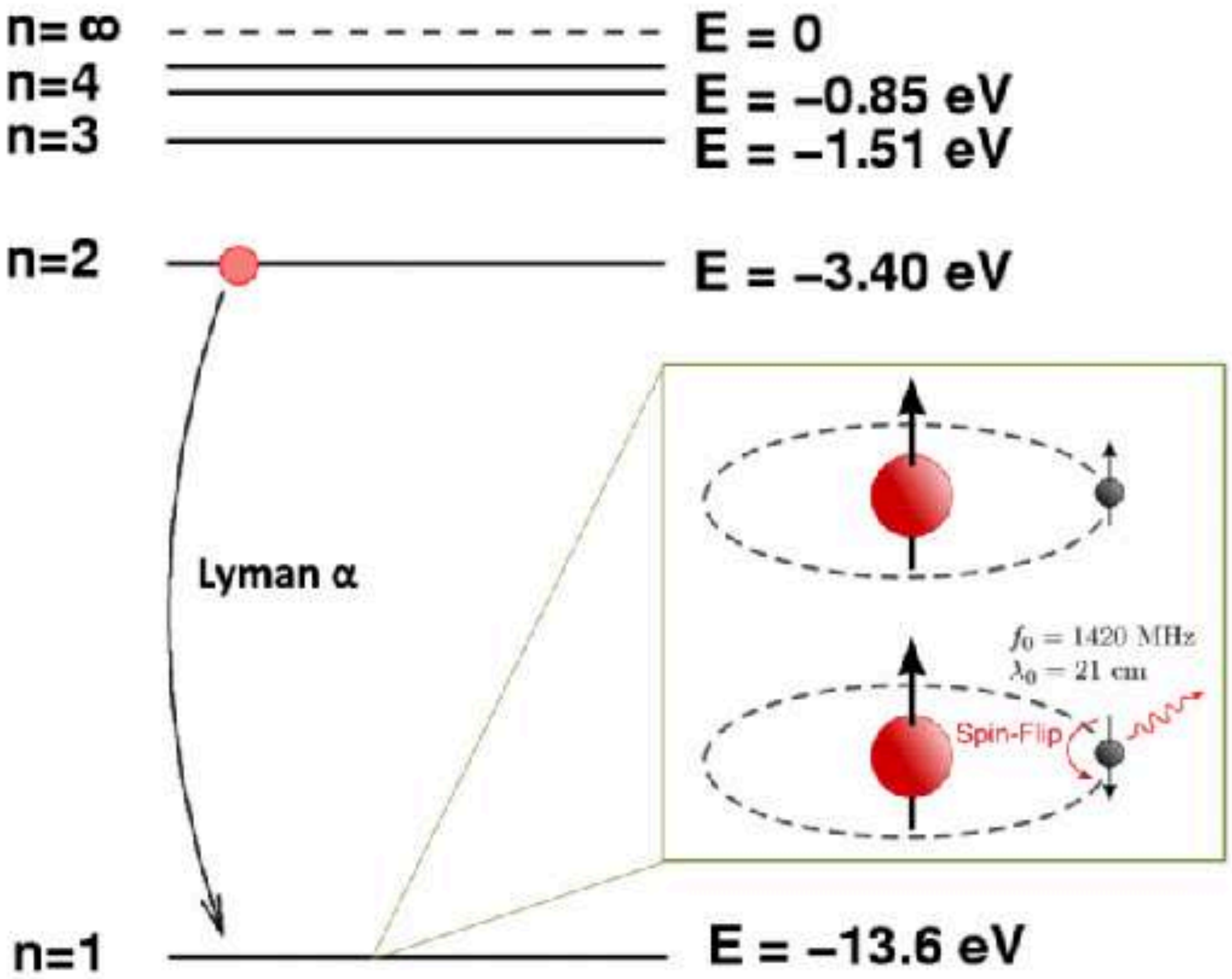
Very important line!!!

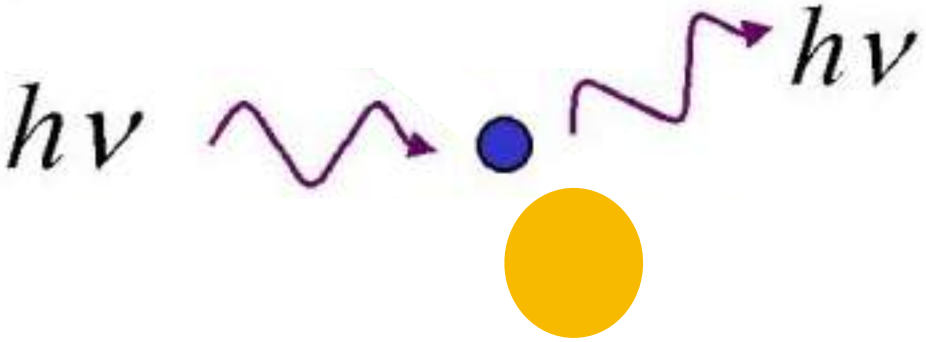
Couples the 21cm line Spin Temperature to the Gas Temperature

Main cooling line for star formation

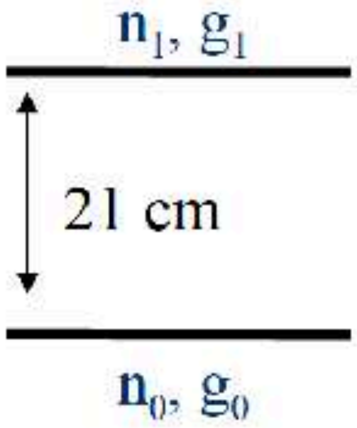
Independent tracer of the high redshift Universe

The Lyman-alpha transition: x_α



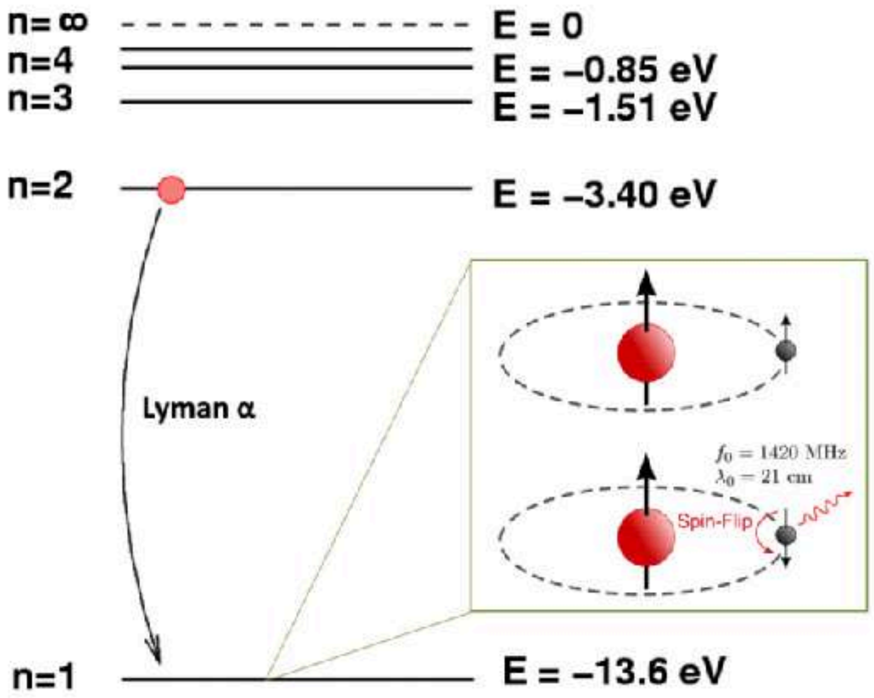


Radiative coupling of TS to TK
due to scattering
(absorption and reemission)
of Lyα photons



The Lyα line has a huge cross section: It is absorbed and reemitted multiple times in a hydrogen cloud

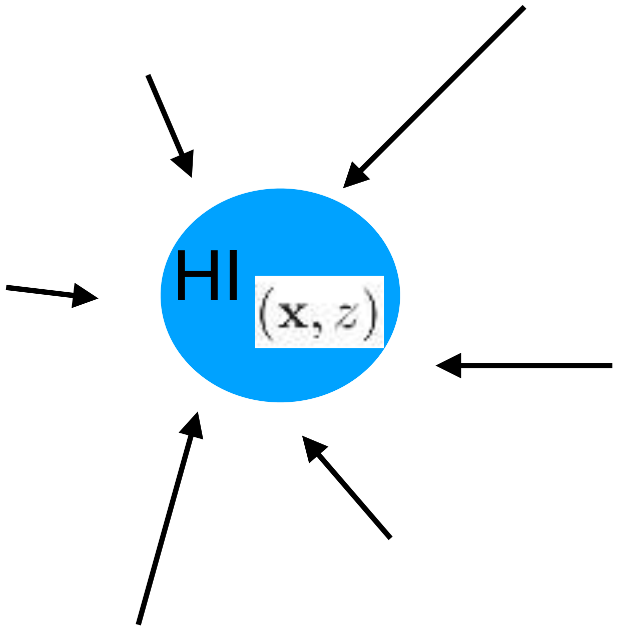
The Lyman-alpha transition: x_α



$$x_\alpha = \frac{S_\alpha J_\alpha}{J_c}$$

$$J_c \equiv \frac{16\pi^2 T_\star e^2 f_\alpha}{27 A_{10} T_\gamma m_e c}$$
$$\approx 5.552 \times 10^{-8}(1+z) \qquad \mathrm{m^{-2} s^{-1} Hz^{-1} sr^{-1}}$$

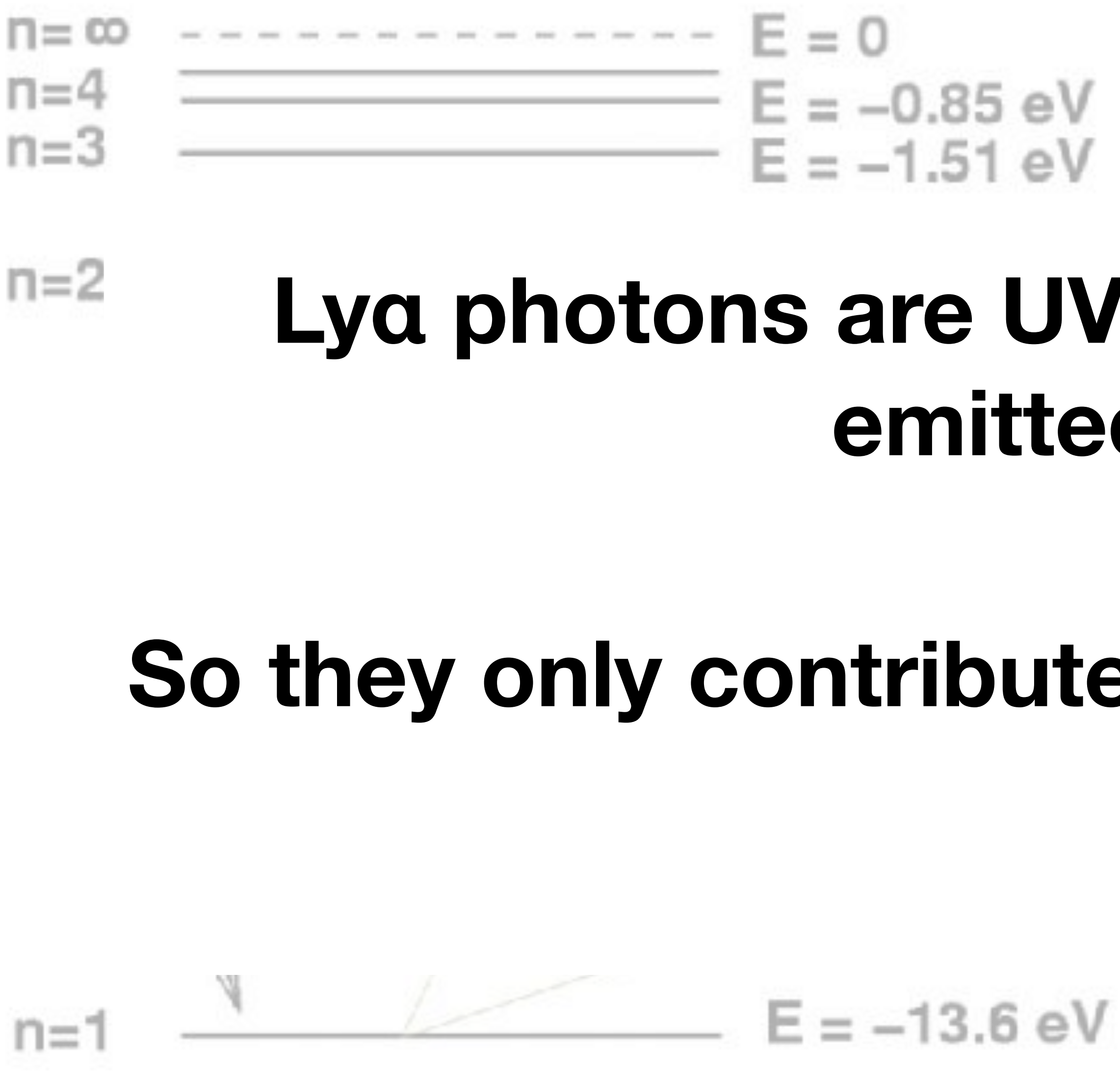
$$T_\star \equiv hc/k\lambda_{21\mathrm{cm}} = 0.0628\,\mathrm{K}$$

