Technical Documentation of OMEGA Brazil

World Bank Group*

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1 Introduction

This document presents OMEGA - an Open-economy Multi-sector Endogenous-Growth Assessment model. It provides a *complete* and *self-contained* characterization of OMEGA.

For the sake of concise model documentation, we generalize sectors to have them symmetric to each other in terms of functional forms. Then, we impose zero restrictions on specific parameters to obtain the desired sector heterogeneity and to match the coded model.[1]

For the sake of brevity, we do not describe the parameters or variables in the text. The notation is reported in Appendix A.

Since the model includes a stochastic trend arising from the accumulation of human capital H_t, we define $\tilde{X}_t \equiv X_t/H_{t-1}$ for any real variable X_t that grows along this stochastic trend. Note that some model parameters x (mainly policy parameters) may follow the stochastic trend. Then, $\tilde{x} \equiv x_t/H_{t-1}$. The nominal variables also follow a trend of the price level. We divide the trending nominal variables by the consumption deflator and define $\hat{X}_t \equiv X_t/H_{t-1}/P_{C,t}$. Note that for the price of a good or service $P_{X,t}$ and the deflator of a quantity index $P_{Y,t}$, we define $p_{X,t} \equiv P_{X,t}/P_{C,t}$ and $p_{Y,t} \equiv P_{Y,t}/P_{C,t}$, respectively. Note further that bonds and equity shares issued in foreign currency are detrended by the foreign price level but by the domestic stochastic trend.

The structure of the appendix follows the sector decomposition of the economy. The model features high- and low-skilled household sectors (Sections 2.1 and 2.2) as well as four final good producing sectors, four material value-added sectors and four energy-related value-added sectors (Section 3.1). Various additional types of firms provide the microfoundation of important model features: sector-specific firms that homogenize the output of firms participating in voluntary and compliance carbon markets (Section 3.2), a banking sector (Section 3.3), an employment firm that links employment to hours worked (Section 3.4), sector-specific firm value expropriators that allow for endogenous equity risk (Section 3.5), a carbon-offset firm that certifies the households' reforestation-backed carbon sequestration into tradable assets (Section 3.6), input-specific importers (Section 3.7), and foreign-owned domestic firms that sell a capital service to value-added sectors (Section 3.8). The Rest of the World (RW) is discussed in Section 4. The government sector including fiscal policy, monetary policy, low-skilled wage policy, and emission cap policy are discussed in Section 5. Section 6 states all market-clearing conditions. Section 7 states quantity indices and relative price deflators. Finally, Section 8 states the zero restrictions required for OMEGA to collapse to the version used in the code.

2 The household sector (HHS)

Two types of households populate the model economy: *high-skilled labor* HS households and liquidity-constrained *low-skilled labor* LS households. We define $HHS \equiv \{HS, LS\}$.

2.1 High-skilled labor households (HS)

There is a continuum of HS households indexed by $i \in [0,1]$. HS households supply a differentiated high-skilled labor variety i over which they have price-setting power facing a nominal rigidity along the lines of Calvo (1983). As shareholders, they exclusively own all the domestically owned firms (as opposed to foreign owned firms).

HS households engage in deforestation which transforms forest area into land area and releases CO2 into the atmosphere. Land is rented out to the value-added sectors. However, deforestation is illegal and incurs convex costs.

[1] For instance, only the fossil fuel sector uses carbon as a production input. We modeled this by allowing carbon to enter every value-added sector symmetrically, but then restricting the carbon input share to zero for all sectors except fossil fuels.

The model features an endogenous growth mechanism along the lines of Lucas (1988). HS households spend hours on education building human capital, which directly affects labor-embodied productivity. It rewards the household with a higher return on labor, but spending hours on education involves a monetary cost and yields disutility equivalent to supplying labor hours. There are three important external effects on labor-embodied productivity which imply that the decentralized market solution will lead to suboptimal results. First, the average human capital of the economy improves the high-skilled household's ability to accumulate human capital. Second, the stocks of domestically owned and foreign-owned private capital also improve the rate of accumulation of human capital. Finally, human capital also affects the productivity of low-skilled households, which we assume are unable to invest in human capital.

Households derive utility from holding wealth. They hold their wealth as interest-free money balances, bank deposits, government and foreign bonds, as well as equity shares. To motivate interest-free money balances, we follow Coenen et al. (2008) and assume transaction costs that are proportional to consumption and depend on the consumption-based velocity of money.

Government bonds are issued either in domestic (d) or foreign (f) currency. Foreign bonds are issued in foreign currency by the Rest of the World (RW). To capture the average maturity of the debt, we follow Adrian et al. (2022) and assume domestic bonds with geometrically decaying coupon payments.

Following Christoffel et al. (2008) and Coenen et al. (2008), holding bonds or equity shares is associated with financial intermediation costs that capture risk premia included in asset returns. A risk premium introduces a wedge between the return of the risk-free bank deposits and the respective bond or equity return. Risk premia are endogenous. The risk premium on government bonds increase with public debt. The external risk premium increases with the net asset position of the country vis-a-vis the RW. The risk premium on equity shares is modeled in a more elaborate way: As discussed in section 3.5, it is determined by firm-value expropriation risks similar to Germaschewski et al. (2021).

Transaction and financial intermediation costs as well as the expropriated firm value are rebated to the households in a lump-sum manner. This assumption eliminates any corresponding flow of funds in the aggregate but maintains the incentives captured by the first-order conditions (FOCs).

Consumption, education, and asset allocation. Given the budget constraint, transaction and financial intermediation costs as well as the cost of education, HS households maximize inter-temporal utility by choosing optimal inter-temporal paths for consumption, hours spent on education, human capital, deforestation, land, money balances, domestic bank deposits, governments bonds in domestic currency of vintage v, government bonds in foreign currency of vintage v, one-period foreign bonds, and equity shares of the unity continuum of firms in value-added sector $j \in VAS$ with $VAS \equiv \{AG, IN, TR, SV, RY, FY, RF, FF\}$ and in the emission-trading compliance regime $e \in \{c, v\}$ where c indicates the firm must comply with the regulation of the emission trading system (ETS) and v indicates the firm may participate in voluntary carbon markets (as discussed in subsection 3.1). Per-period utility depends on consumption, labor hours, education hours, and wealth. Utility from consumption is subject to external habit formation. Formally, household i seeks to

$$\max_{\left\{Z^{HS,i}_{CH,s}, E^{HS,i}_s, H^i_s, DF^{HS,i}_s, LD^{HS,i}_s, M^{HS,i}_s, D^{HS,i}_{d,s}, \{B^{GV,i}_{d,v,s}\}^t_{v=0}, \{B^{GV,i}_{f,v,s}\}^t_{v=0}, B^{RW,i}_{f,s}, \{\int_0^1 S^{HS,i}_{j,e,k,s} dk\}_{j\in VAS \ \& \ e\in\{c,v\}}\right\}^{\infty}_{s=t}} \mathrm{E}_t \sum_{s=t}^{\infty} \beta^{s-t} U^{HS,i}_s,$$

with per-period utility,

$$\begin{aligned} U^{HS,i}_s = {} & \varepsilon^{HS}_{C,s} \psi_C \left(H_{s-1}\right)^{\varrho} \frac{\left(Z^{HS,i}_{CH,s} - \kappa Z^{HS}_{CH,s-1}\right)^{1-\varrho}}{1-\varrho} - \varepsilon_{H,s} \psi^{HS}_N H_{s-1} \frac{\left(Y^{HS,i}_s\right)^{1+\eta^{HS}}}{1+\eta^{HS}} - \\ & - \varepsilon_{H,s} \psi_H H_{s-1} \frac{\left(E^{HS,i}_s\right)^{1+\eta^{HS}}}{1+\eta^{HS}} + \psi_A \frac{A^{HS,i}_s}{P_{C,s}}, \end{aligned}$$

subject to the inter-temporal budget constraint which equates the uses of funds (consumption plus the consumption tax and transaction costs, education expenditures, end-of-period wealth plus the wealth tax, end-of-period money balances, capital gains and income taxes, labor income taxes, and land taxes) and sources of funds (beginning-of-period bank deposits plus interest, beginning-of-period bonds plus coupon payments net of financial intermediation costs, beginning-of-period equity shares plus dividends net of firm-value expropriation, beginning-of-period money balances, high-skilled labor income, rental income from land, royalties from carbon mining, the profits of the final good sectors and importers, and the lump-sum rebate of transaction and financial intermediation costs as well as expropriated dividends and deforestation penalties),