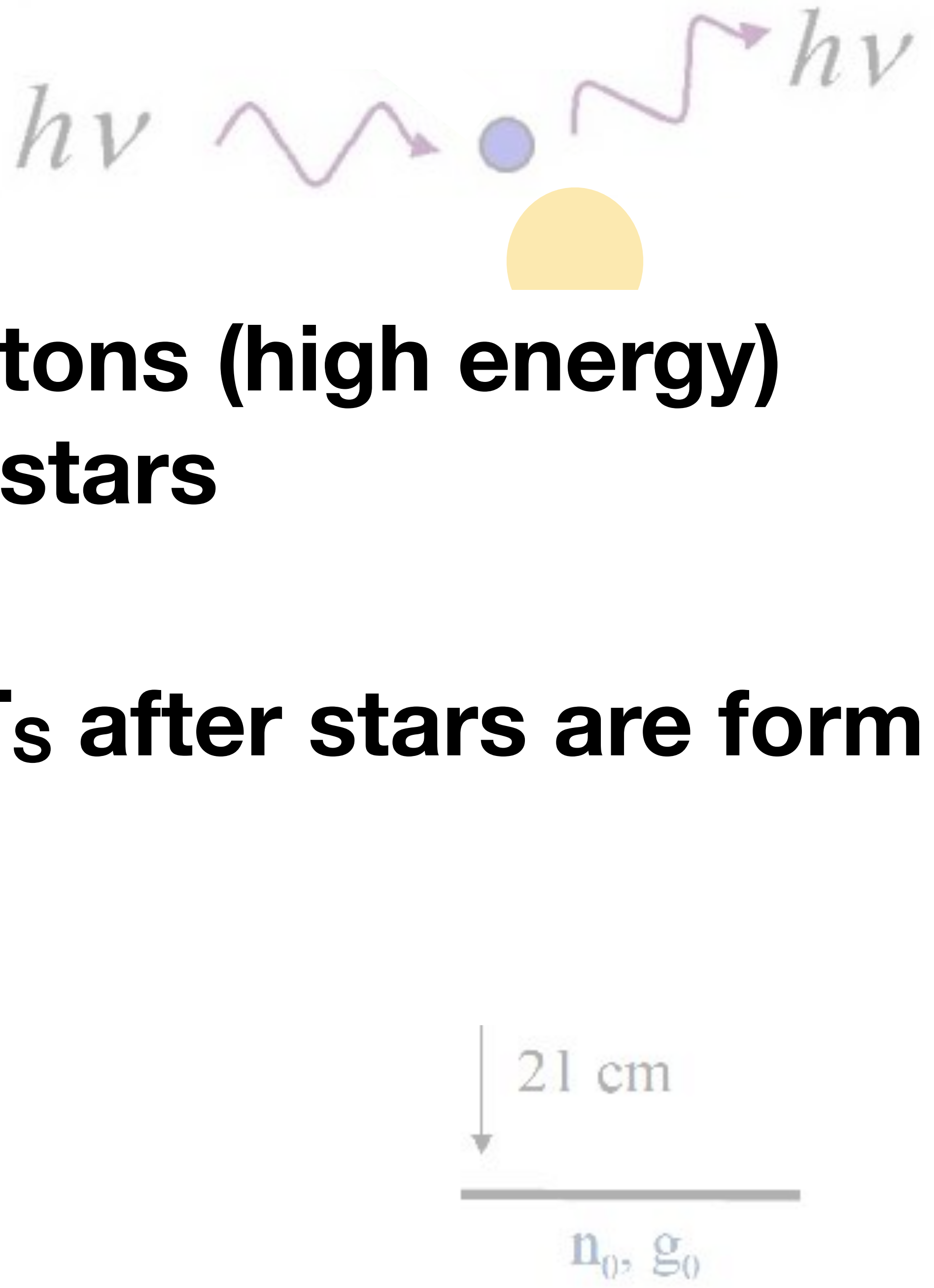


$$J_\alpha(\mathbf{x}, z) = \frac{(1+z)^2}{4\pi} \sum_{n=2}^{n_{\max}} f_{\mathrm{rec}}(n) \times$$
$$\times \int \frac{d\Omega'}{4\pi} \int_0^{x_{\max}(n)} dx' \; \epsilon_\alpha(\mathbf{x} + \mathbf{x}', \nu'_n; z')$$

$$J_\alpha(z) = \frac{(1+z)^2}{4\pi} \sum_{n=2}^{n_{\max}} f_{\mathrm{rec}}(n) \int_z^{z_{\max}(n)} \frac{c\,dz'}{H(z')} \epsilon_\alpha(\nu'_n, z')$$

The Lyman-alpha transition: x_α





Lyα photons are UV photons (high energy) emitted by stars

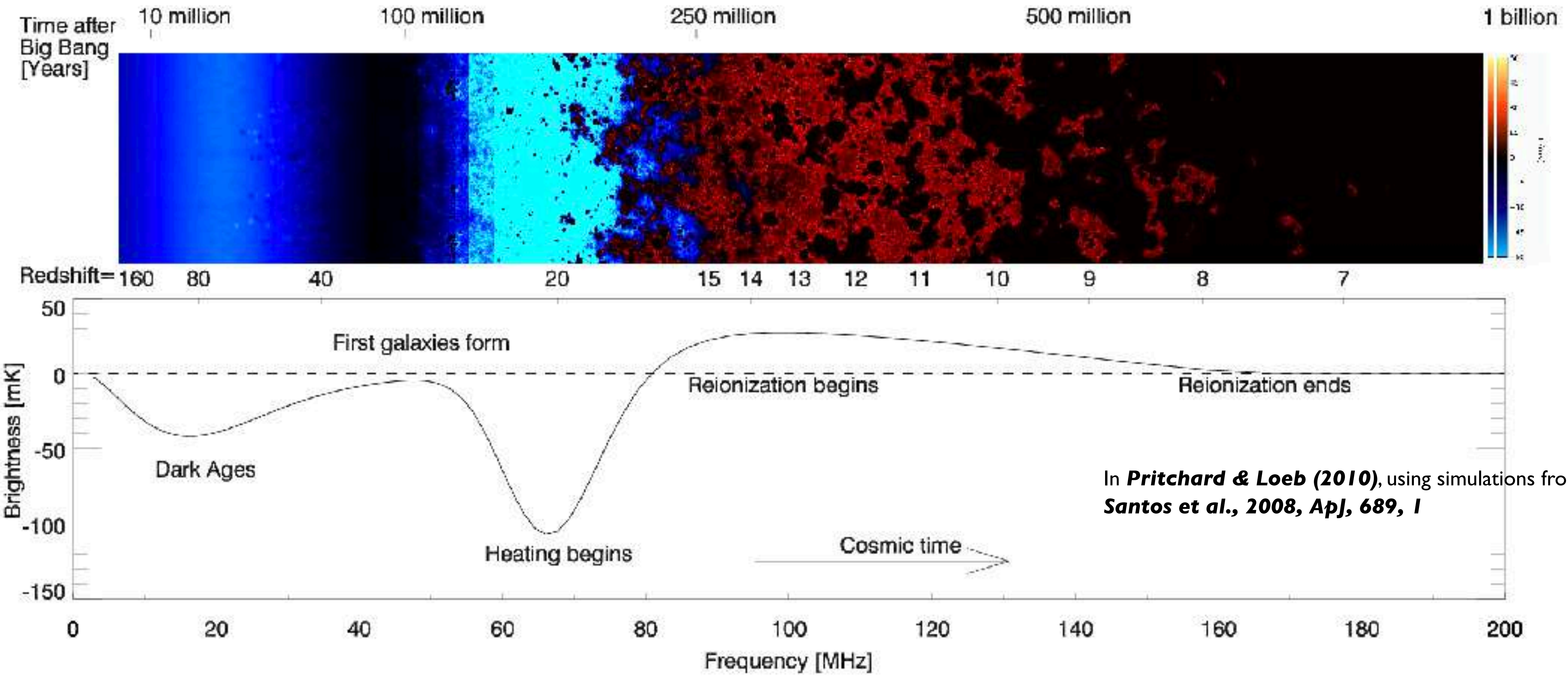
⬇

So they only contribute to T_S after stars are form

TK

n)

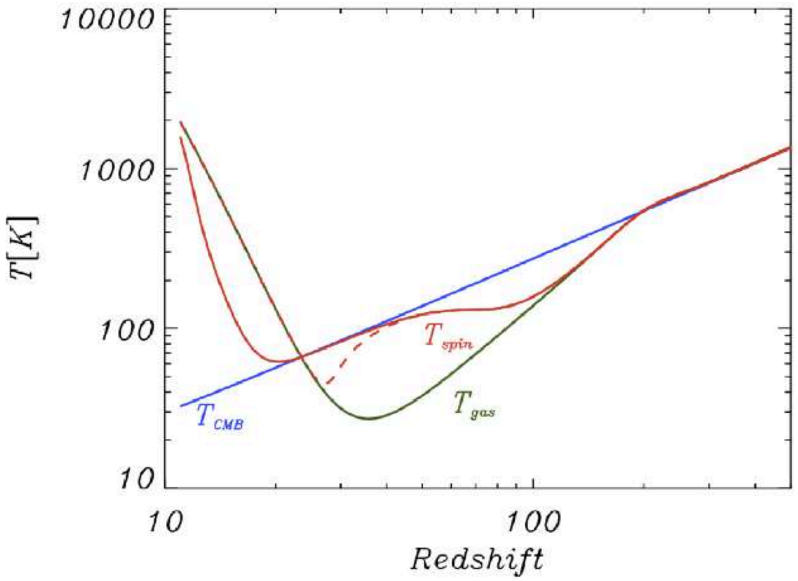
The global 21cm signal



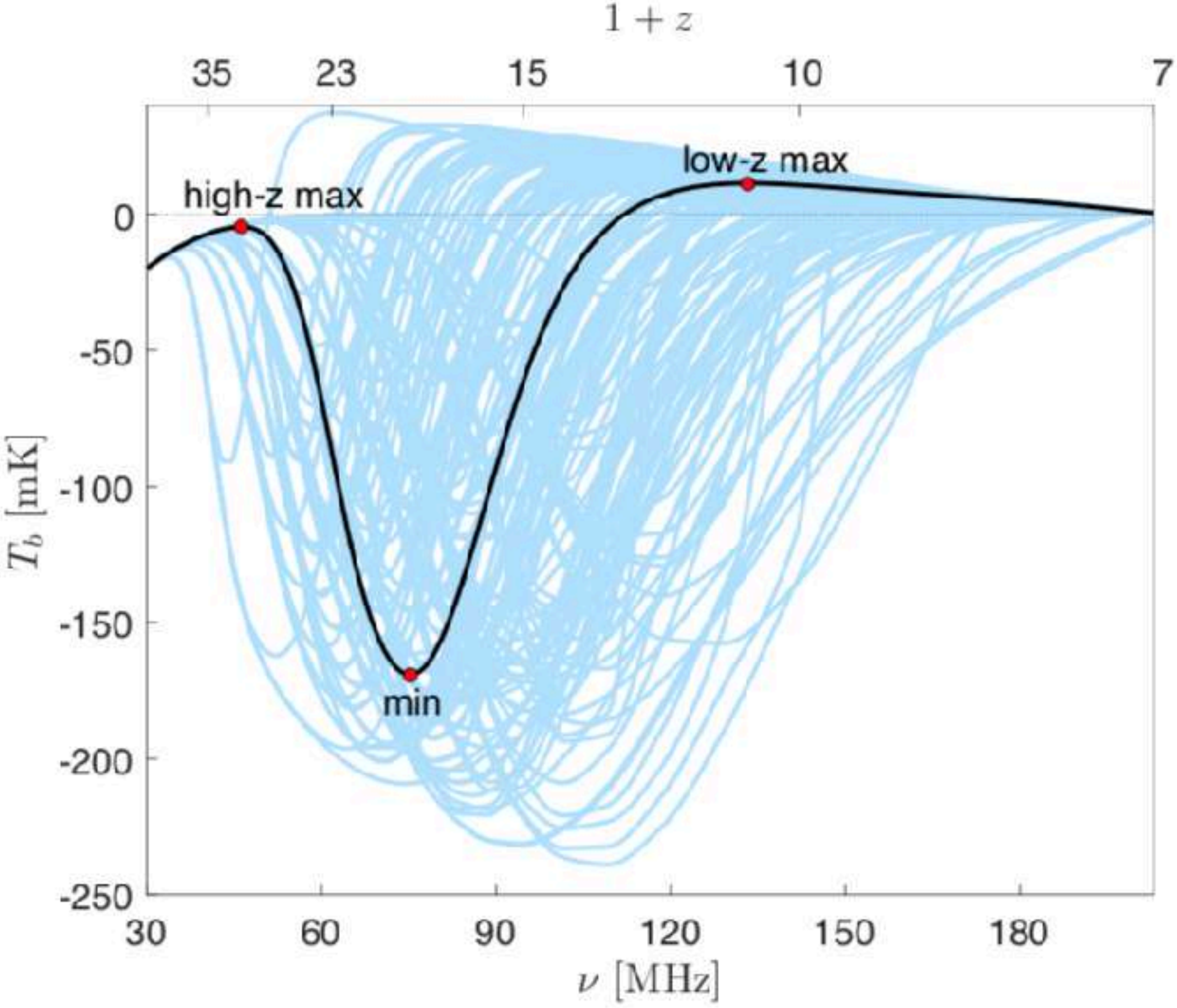
In ***Pritchard & Loeb (2010)***, using simulations from ***Santos et al., 2008, ApJ, 689, 1***

$$\delta T_{\mathrm{b}} = \propto \frac{T_S - T_{\mathrm{CMB}}}{T_S}$$

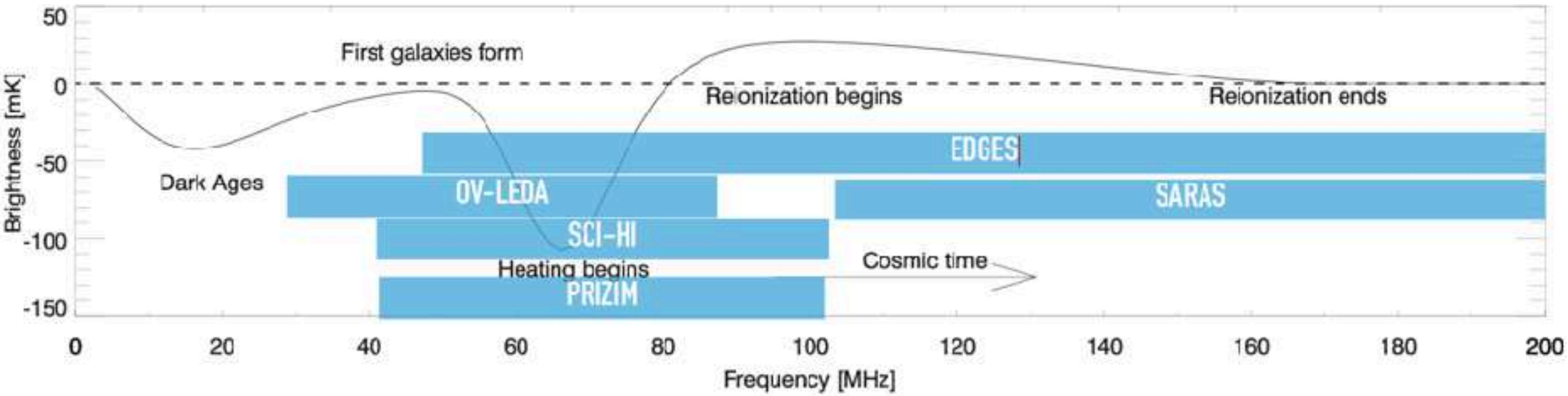
$$T_s^{-1} = \frac{T_{CMB}^{-1} + x_c T_k^{-1} + x_\alpha T_k^{-1}}{1 + x_c + x_\alpha}$$