$$
\begin{aligned}
&\left(1+t_C+\Gamma_{v,s}^{HS,i}\right) P_{Y,s}^{CH} Z_{CH,s}^{HS,i} + P_{E,s} E_s^{HS,i} + \\
&+ \left(1+t_A\right) A_s^i + M_s^{HS,i} + \\
&+ \Psi_{DF,s}^{HS,i} + \\
&+ T_{R,s}^{HS,i} + t_L^{HS} P_{YH,s}^{HS,i} Y_s^{HS,i} H_{s-1}^i + t_{LD}^{HS} LD_{s-1}^{HS,i} + \left(1-\Gamma_{Bd,s-1}^{GV}\right) \sum_{v=1}^{s-1} \left(P_{Bd,v,s}^{GV} + J_{Bd,v,s}^{GV}\right) B_{d,v,s-1}^{GV,i} + \\
&\qquad + \left(1-\Gamma_{Bf,s-1}^{GV}\right) \mathcal{S}_s \sum_{v=1}^{s-1} \left(P_{Bf,v,s}^{GV} + J_{Bf,v,s}^{GV}\right) B_{f,v,s-1}^{GV,i} + \\
&\qquad + \left(1-\Gamma_{Bf,s-1}^{RW}\right) \mathcal{S}_s J_{Bf,s}^{RW} B_{f,s-1}^{RW,i} + \\
&\qquad + \sum_{j\in VAS} \sum_{e\in\{c,v\}} \left(1-\Gamma_{S,s}^{j,e}\right) \int_0^1 \left(P_{S,s}^{j,e,k} + J_s^{j,e,k}\right) S_{j,e,k,s-1}^{HS,i} dk + \\
&\qquad + M_{s-1}^{HS,i} + P_{YH,s}^{HS,i} Y_s^{HS,i} H_{s-1}^i + \frac{r_{LD,s}}{\psi_{LD}} LD_{s-1}^{HS,i} + \\
&\qquad + \left(1-\omega_G^{CB}\right) \int_0^1 P_{Y,s}^{CB,k} Y_s^{CB,k} dk \\
&\qquad + \sum_{j\in FGS} \int_0^1 J_s^{j,k} dk + \sum_{j\in VAS} \int_0^1 J_s^{j,kM} dk + \Psi_s^{HS},
\end{aligned}
$$

where

$$
\Gamma_{v,s}^{HS,i} = \tau_{v1}^{HS} v_s^{HS,i} + \frac{\tau_{v2}^{HS}}{v_s^{HS,i}} - 2\sqrt{\tau_{v1}^{HS}\tau_{v2}^{HS}}
$$

are the transaction costs,

$$
v_s^{HS,i} = (1+t_C)\frac{P_{Y,s}^{CH} Z_{CH,s}^{HS,i}}{M_s^{HS,i}}
$$

is the velocity of money,

$$
\begin{aligned}
A_s^{HS,i} = {} & D_{d,s}^{HS,i} + \sum_{v=1}^{s} P_{Bd,v,s}^{GV} B_{d,v,s}^{GV,i} + \mathcal{S}_s \sum_{v=1}^{s} P_{Bf,v,s}^{GV} B_{f,v,s}^{GV,i} + \mathcal{S}_s P_{Bf,s}^{RW} B_{f,s}^{RW,i} + \\
& + \sum_{j\in VAS} \sum_{e\in\{c,v\}} \int_0^1 P_{S,s}^{j,e,k} S_{j,e,k,s}^{HS,i} dk
\end{aligned}
$$

is end-of-period financial wealth comprising domestic bank deposits, government bonds in domestic currency of all vintages, government bonds in foreign currency of all vintages expressed in domestic currency, foreign one-period bonds expressed in domestic currency, and equity shares of the value-added firms,

$$
\Psi_{DF,s}^{HS,i} = \frac{\psi_{DF}^{HS}}{\varepsilon_{DF,s}\phi_{DF}^{HS}} \left(DF_s^{HS,i}\right)^{\phi_{DF}^{HS}}
$$

are convex costs of illegal deforestation which are rebated to the HS sector in a lump-sum manner,

$$T_{R,s}^{HS,i} = t_R \begin{bmatrix} \left(R_{Dd,s-1} - 1\right) D_{d,s-1}^{HS,i} + \\ + \left(1 - \Gamma_{Bd,s-1}^{GV}\right) \sum_{v=1}^{s-1} \left(P_{Bd,v,s}^{GV} - P_{Bd,v,s-1}^{GV} + J_{Bd,v,s}^{GV}\right) B_{d,v,s-1}^{GV,i} + \\ + \left(1 - \Gamma_{Bf,s-1}^{GV}\right) \mathcal{S}_s \sum_{v=1}^{s-1} \left(P_{Bf,v,s}^{GV} - P_{Bf,v,s-1}^{GV} + J_{Bf,v,s}^{GV}\right) B_{f,v,s-1}^{GV,i} + \\ + \left(1 - \Gamma_{Bf,s-1}^{RW}\right) \mathcal{S}_s \left(1 - P_{Bf,s-1}^{RW}\right) B_{f,s-1}^{RW,i} + \\ + \sum_{j \in VAS} \sum_{e \in \{c,v\}} \left(1 - \Gamma_{S,s}^{j,e}\right) \int_0^1 \left(P_{S,s}^{j,e,k} - P_{S,s-1}^{j,e,k} + J_s^{j,e,k}\right) S_{j,e,k,s-1}^{HS,i} dk + \\ + \left(1 - \omega_G^{CB}\right) \mathcal{S}_s \int_0^1 P_{Y,s}^{CBW,k} Y_s^{CB,k} dk + \left(\Psi_s^{HS} - \Gamma_{v,s}^{HS} P_{Y,s}^{CH} Z_{CH,s}^{HS}\right) \end{bmatrix}$$

are taxes on capital gains, dividends, and interest income,

$$\Gamma_{W,s} = \tau_W \left(\varepsilon_{\Gamma W,s}^{\frac{1}{\tau_W}} \exp\left(\frac{\mathcal{S}_s \left(B_{Bf,s}^{RW} - B_{Bf,s}^{GV} - D_{f,RW,s}^{RW}\right)}{P_{C,s} \tilde{Y}_s} \right) - 1 \right)^{\phi_W}$$

is the risk premium on foreign bonds, and

$$\begin{aligned} \Psi_s^{HS} &= \Gamma_{v,s}^{HS} P_{Y,s}^{CH} Z_{CH,s}^{HS} + \Gamma_{Bd,s-1}^{GV} \sum_{v=1}^{s-1} \left(P_{Bd,v,s}^{GV} + J_{Bd,v,s}^{GV}\right) B_{d,v,s-1}^{GV} + \\ &+ \Gamma_{Bf,s-1}^{GV} \mathcal{S}_s \sum_{v=1}^{s-1} \left(P_{Bf,v,s}^{GV} + J_{Bf,v,s}^{GV}\right) B_{f,v,s-1}^{GV} + \Gamma_{Bf,s-1}^{RW} \mathcal{S}_s J_{Bf,s}^{RW} B_{f,s-1}^{RW} + \\ &+ \sum_{j \in VAS} \sum_{e \in \{c,v\}} \Gamma_{S,s}^{j,e} \int_0^1 \left(P_{S,s}^{j,e,k} + J_s^{j,e,k}\right) S_{j,e,k,s-1}^{HS} dk + \Psi_{DF,s}^{HS} \end{aligned}$$

is the lump-sum refund of consumption transaction costs and financial intermediation costs including the expropriated dividends, and deforestation costs, as well as subject to the law of motion of human capital accumulation

$$\begin{aligned} H_s^i &= (1 - \delta_H) H_{s-1}^i + \\ &+ \varepsilon_{H,s} \psi_H \left(E_s^{HS,i} H_{s-1}^i\right)^{\alpha_H} \left(\left(\sum_{k \in VAS} \sum_{e \in \{c,v\}} K_{Pg,s-1}^{k,e} \right)^{\alpha_{Kd}} \left(K_{Pg,s-1}^{FV}\right)^{\alpha_{Kf}} \left(H_{s-1}\right)^{1-\alpha_{Kd}-\alpha_{Kf}} \right)^{1-\alpha_H}. \end{aligned}$$

and subject to the law of motion of HS land

$$LD_s^{HS,i} = DF_s^{HS,i} + (1 - \delta_{LD}) LD_{s-1}^{HS,i}.$$

The first-order condition (FOC) w.r.t. $Z_{CH,s}^{HS,i}$ relates the shadow price of an additional unit of income to the marginal utility. Aggregating over all households and normalizing by human capital yields