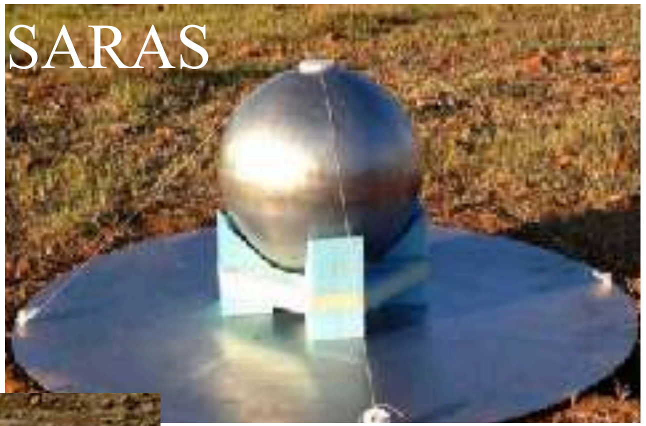


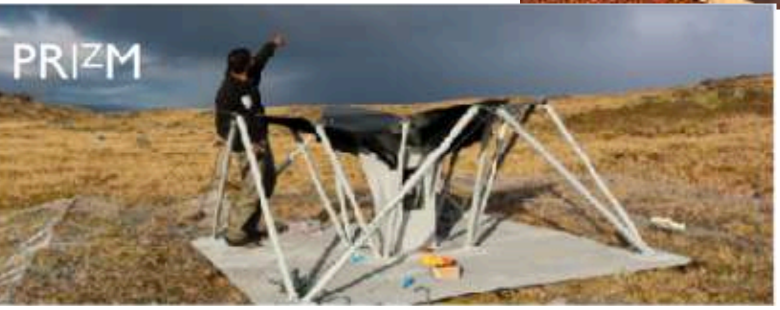
The global 21cm signal



High-z 21cm Projects Under Way



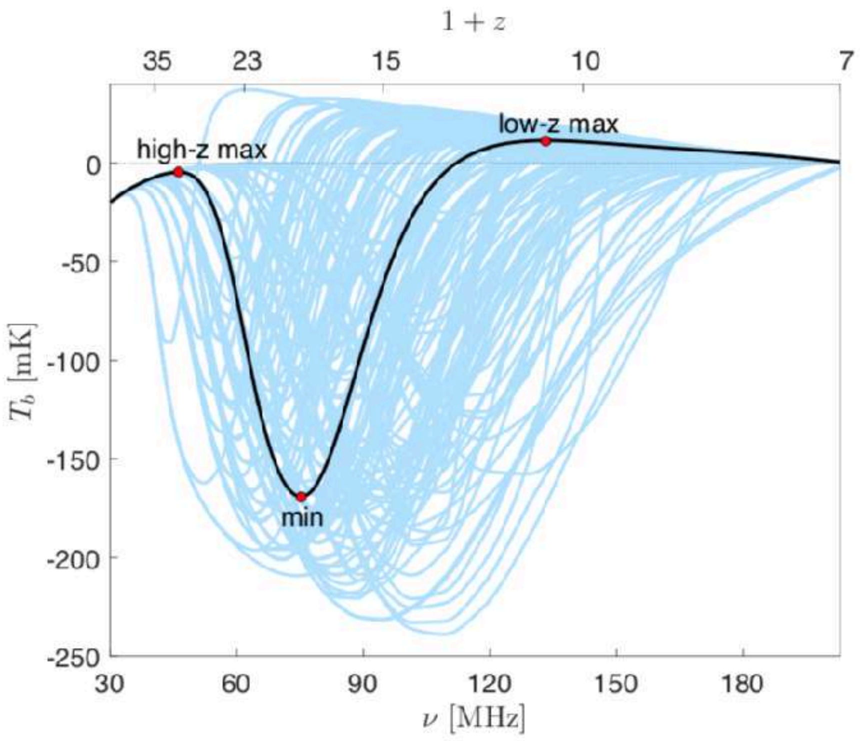


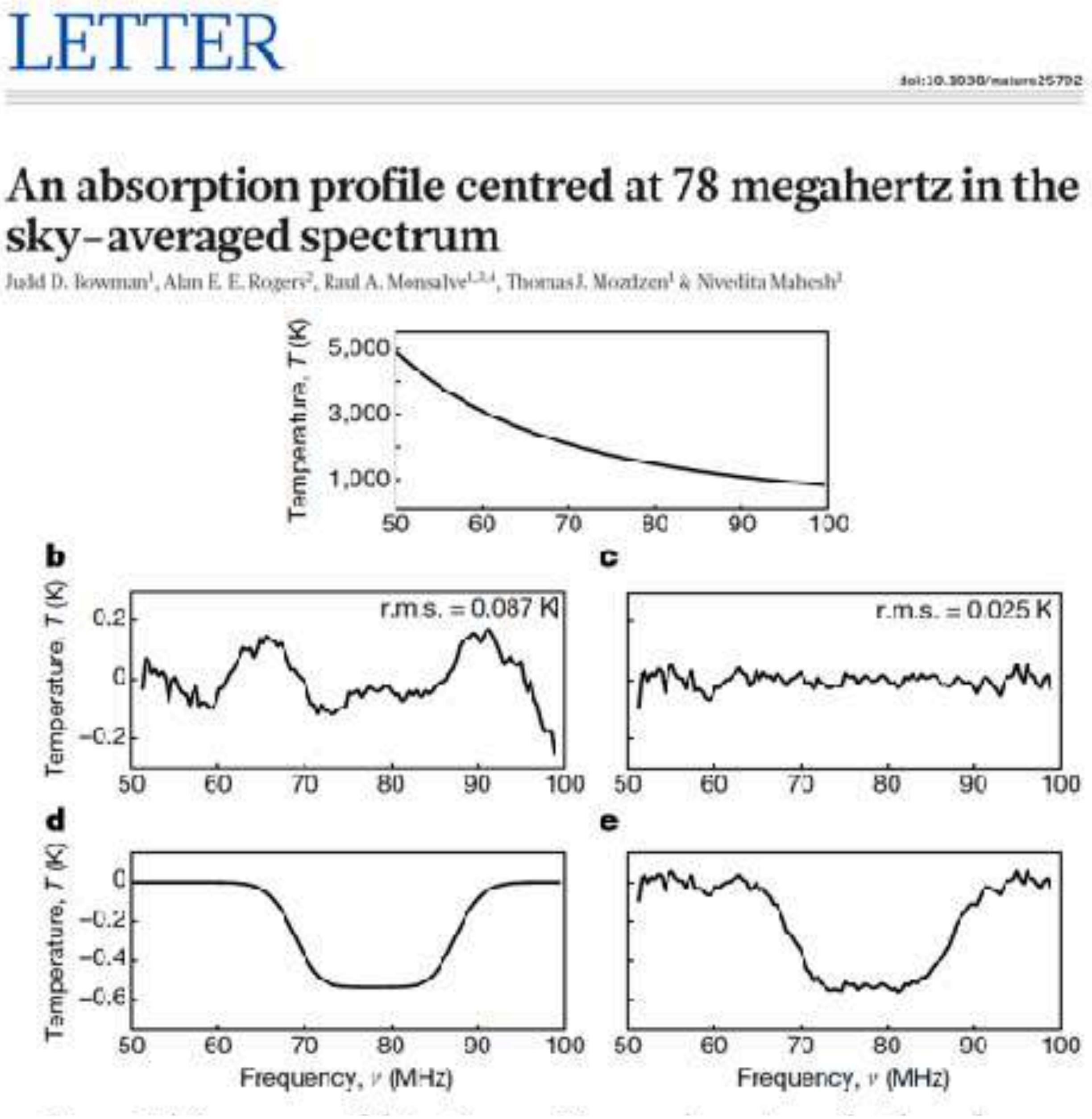


EDGES

Experiment to Detect the Global Epoch of Reionization Signature

50-100MHz





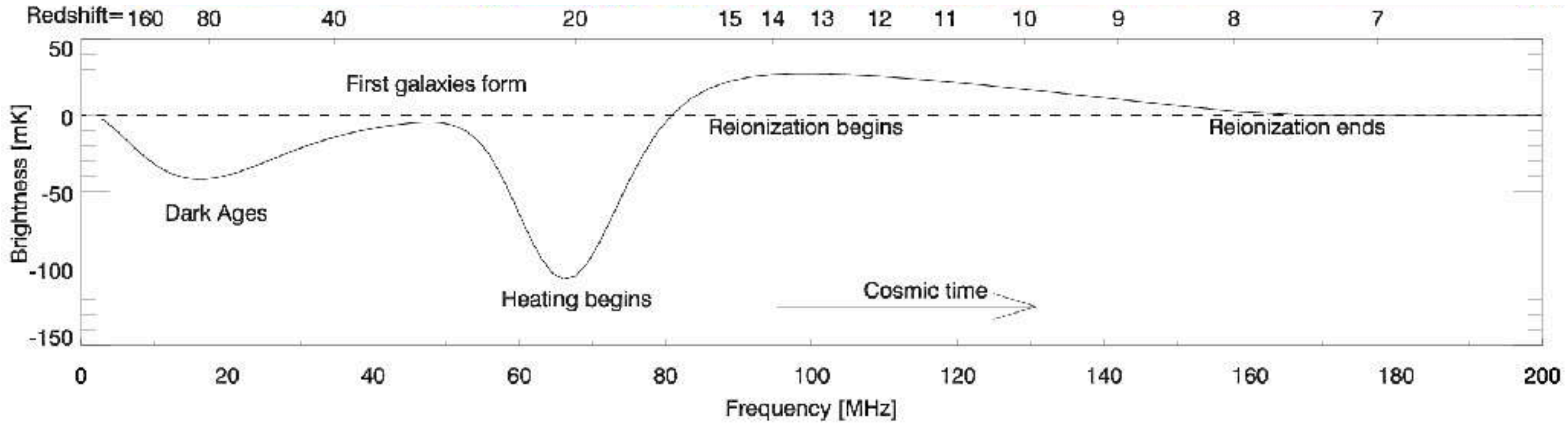
LETTER

doi:10.1038/nature25792

An absorption profile centred at 78 megahertz in the sky-averaged spectrum

Judd D. Bowman[1], Alan E. E. Rogers[2], Raul A. Monsalve[1,3,4], Thomas J. Mozdzen[1] & Nivedita Mahesh[1]

How could this feature absorption be so strong?



Thermal equilibrium at
z≈200-150 ⇒ T_K=TCMB

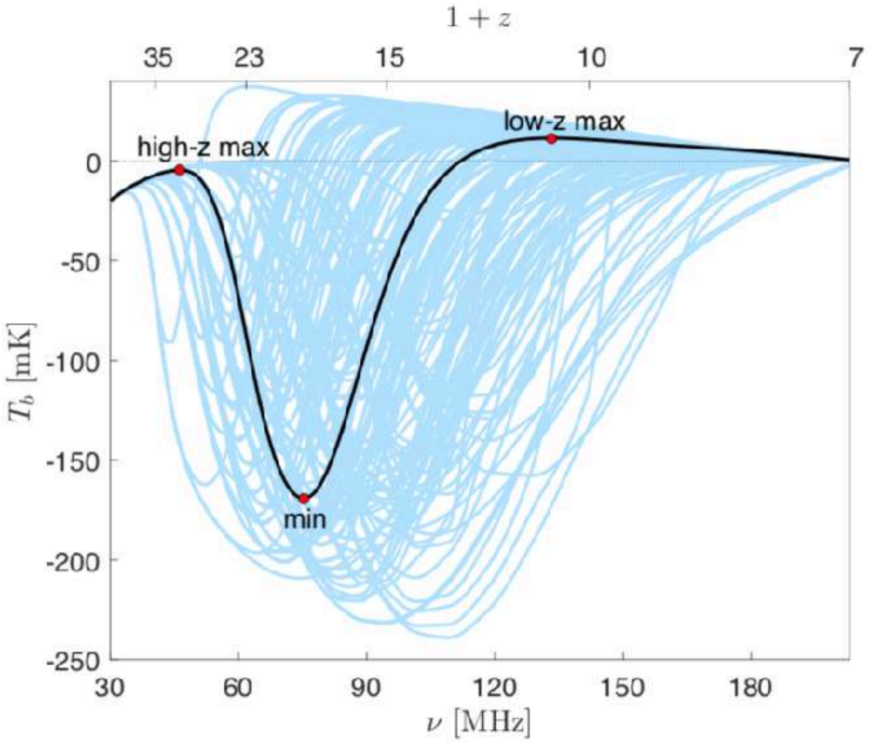
$$\delta T_{\rm b} = \propto \frac{T_S - T_{\rm CMB}}{T_S}$$

$$\delta T_{21}(z) \propto \left[1 - \frac{T_{\rm R}(z)}{T_{\rm S}(z)}\right]$$

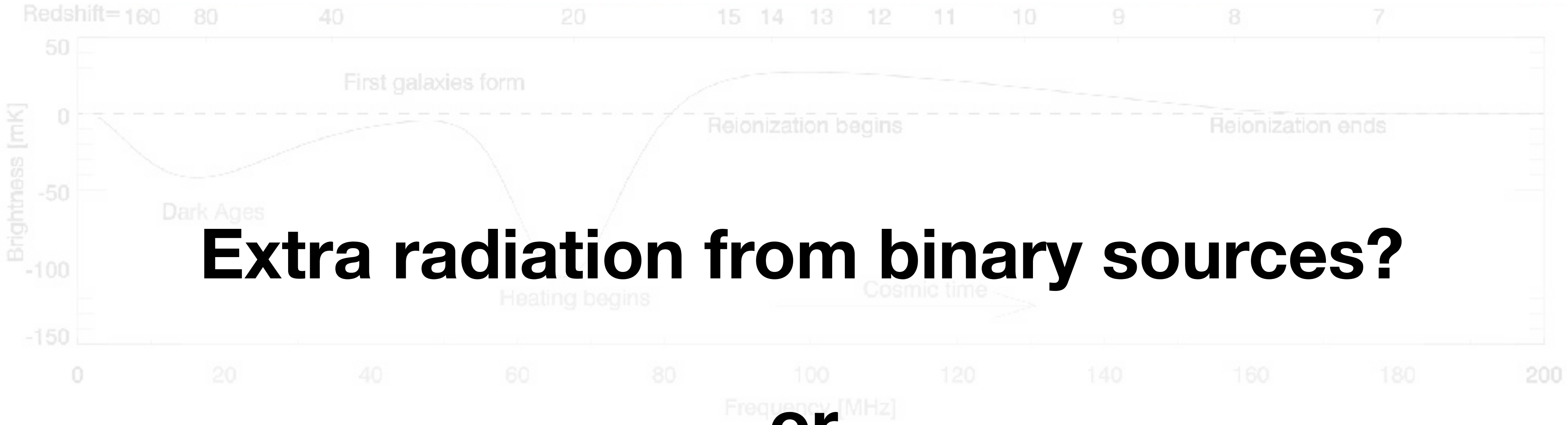
$$\delta T_{21} \propto \frac{x_\alpha + x_c}{1 + x_\alpha + x_c}\left(1 - \frac{T_{\rm R}}{T_{\rm gas}}\right)$$

$$T_{\rm gas} \propto (1+z)^2$$

$$T_R \propto (1+z)$$



How could this feature absorption be so strong?



Extra radiation from binary sources?

or

Thermal equilibrium at
z≈150 ⇒ T_K=TCMB

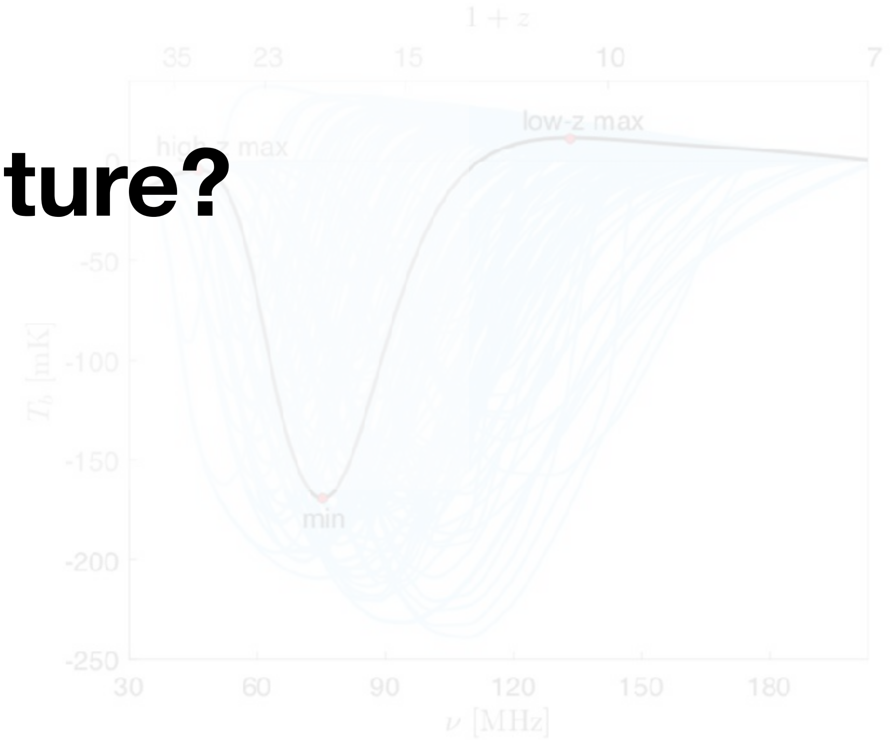
$$T_{\rm gas} \propto (1+z)^2$$

A colder gas Temperature?

$$\delta T_{\rm b} = \propto \frac{T_S - T_{\rm CMB}}{T_S}$$

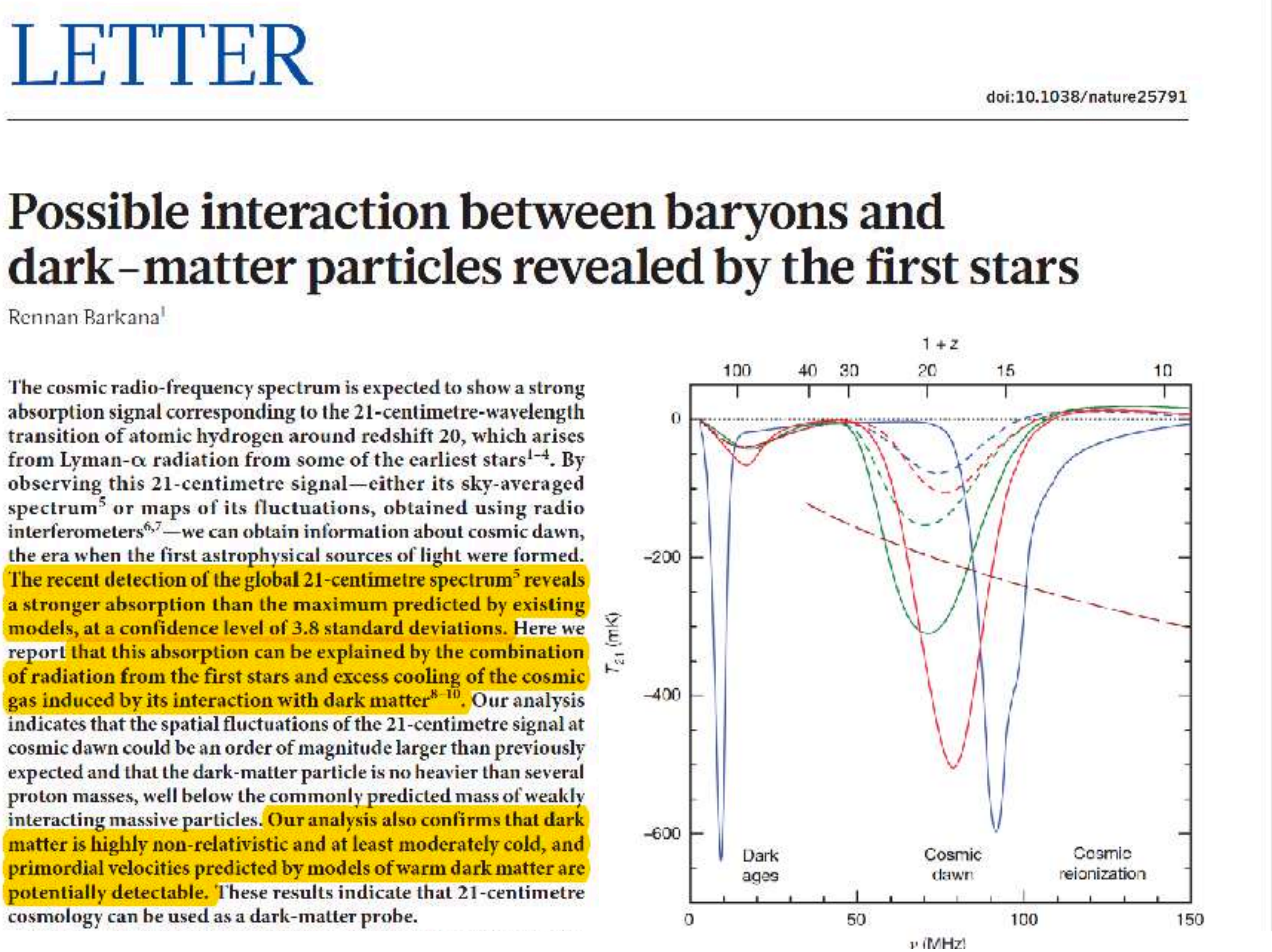
$$T_R \propto (1+z)$$

$$\delta T_{21}(z) \propto \left[1 - \frac{T_{\rm R}(z)}{T_{\rm S}(z)}\right]$$



$$\delta T_{21} \propto \frac{x_\alpha + x_c}{1 + x_\alpha + x_c}\left(1 - \frac{T_{\rm R}}{T_{\rm gas}}\right)$$

DM



LETTER

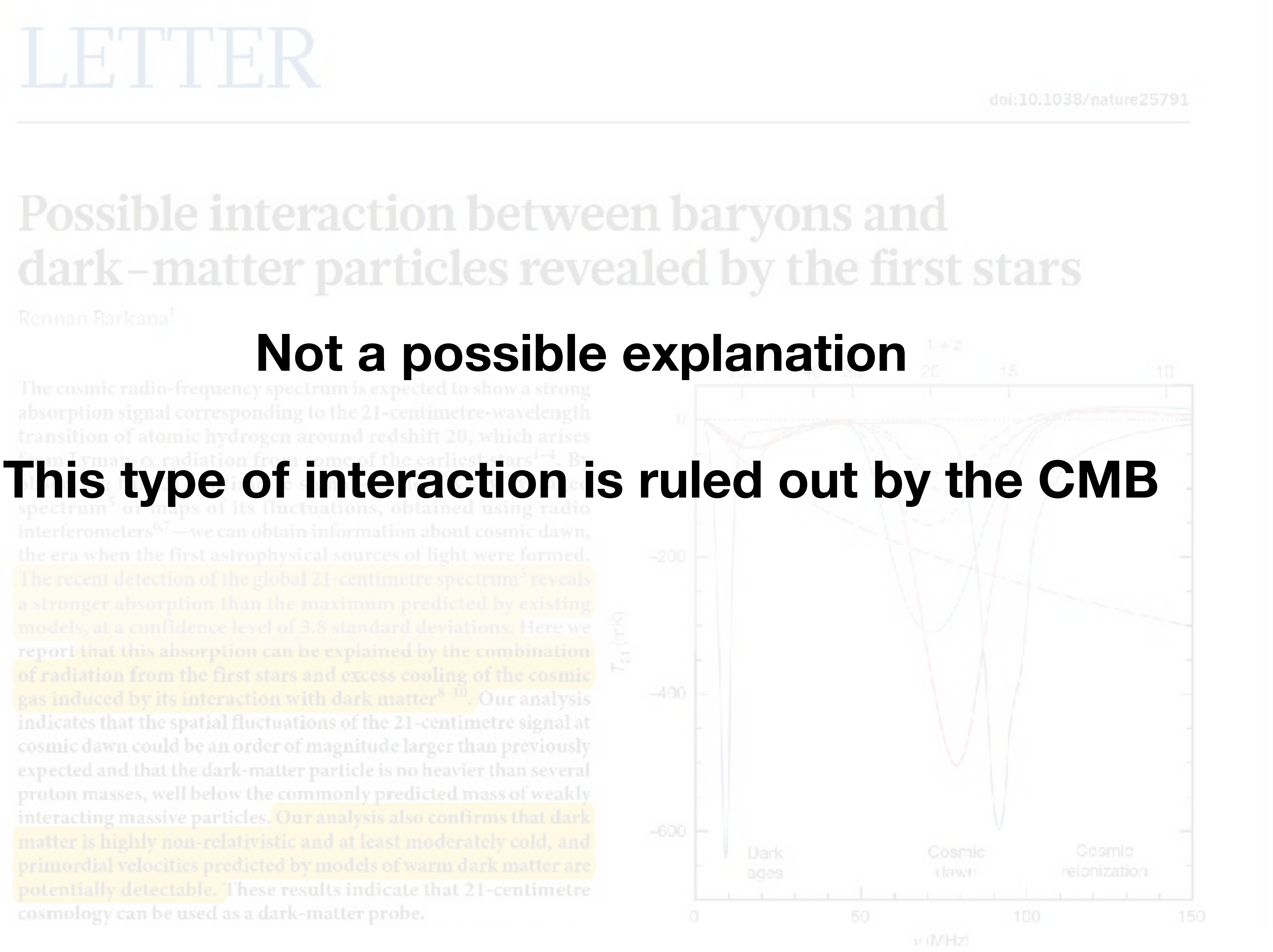
doi:10.1038/nature25791

Possible interaction between baryons and dark-matter particles revealed by the first stars

Rennan Barkana[1]

The cosmic radio-frequency spectrum is expected to show a strong absorption signal corresponding to the 21-centimetre-wavelength transition of atomic hydrogen around redshift 20, which arises from Lyman-α radiation from some of the earliest stars[1–4]. By observing this 21-centimetre signal—either its sky-averaged spectrum[5] or maps of its fluctuations, obtained using radio interferometers[6,7]—we can obtain information about cosmic dawn, the era when the first astrophysical sources of light were formed. The recent detection of the global 21-centimetre spectrum[5] reveals a stronger absorption than the maximum predicted by existing models, at a confidence level of 3.8 standard deviations. Here we report that this absorption can be explained by the combination of radiation from the first stars and excess cooling of the cosmic gas induced by its interaction with dark matter[8–10]. Our analysis indicates that the spatial fluctuations of the 21-centimetre signal at cosmic dawn could be an order of magnitude larger than previously expected and that the dark-matter particle is no heavier than several proton masses, well below the commonly predicted mass of weakly interacting massive particles. Our analysis also confirms that dark matter is highly non-relativistic and at least moderately cold, and primordial velocities predicted by models of warm dark matter are potentially detectable. These results indicate that 21-centimetre cosmology can be used as a dark-matter probe.