$$\left(1 + t_C + \Gamma_{v,t}^{HS} + \Gamma_{v,t}^{\prime HS} v_t^{HS}\right) p_{Y,t}^{CH} \lambda_t^{HS} = \varepsilon_{C,t}^{HS} \psi_C \left(\tilde{Z}_{CH,t}^{HS} - \kappa \tilde{Z}_{CH,t-1}^{HS} / g_{t-1}\right)^{-\varrho} \tag{1}$$

where

$$\Gamma_{v,t}^{HS} = \tau_{v1}^{HS} v_t^{HS} + \frac{\tau_{v2}^{HS}}{v_t^{HS}} - 2\sqrt{\tau_{v1}^{HS} \tau_{v2}^{HS}} \tag{2}$$

$$\Gamma'^{HS}_{v,t} = \tau^{HS}_{v1} - \frac{\tau^{HS}_{v2}}{\left(v^{HS}_t\right)^2} \tag{3}$$

$$v^{HS}_t = (1 + t_C)\frac{p^{CH}_{Y,t}\tilde{Z}^{HS}_{CH,t}}{\hat{M}^{HS}_t}. \tag{4}$$

Eq. (1) states that the shadow price of an additional unit of income, λ^{HS}_t, is equal to the marginal utility of the amount of consumption which this additional unit of income can buy after paying consumption taxes and transaction costs. The parameter κ controls the habit persistence. We introduce and calibrate preference scaling parameters such as ψ_C to have relations between core variables match the empirical data. The elasticity of inter-temporal substitution $1/\varrho$ controls how strongly consumption is postponed to the future in response to an increase in the real interest rate.

The FOC w.r.t. education hours, $E^{HS,i}_s$, equates the marginal utility-costs of one hour spent on education (disutility from spending time on education) to the marginal utility of that hour (the utility from additional consumption in the future allowed by higher wage income due to an increased productivity). After aggregation and detrending, the FOC reads

$$\varepsilon_{H,t}\psi_H\left(E^{HS}_t\right)^{\eta^{HS}} + \lambda^{HS}_t\tilde{p}_{E,t} = Q^{HS}_{H,t}\varepsilon_{H,t}\psi_H\alpha_H\left(E^{HS}_t\right)^{\alpha_H-1}\left(\tilde{K}^{ext}_{t-1}/g_{t-1}\right)^{1-\alpha_H} \tag{5}$$

where

$$\tilde{K}^{ext}_t \equiv \left(\sum_{k\in VAS}\sum_{e\in\{c,v\}}\tilde{K}^{k,e}_{Pg,t}\right)^{\alpha_{Kd}}\left(\tilde{K}^{FV}_{Pg,t}\right)^{\alpha_{Kf}} \tag{6}$$

is the capital externality on human capital accumulation. Noting that the investment in education is identical across households leading to identical stocks of human capital, the FOC w.r.t. human capital, H^i_s, implies

$$Q^{HS}_{H,t} = \beta\lambda^{HS}_{t+1}\tilde{p}^{HS}_{Y,t+1}Y^{HS}_{t+1} + \beta\mathrm{E}_tQ^{HS}_{H,t+1}\left((1-\delta_H) + \varepsilon_{H,t+1}\psi_H\alpha_H\left(E^{HS}_{t+1}\right)^{\alpha_H}\left(\tilde{K}^{ext}_t/g_t\right)^{1-\alpha_H}\right) \tag{7}$$

where $P^{HS,i}_{Y,t} = P^{HS,i}_{YH,t}H^i_{t-1}$ is the effective wage for an hour of labor supplied. The FOC states that the shadow price of human capital in t is the discounted consumption-utility of the additional wage income in $t+1$ plus the discounted value of the stock of human capital in $t+1$ which is reduced by depreciation and increased by education.

Detrending the law of motion of human capital yields the endogenous, labor-embodied productivity growth rate, $g_t \equiv H_t/H_{t-1}$, as

$$g_t = (1-\delta_H) + \varepsilon_{H,t}\psi_H\left(E^{HS}_t\right)^{\alpha_H}\left(\tilde{K}^{ext}_{t-1}/g_{t-1}\right)^{1-\alpha_H}. \tag{8}$$

The FOC w.r.t. deforestation $DF^{HS,i}_s$, measured in millions of hectares (mha), implies

$$\tilde{Q}^{HS}_{LD,t} = \frac{\tilde{\psi}^{HS}_{DF}}{\varepsilon_{DF,t}}\lambda^{HS}_t\left(DF^{HS,i}_t\right)^{\phi^{HS}_{DF}-1} \tag{9}$$

which equates the value of HS-land (which constitutes the marginal value of deforestation) and the marginal cost of deforestation expressed in units of utility.[2]

[2] Note that, facing a trending value of land, the policy stance on deforestation needs to permanently tighten in order to keep deforestation constant.

The FOC w.r.t. HS-land $LD_s^{HS,i}$ implies the value of land as

$$\tilde{Q}_{LD,t}^{HS} = \beta \mathrm{E}_t \left(\lambda_{t+1}^{HS} \left(\frac{\tilde{r}_{LD,t+1}}{\psi_{LD}} g_t - \tilde{t}_{LD}^{HS} \right) + (1 - \delta_{LD}) \tilde{Q}_{LD,t+1}^{HS} g_t \right) \tag{10}$$

which states recursively that the value of HS-land is the discounted rental income expressed in units of utility plus the next period's discounted value of the fraction of HS-land which natural forces have not converted back to forest area (non-depreciated land).[3]

The aggregated law of motion of HS-land is

$$LD_t^{HS} = DF_t^{HS} + (1 - \delta_{LD}) LD_{t-1}^{HS}. \tag{11}$$

The FOC w.r.t. money balances, $M_s^{HS,i}$, determines optimal money balances and implies

$$\left(1 - \Gamma_t'^{HS} \frac{\left(v_t^{HS}\right)^2}{1 + t_C} \right) \lambda_t^{HS} = \beta \mathrm{E}_t \frac{1}{\Pi_{C,t+1}} \lambda_{t+1}^{HS} \tag{12}$$

which equates the marginal disutility of an additional unit of liquidity (forgone consumption utility) and its marginal utility (lower transaction costs and higher budget in the next period).

The FOC w.r.t. safe domestic bank deposits, $D_{d,s}^{HS,i}$, implies

$$(1 + t_A)\lambda_t^{HS} = \psi_A + \beta \mathrm{E}_t \frac{R_{Dd,t} - t_R(R_{Dd,t} - 1)}{\Pi_{C,t+1}} \lambda_{t+1}^{HS} \tag{13}$$

which is a variant of the Euler equation and determines how the households smooth consumption optimally. It equates the marginal disutility of an additional unit of wealth (forgone consumption utility) and its marginal utility (direct utility derived from holding wealth and higher budget in the next period). The utility scaling parameter drives a wedge into this otherwise standard Euler relationship because in the current model financial wealth in itself yields utility.

The FOC w.r.t. government bonds of vintage v issued in domestic currency, $B_{d,v,s}^{GV,i}$, implies

$$\begin{aligned} \psi_A \frac{P_{Bd,v,t}^{GV}}{P_{C,t}} - \frac{\lambda_t^{HS,i}}{P_{C,t}} (1 + t_A) P_{Bd,v,t}^{GV} + \beta \mathrm{E}_t \frac{\lambda_{t+1}^{HS,i}}{P_{C,t+1}} \left(1 - \Gamma_{Bd,t}^{GV}\right) \left(P_{Bd,v,t+1}^{GV} + J_{Bd,v,t+1}^{GV}\right) - \\ - \beta \mathrm{E}_t \frac{\lambda_{t+1}^{HS,i}}{P_{C,t+1}} t_R \left(1 - \Gamma_{Bd,t}^{GV}\right) \left(P_{Bd,v,t+1}^{GV} - P_{Bd,v,t}^{GV} + J_{Bd,v,t+1}^{GV}\right) = 0 \\ \forall v = 1, \ldots, t. \end{aligned}$$

The yield-to-maturity $R_{Bd,t}^{GV}$ of a perpetual government bond satisfies

$$P_{Bd,v,t}^{GV} = \sum_{s=t}^{\infty} \frac{J_{Bd,v,s+1}^{GV}}{{R_{Bd,t}^{GV}}^{s-t+1}}$$

which implies for the initial price of a bond issued in v that

$$P_{Bd,v,v}^{GV} = \sum_{s=v}^{\infty} \frac{J_{Bd,v,s+1}^{GV}}{{R_{Bd,v}^{GV}}^{s-v+1}}.$$

[3] For a well-defined steady state of deforestation to exist, we assume a small natural rate of reforestation δ_{LD}. Note further that, on the supply side, the unit of land is millions of hectares. For convenience, we choose the unit of land on the demand side such that $r_{LD,t} = 1$ at the steady state. The conversion rate ψ_{LD} is restricted accordingly.