DM



LETTER

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Not a possible explanation

This type of interaction is ruled out by the CMB

Is the detection real?

Discussion

- Is it a real detection?
  - Foreground removal artefact due to the simple polynomial foreground model
  - Multi path propagation
  - Ionosphere
  - Absorption line in the galaxy

To be confirmed soon!!!

SARAS 2

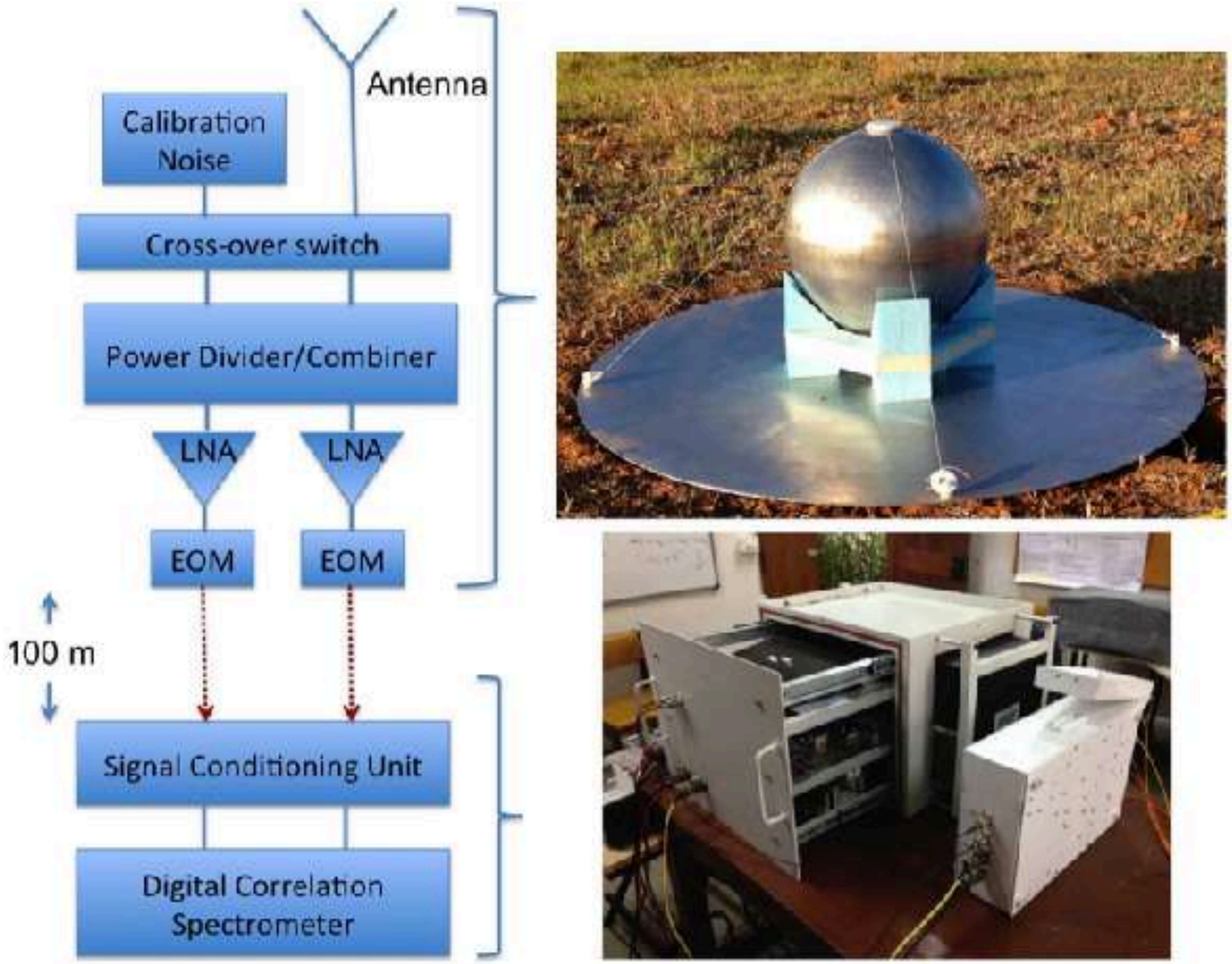
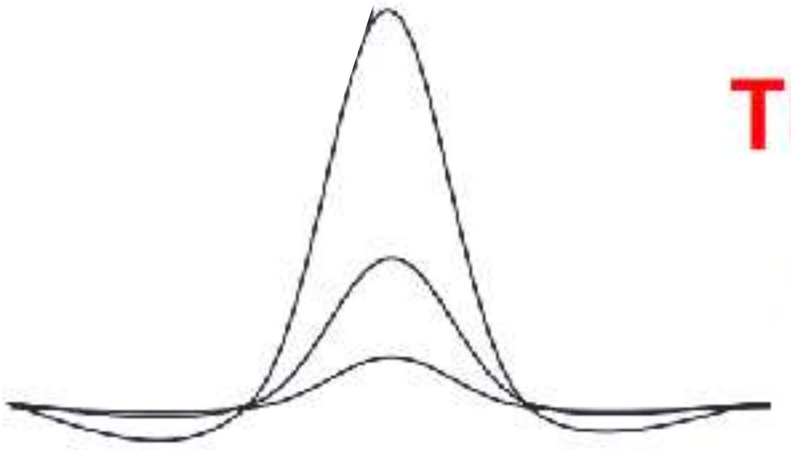


Figure 1. SARAS 2: In the schematic, LNA refers to Low-Noise Amplifiers while EOM are Electro-Optical Modulators. The upper right image shows the sphere-disk monopole, with the sphere supported using styrofoam, cotton strings and teflon fasteners. The lower right image shows the spectrometer.

Spectral radiometer designed for measuring the all-sky global 21-cm spectral distortions, located in Southern India

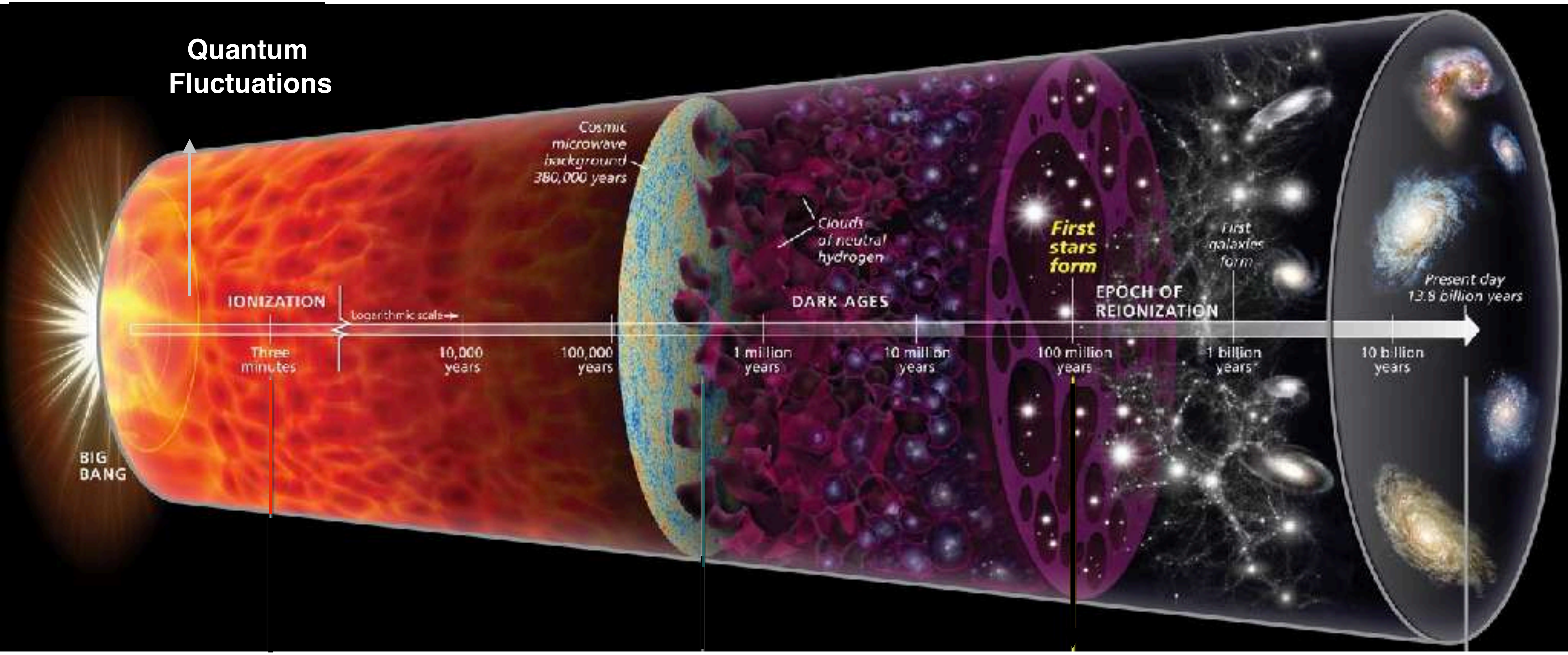
Next: The first light sources

From quantum fluctuations to the first bound structures



The rich gets richer and the poor gets poorer

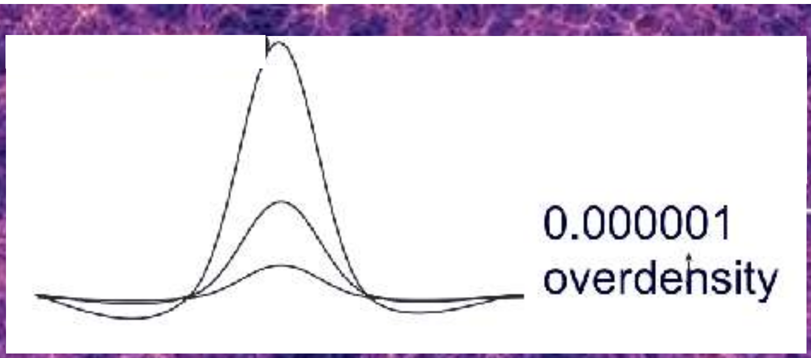
The first light in the Universe



Quantum fluctuations evolve into classic density fluctuations and later into large scale structure

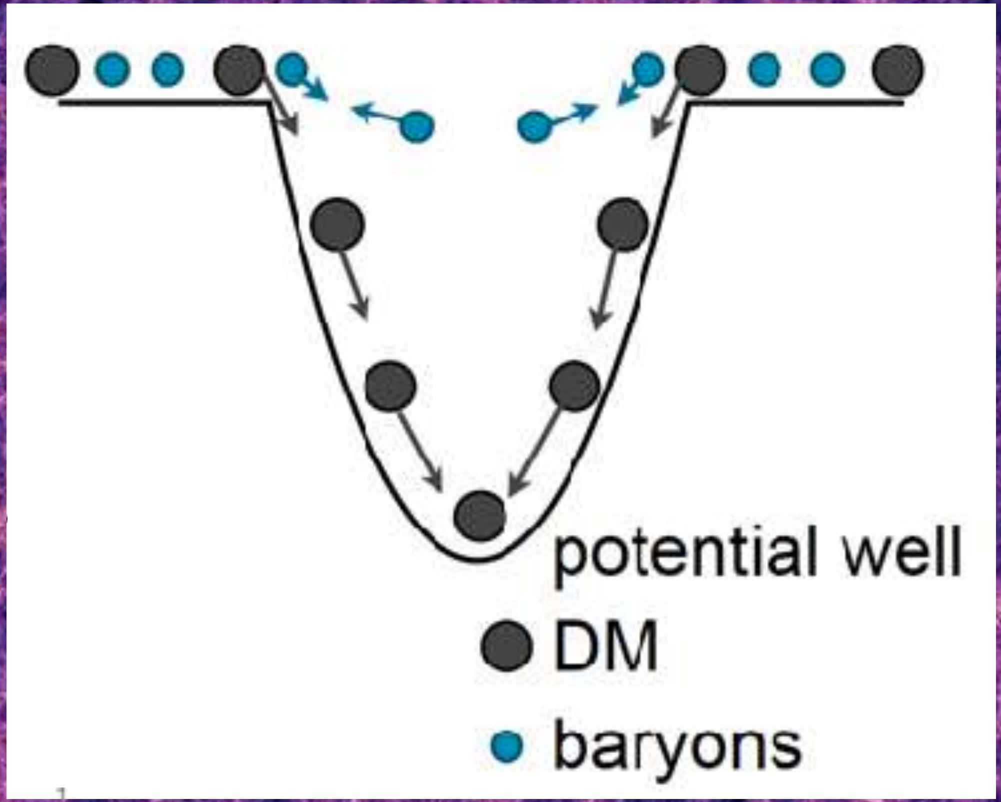
Credit - http://discovermagazine.com/~/media/Images/Issues/2014/April/21-centimeter-cosmology.jpg

From density fluctuations to star formation



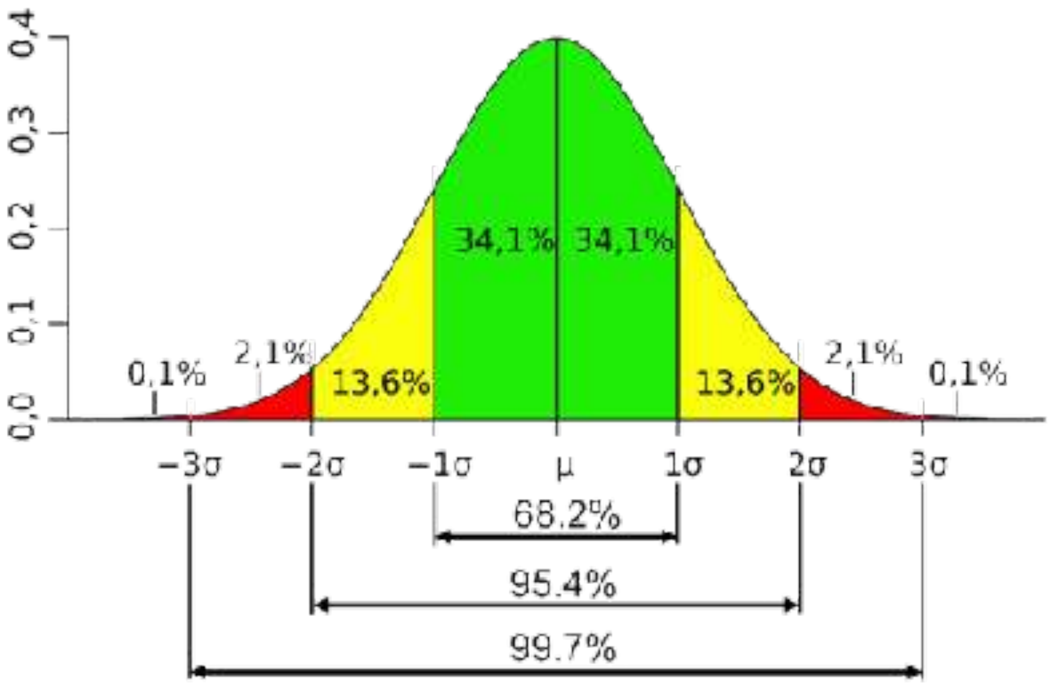
Overdense regions become more and more overdense and form DM halos

First Stars formed in the peaks of the density field



Structure Formation: Dark Matter halos

i) First density fluctuations are gaussian
(confirmed by the CMB)



Credit - https://kanbanize.com/blog/normal-gaussian-distribution-over-cycle-time/

ii) The Universe is flat

$$\delta = \frac{\rho - \bar{\rho}}{\bar{\rho}} \qquad \bar{\rho} = \Omega_m \rho_{\rm crit}$$

$$\rho_c = \frac{3H^2}{8\pi G} = 10^{-26}\ {\rm kg/m^3}$$

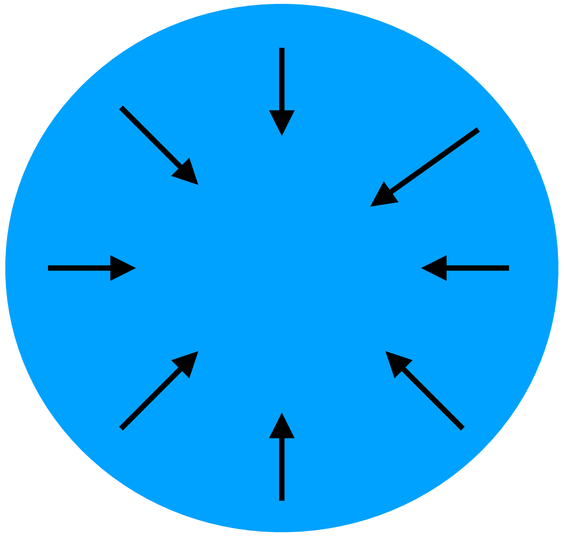
(10 hydrogen atoms per cubic metre)

Fluctuations become non linear at $\delta \approx 1$

iii) For non-linear fluctuations

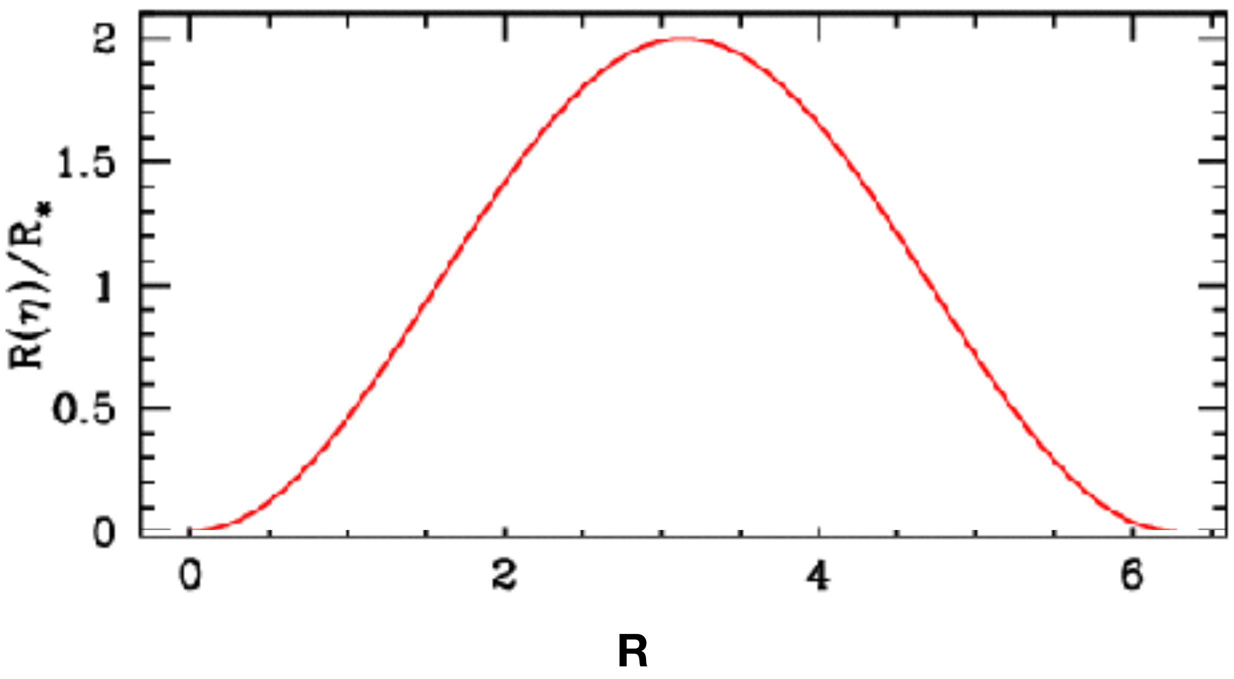
↓

Spherical collapse
(Press & Schechter formalism)



Dark Matter Halos Formation

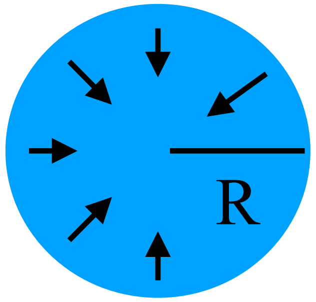
- Consider a spherically symmetric overdensity in an expanding background
- By Birkhoff's Theorem, it can treat as an independent and scaled version of the Universe
- Initial expansion with Hubble flow, turnaround, collapse and virialisation



Credit - Chris Power

Spherical collapse: Press & Schechter Formalism

Mass-density field smoothed over a scale R correspondent to a mass M is also Gaussian



$$P(\delta > \delta_{\rm crit}|M) = \frac{1}{\sigma(M)\sqrt{2\pi}} \int_{\delta_{\rm crit}}^{\infty} d\delta' \; \exp\left(-\frac{\delta'^2}{2\sigma^2(M)}\right)$$

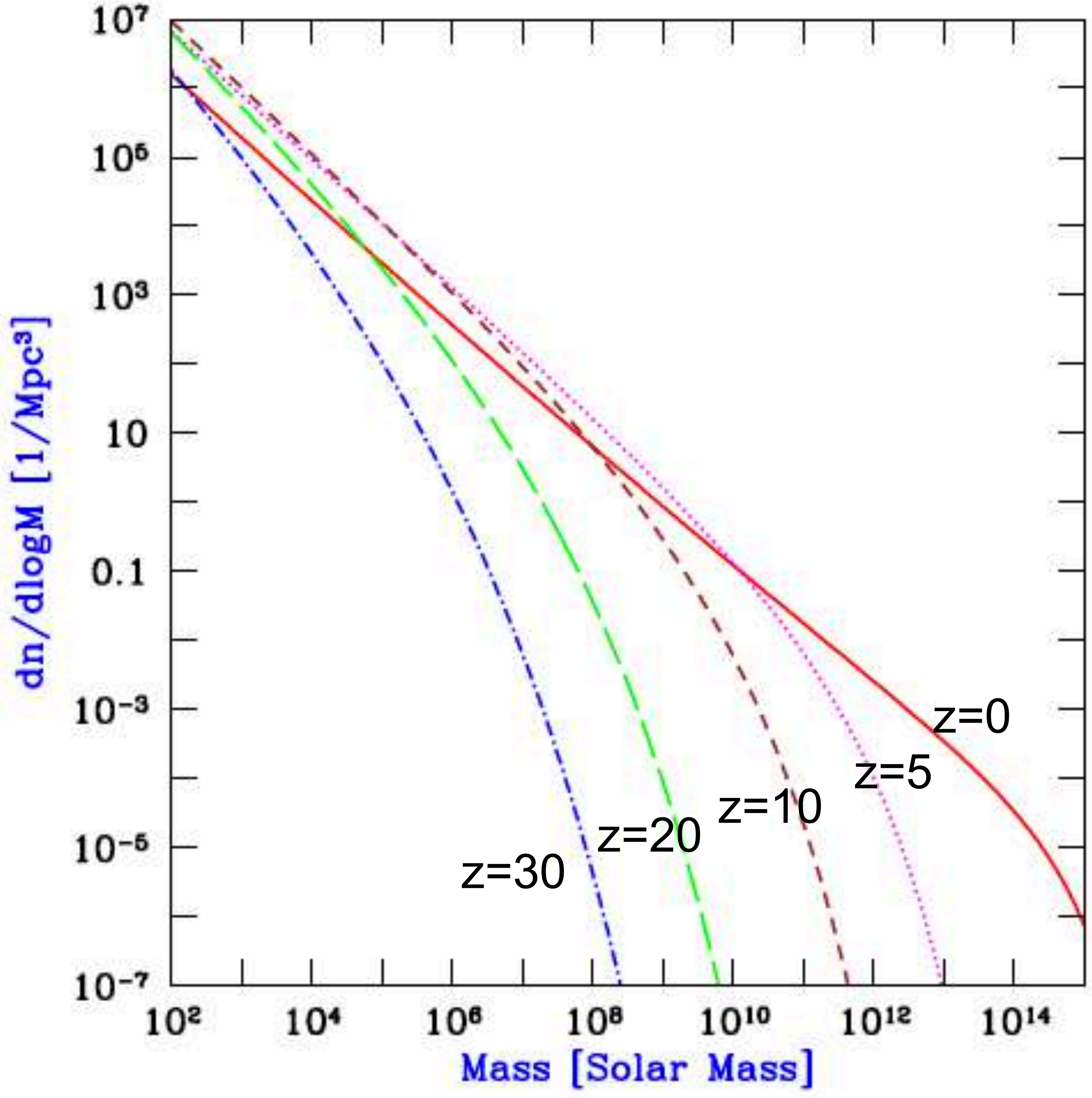
$\sigma^2(M)$ denotes the variance of the mass-density field smoothed over scale M

$$P(>M) = P(\delta > \delta_{\rm crit}|M) = \frac{1}{2}\left[1 - \mathrm{erf}\left(\frac{\nu}{\sqrt{2}}\right)\right] \qquad \nu \equiv \frac{\delta_{\rm crit}}{\sigma(M)}$$

Letting dn be the comoving number density of halos of mass between M and $M + dM$

$$\frac{dn}{dM} = \sqrt{\frac{2}{\pi}}\, \frac{\rho_m}{M}\, \frac{-d(\ln\sigma)}{dM}\, \nu_c\, e^{-\nu_c^2/2}$$

Spherical collapse: Press & Schechter Formalism



$$\frac{dn}{dM} = \sqrt{\frac{2}{\pi}}\,\frac{\rho_m}{M}\,\frac{-d(\ln\sigma)}{dM}\,\nu_c\,e^{-\nu_c^2/2}$$

$$\nu \equiv \frac{\delta_{\mathrm{crit}}}{\sigma(M)}$$

Credit: Barkana & Loeb 2001

“Let there be Light”
The first stars and Galaxies

POP III stars : Formed from primordial gas with only Hydrogen and Helium

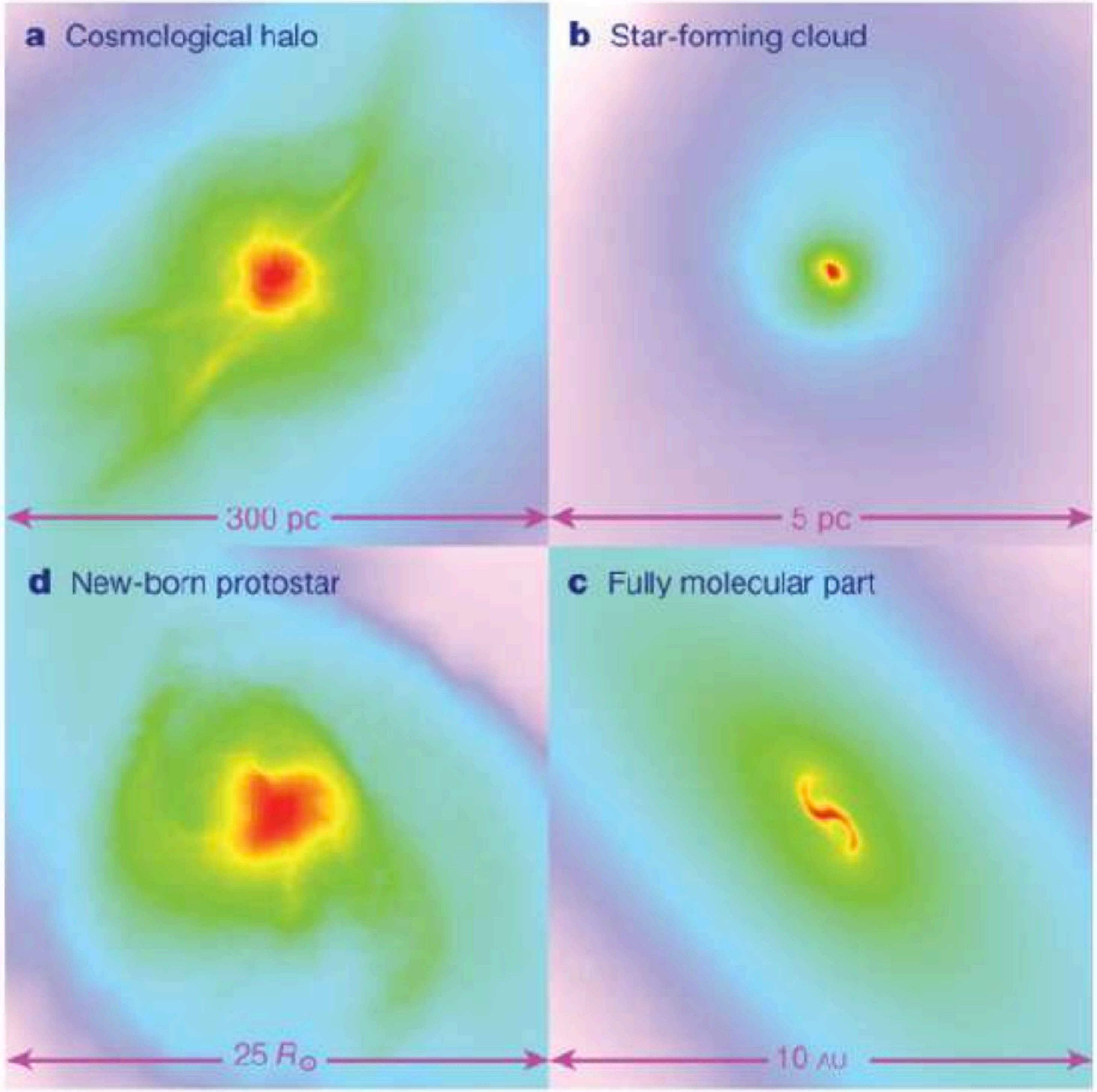
Cooling without metals

The fragmentation issue

Impact on the IGM of the first stars

Credit: Saleem Zaroubi

Stellar Formation in a Halo



Projected gas distribution around a primordial protostar.