Note that we assume geometric decay of coupons as in $J^{GV}_{Bd,v,s+1} = {\zeta^{GV}_{Bd}}^{s-v} J^{GV}_{Bd,v,v+1}$ where $1-\zeta^{GV}_{Bd}$ is the rate of geometric decay. Substituting this into the previous equation and solving the geometric series yields

$$
\begin{aligned}
P^{GV}_{Bd,v,v} &= \sum_{s=v}^{\infty} \frac{{\zeta^{GV}_{Bd}}^{s-v} J^{GV}_{Bd,v,v+1}}{{R^{GV}_{Bd,v}}^{s-v+1}} \\
P^{GV}_{Bd,v,v} &= \sum_{s=v}^{\infty} \left(\frac{\zeta^{GV}_{Bd}}{R^{GV}_{Bd,v}} \right)^{s-v} \frac{J^{GV}_{Bd,v,v+1}}{R^{GV}_{Bd,v}} \\
P^{GV}_{Bd,v,v} &= \frac{J^{GV}_{Bd,v,v+1}}{R^{GV}_{Bd,v}} \frac{1}{1-\frac{\zeta^{GV}_{Bd}}{R^{GV}_{Bd,v}}} \\
(R^{GV}_{Bd,v} - \zeta^{GV}_{Bd}) P^{GV}_{Bd,v,v} &= J^{GV}_{Bd,v,v+1} \\
J^{GV}_{Bd,v,s+1} &= \left(R^{GV}_{Bd,v} - \zeta^{GV}_{Bd}\right) {\zeta^{GV}_{Bd}}^{s-v} P^{GV}_{Bd,v,v}
\end{aligned}
$$

Assuming that each bond raises one unit of domestic currency upon issue, $P^{GV}_{Bd,v,v} = 1$, we obtain a relationship between the bond return and the rate of decay as

$$
1 = \sum_{s=v}^{\infty} \frac{\left(R^{GV}_{Bd,v} - \zeta^{GV}_{Bd}\right) {\zeta^{GV}_{Bd}}^{s-v}}{{R^{GV}_{Bd,v}}^{s-v+1}}.
$$

The FOC for the bond issued in $v = t$ then implies

$$
\begin{aligned}
&(1+t_A)\lambda^{HS}_t = \psi_A + \\
&+ \beta(1-\Gamma^{GV}_{Bd,t}) \mathrm{E}_t \frac{\left(R^{GV}_{Bd,t} - \zeta^{GV}_{Bd}\right)\left(1 + \frac{\zeta^{GV}_{Bd}}{R^{GV}_{Bd,t+1} - \zeta^{GV}_{Bd}}\right) - t_R \left(\left(R^{GV}_{Bd,t} - \zeta^{GV}_{Bd}\right)\left(1 + \frac{\zeta^{GV}_{Bd}}{R^{GV}_{Bd,t+1} - \zeta^{GV}_{Bd}}\right) - 1\right)}{\Pi_{C,t+1}} \lambda^{HS}_{t+1}.
\end{aligned}
\tag{14}
$$

It is easy to see that this equation collapses to the FOC of a standard one-period bond if the rate of decay is 1, i.e. $\zeta^{GV}_{Bd} = 0$. The risk premium on government bonds issued in domestic currency satisfies

$$
\Gamma^{GV}_{Bd,t} = 0. \tag{15}
$$

Applying the equivalent reasoning, the FOC w.r.t. government bonds of vintage $v = t$ issued in foreign currency, $B^{GV,i}_{f,v,s}$, implies

$$
\begin{aligned}
&(1+t_A)\lambda^{HS}_t = \psi_A + \\
&+ \beta(1-\Gamma^{GV}_{Bf,t}) \mathrm{E}_t \frac{\left(R^{GV}_{Bf,t} - \zeta^{GV}_{Bf}\right)\left(1 + \frac{\zeta^{GV}_{Bf}}{R^{GV}_{Bf,t+1} - \zeta^{GV}_{Bf}}\right) - t_R \left(\left(R^{GV}_{Bf,t} - \zeta^{GV}_{Bf}\right)\left(1 + \frac{\zeta^{GV}_{Bf}}{R^{GV}_{Bf,t+1} - \zeta^{GV}_{Bf}}\right) - 1\right)}{\Pi_{C,t+1}} \lambda^{HS}_{t+1} \Delta_{\mathcal{S},t+1}
\end{aligned}
\tag{16}
$$

where

$$
\Gamma^{GV}_{Bf,t} = 0 \tag{17}
$$

is the risk premium on government bonds issued in foreign currency, where

$$
\Delta_{\mathcal{S},t} \equiv \frac{\mathcal{S}_t}{\mathcal{S}_{t-1}} = \frac{\mathcal{E}_t}{\mathcal{E}_{t-1}} \frac{\Pi_{C,t}}{\Pi^*_{C,t}} \tag{18}
$$

is the rate of depreciation of the nominal exchange rate with the real exchange rate defined as $\mathcal{E}_t = \mathcal{S}_t P^*_{C,t} / P_{C,t}$, and where the yield-to-maturity $R^{GV}_{Bf,t}$ satisfies

$$
P^{GV}_{Bf,v,t} = \sum_{s=t}^{\infty} \frac{J^{GV}_{Bf,v,s+1}}{{R^{GV}_{Bf,t}}^{s-t+1}}.
$$

The FOC w.r.t. foreign one-period bonds, $B_{f,s}^{RW,i}$, denominated in foreign currency implies

$$(1+t_A)\lambda_t^{HS} = \psi_A + \beta(1-\Gamma_{W,t})\mathrm{E}_t \frac{R_{Bf,t}^{RW} - t_R(R_{Bf,t}^{RW}-1)}{\Pi_{C,t+1}} \lambda_{t+1}^{HS} \Delta_{\mathcal{S},t+1} \tag{19}$$

where the bond return $R_{Bf,t}^{RW}$ satisfies

$$1 = \frac{J_{Bf,t+1}^{RW}}{R_{Bf,t}^{RW}}.$$

Note that we consider one-period bonds that raise one unit of foreign currency upon issue, $P_{Bf,t}^{RW} = 1$. Eq. (19) is the risk-adjusted uncovered interest parity condition and it pins down the change of the nominal exchange rate which adjusts to eliminate any arbitrage opportunity making the household indifferent between holding foreign debt or any other asset. The external risk premium as a function of aggregated variables reads

$$\Gamma_{W,t} = \tau_W \left(\varepsilon_{\Gamma W,t}^{\frac{1}{\tau_W}} \exp\left(\frac{\mathcal{E}_t \left(\hat{B}_{Bf,s}^{RW} - \hat{B}_{Bf,s}^{GV} - \sum_{k\in VAS} \sum_{e\in\{c,v\}} \hat{L}_{f,s}^{k,e} \right)}{\tilde{Y}_t} \right) - 1 \right)^{\phi_W}. \tag{20}$$

The FOCs w.r.t. equity shares of sectors $j \in VAS$ and regimes $e \in \{c,v\}$, $S_{j,e,s}^{HS,i}$, are

$$\begin{aligned}(1+t_A)\lambda_t^{HS} = \psi_A + \beta\mathrm{E}_t(1-\Gamma_{S,t+1}^{j,e}) \frac{R_{S,t}^{j,e} - t_R(R_{S,t}^{j,e}-1)}{\Pi_{C,t+1}} \lambda_{t+1}^{HS} \\ \forall j \in VAS \text{ and } \forall e \in \{c,v\}\end{aligned} \tag{21}$$

where the sector-specific and emission-trading-regime-specific equity returns $R_{S,t}^{j,e}$ satisfy

$$\int_0^1 P_{S,t}^{j,e,k} dk = \frac{\int_0^1 P_{S,t+1}^{j,e,k} + J_{t+1}^{j,e,k} dk}{R_{S,t}^{j,e}}.$$

The FOCs eliminate arbitrage opportunities and pin down the inter-temporal evolution of asset prices for each sector's equity shares such that households are indifferent, at the margin, between holding shares of a sector's equity or any other asset. Equity returns exceed the risk-free bond rate by an equity risk premium, $\Gamma_{S,t}$, to be specified in section 3.5.