Bromm *et al.* 2009

1 pc = 3.086 x 10^{16} m

1 $R_\odot$ = 6.09 x 10^{15} m

1 Au = 1.496 x 10^{11} m

Star Formation in a Halo: Stability of gas clouds

Mean densities:

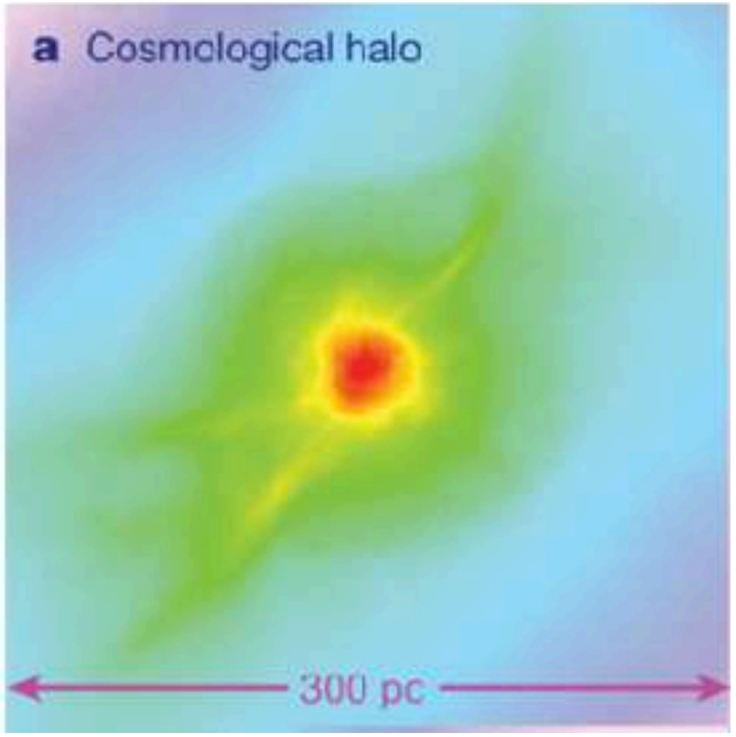
Sun: 1.4 g cm^{-3}
MW at solar radius: 10^{-23} g cm-3
Present day average: 5 x 10^{-31} g cm^{-3}

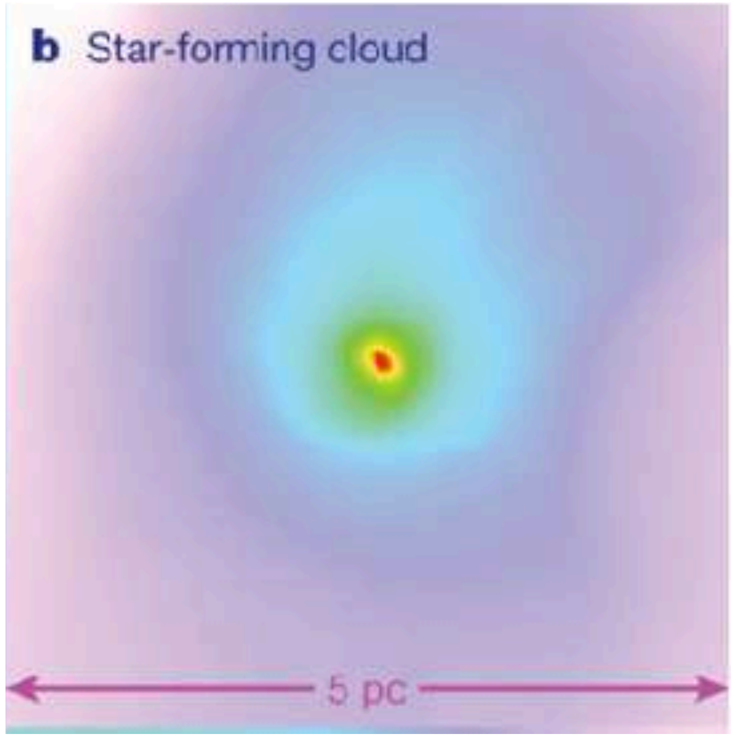
Stability of spherical gas cloud: Jeans criterion

Jeans mass is the critical mass needed
to form a bound object
Gravity vs Gas pressure

$$
\begin{aligned}
M_{\mathrm{J}} &= \frac{\pi^{5/2}}{6}\left(\frac{1}{G}\right)^{3/2} \rho^{-1/2} c_{\mathrm{s}}^{3} \\
&= \frac{\pi^{5/2}}{6}\left(\frac{k}{G}\right)^{3/2}\left(\frac{1}{\mu m_{\mathrm{H}}}\right)^{2} n^{-1/2} T^{3/2} \\
&\approx 50\,\mathrm{M}_{\odot}\,\mu^{-2}\left(\frac{n}{1\mathrm{cm}^{-3}}\right)^{-1/2}\left(\frac{T}{1\mathrm{K}}\right)^{3/2}
\end{aligned}
$$

$\mu \approx 1.22$ for primordial atomic gas $\mu \approx 2.33$ for molecular gas.





Star Formation in a Halo: Cooling

The Jeans Mass scales with redshift and cosmological parameters as:

$$M_{\rm J} \approx 5 \times 10^3 {\rm M}_\odot \left(\frac{\Omega_{\rm m} h^2}{0.14}\right)^{-1/2} \left(\frac{\Omega_{\rm b} h^2}{0.022}\right)^{-3/5} \left(\frac{z+1}{10}\right)^{3/2}$$

The gas needs not only to be bound but also to cool to form stars

$$M_{\rm cool} \approx 6 \times 10^5 {\rm M}_\odot \, h^{-1} \Omega_{\rm m}^{-1/2} \left(\frac{\mu}{1.22}\right)^{-3/2} \left(\frac{z+1}{10}\right)^{3/2}$$

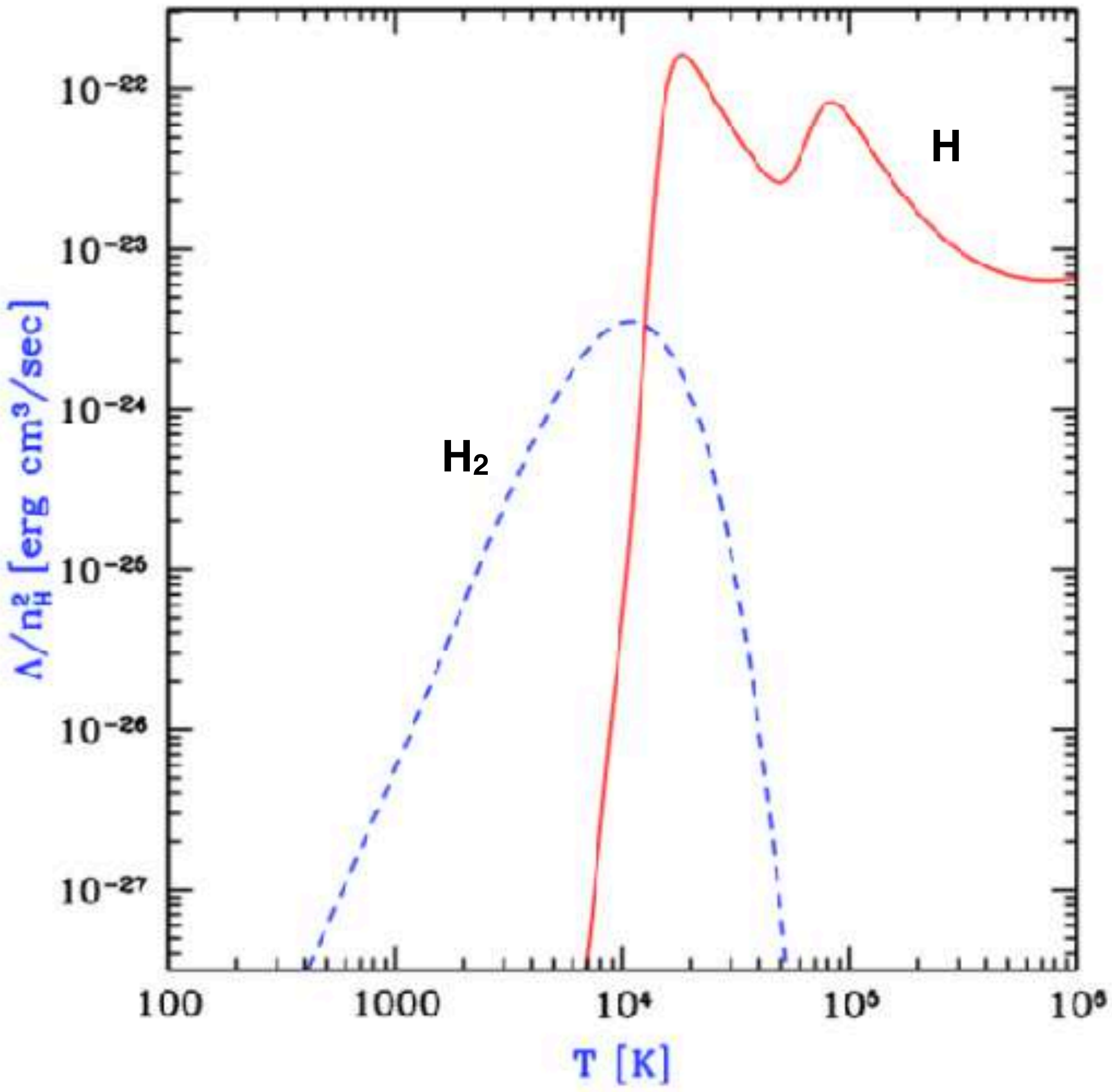
If $M < M_{\rm cool}$ the gas does not cool sufficiently fast (within a Hubble time)

For $z < 40$ $M_{\rm cool} > M_{\rm J}$ ⇒ **Many halos do not form stars**

Cooling rate of Primordial gas

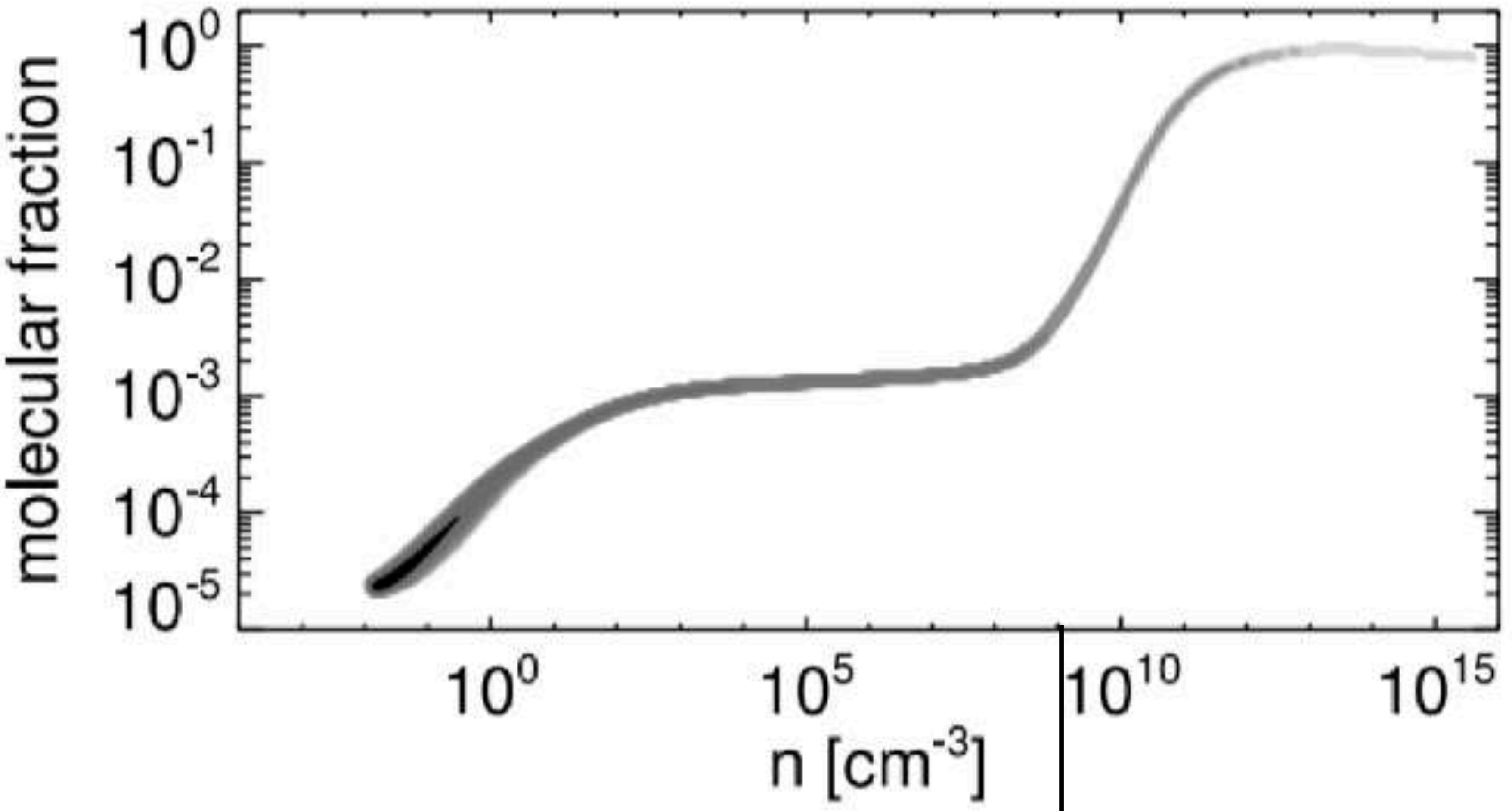
Radiative Cooling for HI alone starts at $T=10^4$K which corresponds to Lyman-alpha transition