By the definition above, the sector-specific equity returns can be expressed in aggregated and detrended variables and solved for the end-of-period asset price. We obtain the asset price equation as

$$\begin{aligned}\hat{p}_{S,t}^{k,e} = \mathrm{E}_t \left(\frac{R_{S,t}^{k,e}}{\Pi_{C,t+1}} \right)^{-1} \left(\hat{p}_{S,t+1}^{k,e} + \hat{J}_{t+1}^{k,e} \right) g_t \\ \forall k \in VAS \text{ and } \forall e \in \{c,v\}.\end{aligned} \tag{22}$$

For aggregation over the various bond vintages and heterogeneous VAS firms, we impose the following bond-market market clearing conditions and portfolio assumptions: First, the income from government bonds in domestic currency is equal to the corresponding debt service payments of the government. This implies that only households hold government bonds in domestic currency. Second, households hold no domestic government bonds in foreign currency. They are held by the RW only. Third, households are the only domestic agents holding foreign bonds. Finally, the number of equity shares for each firm is normalized to one.

$$\sum_{v=1}^{s-1}\left(P^{GV}_{Bd,v,s}+J^{GV}_{Bd,v,s}\right)\int_0^1 B^{GV,i}_{d,v,s-1}di = R^{GV,e}_{Bd,s-1}B^{GV}_{d,s-1}$$

$$\int_0^1 B^{GV,i}_{f,v,s}di = 0$$

$$\int_0^1 B^{RW,i}_{f,s}di = B^{RW}_{f,s}$$

$$\int_0^1 S^{HS,i}_{j,e,k,s}dk = 1 \ \forall j \in VAS \text{ and } \forall e \in \{c,v\}.$$

Under these assumptions and conditions, the budget constraint can be expressed as

$$\begin{aligned}
&p^{CH}_{Y,t}\tilde{Z}^{HS}_{CH,t}+\tilde{p}_{E,t}E^{HS}_t+\hat{A}^{HS}_t+\hat{M}^{HS}_t+\\
&+\hat{T}^{HS}_{C,t}+\hat{T}^{HS}_{A,t}+\hat{T}^{HS}_{L,t}+\hat{T}^{HS}_{R,t}+\hat{T}^{HS}_{LD,t}=\frac{R_{Dd,t-1}}{\Pi_{C,t}}\hat{D}^{HS}_{d,t-1}/g_{t-1}+\frac{R^{GV,e}_{Bd,t-1}}{\Pi_{C,t}}\hat{B}^{GV}_{d,t-1}/g_{t-1}+\\
&\qquad+\frac{R^{RW}_{Bf,t-1}}{\Pi^{RW}_{C,t}}\mathcal{E}_t\hat{B}^{RW}_{f,t-1}/g_{t-1}+\sum_{j\in VAS}\sum_{e\in\{c,v\}}\frac{R^{j,e}_{S,t-1}}{\Pi_{C,t}}\tilde{p}^{j,e}_{S,t-1}/g_{t-1}+\\
&\qquad+\frac{1}{\Pi_{C,t}}\hat{M}^{HS}_{t-1}/g_{t-1}+\tilde{p}^{HS}_{Y,t}Y^{HS}_t+\frac{\tilde{r}_{LD,t}}{\psi_{LD}}LD^{HS}_{t-1}+\left(1-\omega^{CB}_G\right)\mathcal{E}_tp^{CB}_{Y,t}\tilde{Y}^{CB}_t\\
&\qquad+\sum_{j\in FGS}\hat{J}^j_t+\sum_{j\in VAS}\hat{J}^{jM}_t
\end{aligned}\tag{23}$$

where $\hat{T}^{HS}_{C,t}$, $\hat{T}^{HS}_{A,t}$, $\hat{T}^{HS}_{L,t}$, $\hat{T}^{HS}_{R,t}$, and $\hat{T}^{HS}_{LD,t}$ are taxes as specified in section 5.

High-skilled wage setting. HS households supply differentiated labor varieties consisting of *hours weighted by units of human capital*. Hence, they have market power over the price they charge for supplying their labor variety. Production on the firm side, however, uses homogeneous high-skilled labor services. To reconcile the two, we follow standard practice and suppose a high-skilled labor aggregator combines differentiated labor to form a homogeneous high-skilled labor service.

A perfectly competitive high-skilled labor aggregator purchases differentiated high-skilled labor varieties from the high-skilled labor households. Taking as given the price $P^{HS,i}_{YH,t}$ for the labor variety i and the price of homogeneous human-capital weighted high-skilled labor $P^{HS}_{YH,t}$, the representative firm chooses the optimal demand of the labor variety $Z^{HS,i}_t H^i_{t-1}$ in order to minimize costs of producing the homogeneous labor service $Y^{HS}_t H_{t-1}$. The labor aggregator's demand schedule for the high-skilled labor variety can be obtained from the following cost minimization problem:

$$\min_{Z^{HS,i}_t H^i_{t-1}}\int_0^1 P^{HS,i}_{YH,t}Z^{HS,i}_t H^i_{t-1}di$$

subject to a Dixit-Stiglitz aggregator

$$Y^{HS}_t H_{t-1}=\left(\int_0^1\left(Z^{HS,i}_t H^i_{t-1}\right)^{\frac{\sigma^{HS}_{P,t}-1}{\sigma^{HS}_{P,t}}}di\right)^{\frac{\sigma^{HS}_{P,t}}{\sigma^{HS}_{P,t}-1}},$$

where $\sigma^{HS}_{P,t}>1$ is the elasticity of substitution between different high-skilled labor varieties. Noting that the Lagrangian multiplier of the constraint is equal to the aggregate human-capital weighted high-skilled wage index, $P^{HS}_{YH,t}$, the FOC implies

$$Z^{HS,i}_t H^i_{t-1} = \left(\frac{P^{HS,i}_{YH,t}}{P^{HS}_{YH,t}}\right)^{-\sigma^{HS}_{P,t}} Y^{HS}_t H_{t-1}.$$

This demand function for labor variety i imposes a constraint to the high-skilled labor household's problem of choosing the optimal price for its labor variety $P^{HS,i}_{Y,t}$ such that the households supply of its variety matches the aggregator's demand, $Y^{HS,i}_t = Z^{HS,i}_t$.

Following Calvo (1983), Smets and Wouters (2003), and Christoffel et al. (2008), only a fraction $1-\xi^{HS}_P$ of HS households can reset their wage per hour and unit of capital human capital employed, $P^{\#HS,i}_{YH,t}$, in any given period t. The remaining households index their wage per hour and per unit of human capital according to the rule

$$P^{HS,i}_{YH,t} = \Theta^{HS}_{t-1} P^{HS,i}_{YH,t-1}$$

where

$$\Theta^{HS}_t = \left(\Pi^{HS}_{Y,t}/g_{t-1}\right)^{\omega^{HS}} \left(\bar{\Pi}_C\right)^{1-\omega^{HS}}. \tag{24}$$

A household i which is able to reset its wage $P^{\#HS,i}_{YH,t}$ in period t solves the following optimization problem:

$$\max_{P^{\#HS,i}_{YH,t}} \mathrm{E}_t \sum_{s=t}^{\infty} \left(\beta\xi^{HS}_P\right)^{s-t} U^{HS,i}_s$$

where

$$U^{HS,i}_s = \frac{\lambda^{HS,i}_s}{P_{C,s}} \left(1-t^{HS}_L\right) \prod_{r=t+1}^{s} \Theta^{HS}_{r-1} P^{\#HS,i}_{YH,t} \left(\frac{\prod_{r=t+1}^{s} \Theta^{HS}_{r-1} P^{\#HS,i}_{YH,t}}{P^{HS}_{YH,s}}\right)^{-\sigma^{HS}_{P,s}} Y^{HS}_s H_{s-1} -$$
$$- \varepsilon_{H,s}\psi^{HS}_N H_{s-1} \frac{\left(\left(\frac{\prod_{r=t+1}^{s} \Theta^{HS}_{r-1} P^{\#HS,i}_{YH,t}}{P^{HS}_{YH,s}}\right)^{-\sigma^{HS}_{P,s}} Y^{HS}_s \frac{H_{s-1}}{H^i_{s-1}}\right)^{1+\eta^{HS}}}{1+\eta^{HS}}$$

The FOC w.r.t. the high-skilled wage, $P^{\#HS,i}_{YH,t}$, implies

$$\left(\tilde{p}^{\#HS}_{Y,t}\right)^{1+\sigma^{HS}_{P,t}\eta^{HS}} = \frac{\tilde{A}^k_{1,t}}{\tilde{A}^k_{2,t}} \tag{25}$$

where

$$\begin{aligned}\tilde{A}^k_{1,t} &= \sigma^{HS}_{P,t}\varepsilon_{H,t}\psi^{HS}_N \left(\tilde{p}^{HS}_{Y,t}\right)^{\sigma^{HS}_{P,t}\left(1+\eta^{HS}\right)} \left(Y^{HS}_t\right)^{1+\eta^{HS}} \\ &\quad + \beta\xi^k_P \mathrm{E}_t \left(\Theta^{HS}_t\right)^{-\sigma^{HS}_{P,t+1}\left(1+\eta^{HS}\right)} \left(\Pi_{C,t+1}\right)^{\sigma^{HS}_{P,t+1}\left(1+\eta^{HS}\right)} \tilde{A}^k_{1,t+1} g_t\end{aligned} \tag{26}$$

$$\begin{aligned}\tilde{A}^k_{2,t} &= \left(\sigma^{HS}_{P,t}-1\right)\lambda^{HS,i}_t \left(1-t^{HS}_L\right) \left(\tilde{p}^{HS}_{Y,t}\right)^{\sigma^{HS}_{P,t}} Y^{HS}_t \\ &\quad + \beta\xi^k_P \mathrm{E}_t \left(\Theta^{HS}_t\right)^{-\left(\sigma^{HS}_{P,t+1}-1\right)} \left(\Pi_{C,t+1}\right)^{\left(\sigma^{HS}_{P,t+1}-1\right)} \tilde{A}^k_{2,t+1} g_t.\end{aligned} \tag{27}$$

We used the fact that all households able to do so set the same wage and we approximated $\Pi^{1+\sigma^{HS}_{P,t}}_{C,t} \simeq P^{1+\sigma^{HS}_{P,t}}_{C,t} / P^{1+\sigma^{HS}_{P,t-1}}_{C,t-1}$ and $\Pi^{\sigma^{HS}_{P,t}}_{C,t} \simeq P^{\sigma^{HS}_{P,t}}_{C,t} / P^{\sigma^{HS}_{P,t-1}}_{C,t-1}$. Since all households choose the same education, $H^i_t = H_t$. Note further that $\tilde{p}^{HS}_{Y,t} = P^{HS}_{YH,t}$.

The resulting high-skilled wage index of optimal and indexed wages is

$$\tilde{p}_{Y,t}^{HS} = \left(\xi_P^{HS} \left(\Theta_{t-1}^{HS} \frac{\tilde{p}_{Y,t-1}^{HS}}{\Pi_{C,t}} \right)^{1-\sigma_{P,t}^{HS}} + \left(1 - \xi_P^{HS}\right) \left(\tilde{p}_{Y,t}^{\# HS} \right)^{1-\sigma_{P,t}^{HS}} \right)^{\frac{1}{1-\sigma_{P,t}^{HS}}} . \tag{28}$$

The relationship between the elasticity of substitution and the wage mark up is

$$\frac{\sigma_{P,t}^{HS}}{\sigma_{P,t}^{HS} - 1} = \mu_P^{HS} \varepsilon_{P,t}^{HS}. \tag{29}$$

The high-skilled wage inflation rate, consumer price inflation rate, and the high-skilled real wage are related as

$$\frac{\Pi_{Y,t}^{HS}}{\Pi_{C,t}} = \frac{\tilde{p}_{Y,t}^{HS}}{\tilde{p}_{Y,t-1}^{HS}} g_{t-1}. \tag{30}$$

2.2 Low-skilled labor households (LS)

There is a continuum of low-skilled labor (LS) households indexed by $i \in [0, 1]$ each with one member that supplies labor hours and receives a minimum wage which is an administered variable set by a public authority, similar to the policy rate of the central bank as discussed in section 5. LS households have no access to financial markets and, hence, do not own bank deposits or financial securities. They hold interest-free high-powered money which allows them to smooth consumption over time. That is, the LS household's utility maximization problem has a corner solution w.r.t. consumption. LS households are subject to unemployment both at the extensive margin and the intensive margin (hours per person). They internalize the fact that not all of their hours supplied will get employed on the intensive margin.

Similar to HS households, LS households engage in deforestation. In contrast to HS households, LS households do not rent out land for productive use but directly derive utility from owning land. This captures subsistence production not captured in national accounts. As deforestation is illegal, households face convex deforestation costs.

LS households also engage in reforestation which transforms land into forest. Over time, this new forest captures CO2 from the atmosphere up to a given carbon storage capacity. The households sell the per-period sequestration of CO2 to a carbon-credit firm which certifies the sequestration and sells tradable carbon credits on a carbon market. The cost of reforestation is the opportunity cost of the foregone utility of owning land. The adjustment of reforestation generates quadratic costs.

Note that we assume that LS households derive some small utility from reforestation itself. This ensures that reforestation is non-negative even at a carbon price of zero. We calibrate the elasticity of inter-temporal substitution as almost zero implying that the marginal utility does not change much with reforestation unless it converges to zero.

In every period t, the LS household chooses the future paths of consumption, money balances, labor supply, deforestation, LS land, reforestation, and carbon sequestration. The household's i problem is to

$$\max_{\{Z_{CL,s}^{LS,i}, M_s^{LS,i}, Y_s^{LS,i}, DF_s^{LS,i}, LD_s^{LS,i}, RF_s^{LS,i}, G_{Sc,s}^{i}\}_{t=0}^{\infty}} \mathrm{E}_t \sum_{s=t}^{\infty} \beta^{s-t} U_s^{LS,i}$$

with per-period utility

$$
\begin{aligned}
U_s^{LS,i} = {} & \varepsilon_{C,s}^{LS}\psi_C\left(H_{s-1}\right)^{\varrho}\frac{\left(Z_{CL,s}^{LS,i}-\kappa Z_{CL,s-1}^{LS}\right)^{1-\varrho}}{1-\varrho}-\varepsilon_{N,s}^{LS}\psi_N^{LS}H_{s-1}\frac{\left(e_{i,s}Y_s^{LS,i}\right)^{1+\eta^{LS}}}{1+\eta^{LS}} \\
& +\psi_{LD}H_{s-1}\frac{\left(LD_{s-1}^{LS,i}\right)^{1-\varrho_{LD}}}{1-\varrho_{LD}}+\psi_{RF}\frac{\left(RF_s^{LS,i}\right)^{1-\varrho_{RF}}}{1-\varrho_{RF}},
\end{aligned}
$$

subject to the per-period budget constraint

$$
\begin{aligned}
& (1+t_C+\Gamma_{v,s}^{LS,i})P_{Y,s}^{CL}Z_{CL,s}^{LS,i}+M_s^{LS,i}+\Psi_{DF,s}^{LS,i}+\Psi_{RF,s}^{LS,i}+ \\
& \qquad +t_L^{LS}P_{Y,s}^{LS}e_{i,s}Y_s^{LS,i}+t_{LD}^{LS}LD_{s-1}^{LS,i}=P_{Y,s}^{LS}e_{i,s}Y_s^{LS,i}+\Psi_{LS,s}^{GV}+p_{G,s}^{c}G_{Sc,s}+M_{s-1}^{LS,i}+\Psi_s^{LS}
\end{aligned}
$$

where

$$
\Gamma_{v,s}^{LS,i}=\tau_{v1}^{LS}v_s^{LS,i}+\frac{\tau_{v2}^{LS}}{v_s^{LS,i}}-2\sqrt{\tau_{v1}^{LS}\tau_{v2}^{LS}}
$$

are the transaction costs,

$$
v_s^{LS,i}=(1+t_C)\frac{P_{Y,s}^{CL}Z_{CL,s}^{LS,i}}{M_s^{LS,i}}
$$

is the velocity of money,

$$
\Psi_{DF,s}^{LS,i}=\frac{\psi_{DF}^{LS}}{\varepsilon_{DF,s}\phi_{DF}^{LS}}\left(DF_s^{LS,i}\right)^{\phi_{DF}^{LS}}
$$

are convex costs of illegal deforestation which are rebated to the HS sector in a lump-sum manner,

$$
\Psi_{RF,s}^{LS,i}=\frac{\tau_{RF}}{2}\left(\frac{RF_s^{LS,i}}{RF_{s-1}^{LS,i}}-1\right)^2RF_{s-1}^{LS}H_{s-1}
$$

are quadratic reforestation adjustment costs which are rebated to the HS sector in a lump-sum manner, and

$$
\Psi_s^{LS}=\Gamma_{v,s}^{LS}P_{Y,s}^{CL}Z_{CL,s}^{LS}+\Psi_{DF,s}^{LS}+\Psi_{RF,s}^{LS}
$$

is the lump-sum refund of consumption transaction, deforestation, and reforestation adjustment costs, as well as subject to the law of motion of LS land

$$
LD_s^{LS,i}=DF_s^{LS,i}-RF_s^{LS,i}+\left(1-\delta_{LD}\right)LD_{s-1}^{LS,i}
$$

and subject to the law of motion of carbon credits

$$
G_{Sc,s}^{i}=\rho_{GS}G_{Sc,s-1}^{i}+\left(1-\rho_{GS}\right)\psi_{GD}\left(RF_s^{LS,i}\right)^{\phi_{RF}}
$$

where the marginal storage capacity of forest is decreasing.