H2 has vibrational and rotational modes that allow cooling at lower temperatures



Credit: Barkana & Loeb 2001

Molecular gas: H_2 formation

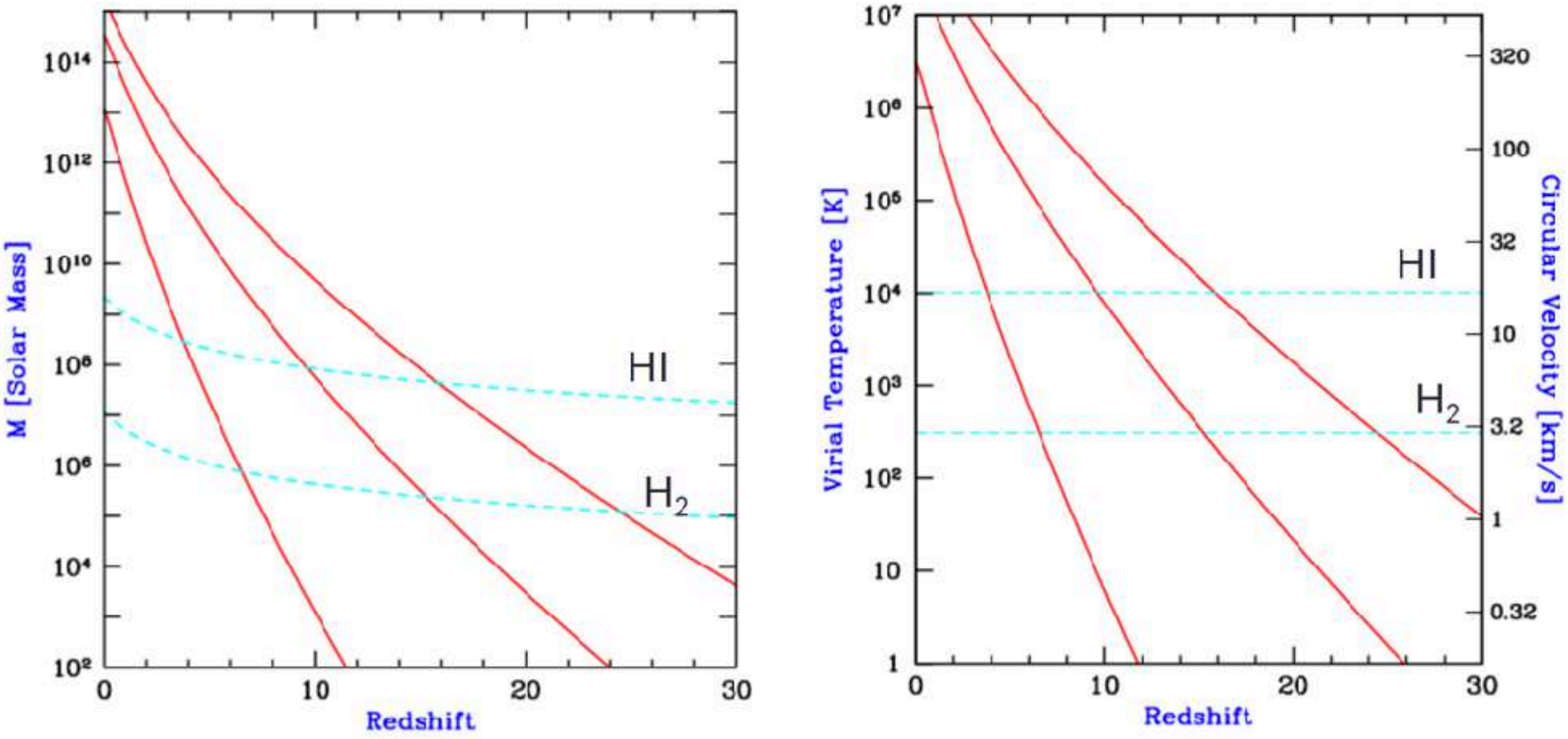


Credit: Yoshida 2006

$$
\begin{aligned}
\mathrm{H} + \mathrm{e}^- &\rightarrow \mathrm{H}^- + \gamma \\
\mathrm{H}^- + \mathrm{H} &\rightarrow \mathrm{H}_2 + \mathrm{e}^-
\end{aligned}
$$

$$\mathrm{H} + \mathrm{H} + \mathrm{H} \rightarrow \mathrm{H}_2 + \mathrm{H}$$

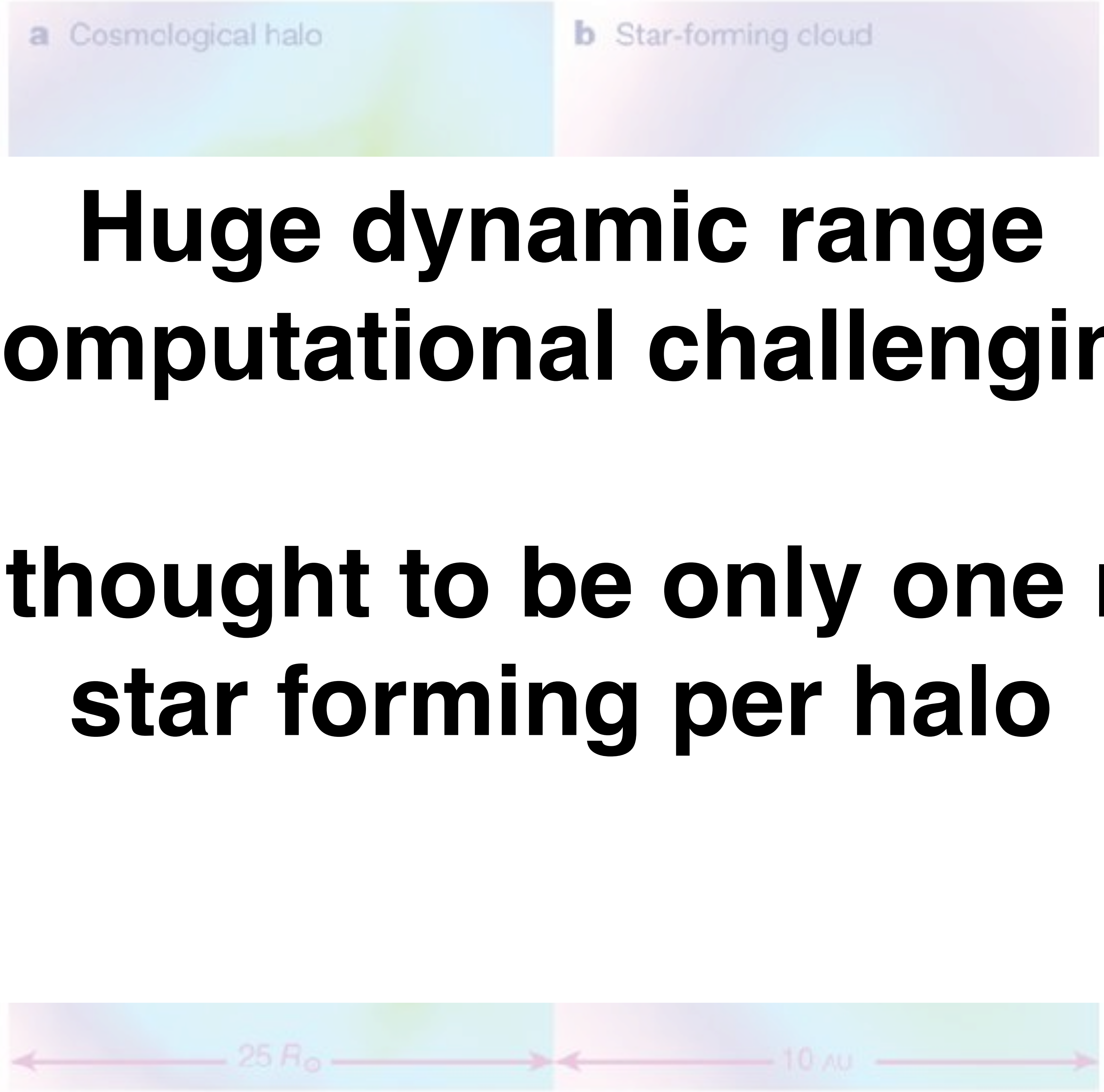
Properties of collapsing halos



$1-\sigma$, $2-\sigma$, and $3-\sigma$ fluctuations (in order from bottom to top)

Credit: Barkana & Loeb 2001

Stellar Formation in a Halo



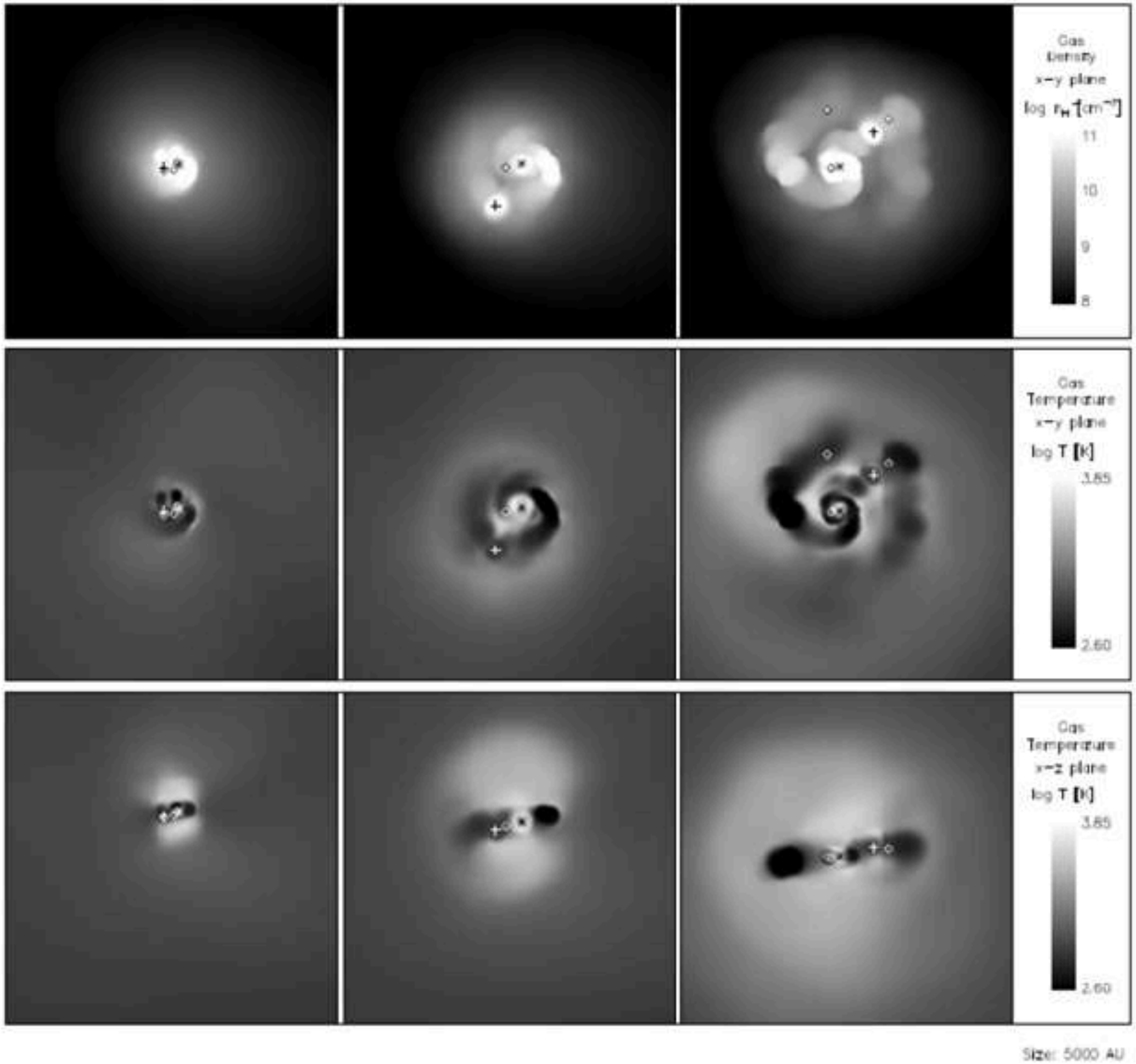
Huge dynamic range
Computational challenging

Initially thought to be only one massive star forming per halo

Projected gas distribution around a primordial protostar.

Bromm *et al.* 2009

Fragmentation



Stacy et al 2009