The FOCs w.r.t. LS consumption, $Z_{CL,s}^{LS,i}$, relates the shadow price of the budget constraint to the marginal utility as

$$
\left(1+t_C+\Gamma_{v,t}^{LS}+\Gamma_{v,t}'^{LS}v_t^{LS}\right)p_{Y,t}^{CL}\lambda_t^{LS}=\varepsilon_{C,t}^{LS}\psi_C\left(\tilde{Z}_{CL,t}^{LS}-\kappa\tilde{Z}_{CL,t-1}^{LS}/g_{t-1}\right)^{-\varrho} \tag{31}
$$

where

$$\Gamma_{v,t}^{LS} = \tau_{v1}^{LS} v_t^{LS} + \frac{\tau_{v2}^{LS}}{v_t^{LS}} - 2\sqrt{\tau_{v1}^{LS}\tau_{v2}^{LS}}, \tag{32}$$

$$\Gamma_{v,t}^{\prime LS} = \tau_{v1}^{LS} - \frac{\tau_{v2}^{LS}}{{v_t^{LS}}^2}, \tag{33}$$

$$v_t^{LS} = (1+t_C)\frac{p_{Y,t}^{CL}\tilde{Z}_{CL,t}^{LS}}{\hat{M}_t^{LS}}. \tag{34}$$

Symmetric to the HS households, the FOCs w.r.t. money balances, $M_t^{LS,i}$, states that

$$\left(1 - \Gamma_t^{\prime LS}\frac{{v_t^{LS}}^2}{1+t_C}\right)\lambda_t^{LS} = \beta \mathrm{E}_t \frac{1}{\Pi_{C,t+1}}\lambda_{t+1}^{LS}. \tag{35}$$

The FOCs w.r.t. labor hours, $Y_t^{LS,i}$, is the labor supply function,

$$\varepsilon_{N,t}^{LS}\psi_N^{LS}\left(e_{i,t}Y_t^{LS}\right)^{\eta^{LS}} = (1-t_L^{LS})\lambda_t^{LS}\tilde{p}_{Y,t}^{LS}. \tag{36}$$

In the optimum, the disutility of supplying one additional unit of labor has to equal the utility it generates through consuming the additional labor income.

The FOC w.r.t. deforestation $DF_s^{LS,i}$ implies

$$\tilde{Q}_{LD,t}^{LS} = \frac{\tilde{\psi}_{DF}^{LS}}{\varepsilon_{DF,t}}\lambda_t^{LS}\left(DF_t^{LS,i}\right)^{\phi_{DF}^{LS}-1} \tag{37}$$

which equates the value of HS-land and the marginal cost of deforestation expressed in units of utility.

The FOC w.r.t. LS land $LD_s^{LS,i}$ implies the value of land as

$$\tilde{Q}_{LD,t}^{LS} = \beta \mathrm{E}_t\left(\frac{\psi_{LD}}{\left(LD_t^{LS}\right)^{\rho_{LD}}} - \lambda_{t+1}^{LS}\tilde{t}_{LD}^{LS} + (1-\delta_{LD})\,\tilde{Q}_{LD,t+1}^{LS}g_t\right) \tag{38}$$

which states recursively that the value of HS-land is the discounted rental income expressed in units of utility plus the next period's discounted value of the fraction of HS-land which natural forces have not converted back to forest area (non-depreciated land).[4]

The FOC w.r.t. reforestation $RF_s^{LS,i}$ implies

$$\begin{aligned}\tilde{Q}_{LD,t}^{LS} + \tau_{RF}\left(\lambda_t^{LS}\left(\frac{RF_t^{LS}}{RF_{t-1}^{LS}} - 1\right) - \mathrm{E}_t\lambda_{t+1}^{LS}\left(\frac{RF_{t+1}^{LS}}{RF_t^{LS}} - 1\right)\frac{RF_{t+1}^{LS}}{RF_t^{LS}}g_t\right)&\\ = \frac{\tilde{\psi}_{RF}}{\left(RF_t^{LS}\right)^{\rho_{RF}}} + \tilde{Q}_{Sc,t}\,(1-\rho_{GS})\,\psi_{GD}\phi_{RF}\left(RF_t^{LS}\right)^{\phi_{RF}-1}.&\end{aligned} \tag{39}$$

The right hand side is the marginal cost of reforestation which consists of the opportunity cost of land (the market value of LS land) and the marginal adjustment cost of reforestation. The left hand side is the

[4] For a well-defined steady state of deforestation to exist, we assume a small natural rate of reforestation δ_{LD}. Note further that, on the supply side, the unit of land is millions of hectares. For convenience, we choose the unit of land on the demand side such that $r_{LD,t} = 1$ at the steady state. The conversion rate ψ_{LD} is restricted accordingly.

marginal benefit of reforestation measured in terms of utility. It consists of the direct marginal utility from reforestation and the value of the new forest.[5]

The FOC w.r.t. carbon credits $G^i_{Sc,s}$ implies

$$\tilde{Q}_{Sc,t} = \lambda_t^{LS} \tilde{p}^c_{G,t} + \beta \rho_{GS} \mathrm{E}_t \tilde{Q}_{Sc,t+1} g_t. \tag{40}$$

The aggregated law of motion of LS land is

$$LD_t^{LS} = DF_t^{LS} - RF_t^{LS} + (1 - \delta_{LD}) LD_{t-1}^{LS}. \tag{41}$$

To aggregate the budget constraint over the continuum of LS households note that only a fraction $e_{e,t}$ of low-skilled labor households are employed on the external margin. Taking integrals over the unity continuum of LS households and detrending, the aggregation reads

$$p^{CL}_{Y,t} \tilde{Z}^{LS}_{CL,t} + \hat{T}^{LS}_{C,t} + \hat{T}^{LS}_{L,t} + \hat{T}^{LS}_{LD,t} + \hat{M}^{LS}_t = e_{e,t} e_{i,t} \tilde{p}^{LS}_{Y,t} Y^{LS}_t + \hat{\Psi}^{GV}_{LS,t} + \tilde{p}^c_{G,t} G_{Sc,t} + \frac{1}{\Pi_{C,t}} \hat{M}^{LS}_{t-1} / g_{t-1} \tag{42}$$

where $e_{e,t}$ and $e_{i,t}$ are the rates of employment at the extensive and intensive margin, respectively.

3 The domestic firm sector

The domestic economy comprises various producing sectors:

- There are four *final good sectors* (FGS) producing high-skilled consumption goods (CH), low-skilled consumption goods (CL), government consumption goods (CG), and investment goods (IV), respectively. We define $FGS \equiv \{CH, CL, CG, IV\}$. Intermediate FGS firms operate under monopolistic competition but do not create value-added. FGS aggregators bundle the respective differentiated FGS goods into homogeneous goods for final use.

- There are eight *value-added sectors* (VAS): agriculture (AG), industry (IN), transport (TR), services (SV), renewable electricity generation (RY), fossil electricity generation (FY), renewable fuels (RF), and fossil fuels (FF). We define $VAS \equiv \{AG, IN, TR, SV, RY, FY, RF, FF\}$. VAS firms are symmetric to FGS firms but, in addition, generate value-added by using capital and labor inputs, as well as natural resources (carbon and land). The price of carbon is exogenous and the supply perfectly elastic. VAS firms rent land from the HS households. Each value-added firm either complies with the regulations of the emission trading system (c) or can participate in a voluntary carbon market (v). Both FGS- and VAS-firms are discussed in Section 3.1.

- In each VAS, there is an *ETS bundler* (EB) which combines the outputs of the *compliance* and *voluntary* segments of the sector using a CES production function. The output of EB-firms in a given sector serves as a domestic input for FGS- and VAS-firms and is sold to the RW as exports. They are discussed in Section 3.2.

- The representative *bank* operates under perfect competition. The lending rate is subject to government regulation. It charges an investment finance premium to compensate for non-performing loans and the negative spread between the lending and deposit rate. The bank is discussed in Section 3.3.

- The employment firm (EM) minimizes the quadratic difference between the employment rate in hours and the employment rate in persons subject to a Calvo-type adjustment rigidity. The EM-firm ensures that employment varies less than hours worked. The EM-firm is discussed in Section 3.4.

[5] To make sense of the last term note the following: $(1 - \rho_{GS}) \psi_{GD} \phi_{RF} \left(RF_t^{LS}\right)^{\phi_{RF}-1}$ is the new forest measured in tCO2e of the carbon it captures in the very first period t. This new forest will capture carbon in the future at a decreasing rate converging to the full carbon storage capacity. The value of a new forest which captures 1 tCO2e in t is $\tilde{Q}_{Sc,t}$.

- Sector-specific *expropriators* steal a share of the firm value subject to prosecution cost. This captures the rule of law and endogenizes the equity risk premium. The expropriators are discussed in Section 3.5.
- A carbon-credit firm certifies the carbon sequestration by LS households and sells tradable carbon credits on the voluntary carbon market. The firm is discussed in Section 3.6.
- Sector-specific *importers* (IMS) purchase differentiated goods on the word market and resell them on domestic markets under monopolistic competition. This impedes the exchange-rate pass-through. We define $IMS \equiv \{AGM, INM, TRM, SVM, RYM, FYM, RFM, FFM\}$. Importers are discussed in section 3.7.
- Finally, there is a *foreign-owned capital-service* (FV) sector to capture foreign direct investment (FDI). FV firms are domestic, but owned by households in the RW. They supply a capital service under monopolistic competition. FV-aggregators sell the homogenized foreign-owned capital service to VAS firms which combine it with their own capital service composite. FV-firms are discussed in Section 3.8.

3.1 Final-good sectors (FGS) and value-added sectors (VAS)

Formally, the equations that describe the FGS-firms are nested in those that describe the VAS-firms. Essentially, FGS firms are VAS firms that do not employ capital, labor, or natural resources and sell their output to final consumers rather than other sectors or the RW. For the sake of a concise exposition, we therefore model them as symmetric to each other and then impose specific zero restrictions (Section 8) to account for differences between them.

As in Albonico et al. (2019) and Adrian et al. (2022), capital is firm-specific rather than owned and rented out by households. Hence, only investment, deinvestment, and depreciation can change the capital stock in the sector k over time.

As in Varga et al. (2022), the model distinguishes between *electricity-specific* (e) capital which is combined with electricity, and *fuel-specific* (f) capital which is combined with fuels, and *labor-specific* (g) capital which is combined with labor to form the capital-labor composite. This distinction is limited to the AG, IN, TR, and SV material sectors. In the other sectors, we will impose zero restrictions on electricity- and fuel-specific capital (as discussed in Section 8). Hence, the model can capture two different types of *green investment*: labor-specific capital investment in the renewable energy sectors and electricity-specific capital investment in the material sectors.

Capital services of use type $j \in \{e, f, g\}$ in sector $k \in VAS$ under the ETS compliance regime $e \in \{c, v\}$ (*compliance* and *voluntary*) are functions of the firm's private capital and public capital. OMEGA features a very granular categorization of public capital as discussed in section 5. We distinguish between public production capital in sector $k \in VAS$ which simply adds to the corresponding stock of private capital and public infrastructure of category $c \in PIC$. We define PIC as the set of categories of infrastructure (such as transportation infrastructure, telecommunication infrastructure, electricity grid, etc.). Similarly to Varga et al. (2022), the public infrastructure has external effects on the productivity of production capital.

Prices are sticky due to the assumption of staggered price contracts (Calvo 1983). In a given period, only a fraction of firms can re-optimize their price. For the remaining firms, prices are indexed in that period. This implies that both FGS- and VAS-firms accrue monopoly rents. In the FGS sectors, we restrict the fixed costs of production such that profits are zero in the steady state.

Adjustments of capital investment and production inputs, as well as capital utilization, are subject to quadratic costs. Emission abatement costs are convex.

Similarly to Varga et al. (2022), the firms' greenhouse gas emissions are a linear function of two sources: combustion of fossil fuel inputs to production and consumption bundles and production activities (process emissions). In general, the parameter ψ^k_{Gj} denotes the emission intensity in sector k of the input from sector j. Note that they are zero for most inputs j.[6] No abatement is possible and the only way to reduce input emissions is to reduce the respective input. The parameter $\psi^k_{GY,m}$ captures process emissions of type $m \in GHG$ in sector k where $GHG \equiv \{CH4, CO2, FG, N2O\}$. Process emissions are independent of production inputs and depend only on the sector output. We distinguish between methane, carbon dioxide, fugitive emissions, and nitrous oxide. They can be reduced by an abatement effort $x^{k,e}_t$ that accrues convex abatement costs as in Nordhaus and Sztorc (2013).

Firms subject to ETS compliance need to purchase emission permits, which can take the form of government-issued emission allowances or carbon credits issued by VAS firms on the voluntary carbon market. The ETS authority introduced in Section 5.4 supplies the government-issued allowances on the compliance market. Allocation of allowances can take different forms, including auctions, free allocation, and grandfathering. Firms that are not subject to ETS compliance can generate carbon credits by abating process emissions and sell them on the voluntary market as carbon offsets. The ETS authority controls the maximum share of carbon offsets allowed in the portfolio of emission permits of regulated entities. Note that we model the absence of an ETS as a fully accommodating ETS authority: that is, all desired emission permits will be provided at zero cost.

Firms pay a finance premium on their investment expenses as a form of intra-period borrowing cost. Firms also face end-of-period borrowing constraints. The domestic and foreign banking sectors will grant intertemporal loans only up to the point where the expected debt obligation in $t+1$ is lower than a fraction of the expected collateral in $t+1$: the value of the non-depreciated stock of capital. The loan-to-value ratios of domestic and foreign borrowing are exogenous.

In each sector $k \in \{FGS, VAS\}$ and each compliance regime $e \in \{c, v\}$, there is a continuum of k-firms indexed by $i \in [0,1]$ producing differentiated intermediate goods $Y^{k,e,i}_t$. Each firm i in sector k and regime e operates under monopolistic competition and, therefore, has price setting power. The intermediate good $Y^{k,e,i}_t$ is sold to a sector-specific aggregator which transforms the varieties into a homogeneous good $Y^{k,e}_t$.

Inputs, investment, capital, utilization, abatement, and debt. Firm i in sector k and regime e chooses the paths of production inputs, investment, the stock of capital, the utilization of capital, the process-emission abatement effort as well as end-of-period domestic and foreign loans in order to maximize its non-expropriated beginning-of-period real value which consists of non-expropriated period-t real dividends and its non-expropriated end-of-period-t value,

$$\left(1 - \Gamma^{k,e}_{S,t}\right)\left(\frac{J^{k,e,i}_t}{P_{C,t}} + \mathcal{V}^{k,e,i}_t\right)$$

with

$$\mathcal{V}^{k,e,i}_t = \mathrm{E}_t \Lambda_{t,t+1}\left(1 - \Gamma^{k,e}_{S,t+1}\right)\left(\frac{J^{k,e,i}_{t+1}}{P_{C,t}} + \mathcal{V}^{k,e,i}_{t+1}\right)$$

where

$$\Lambda_{t-1,t} \equiv \beta \frac{\lambda^{HS}_t}{\lambda^{HS}_{t-1}} \tag{43}$$

is the standard stochastic discount factor which expresses expected dividends in the future period $t+1$ as the hypothetical dividends in t which the households would value the same.[7] The expropriated firm share

[6] The presentation of the model allows for emissions from any non-labor and non-capital input. The zero restrictions on emission intensities are reported in section 8.

[7] Note that the end–of-period equity share price specified in eq. (22) is different from the end-of-period firm value, $p^{k,e}_{S,t} \neq \mathcal{V}^{k,e}_{S,t}$, because the share price and the firm value are subject to different discounting, $\left(R^{k,e}_{S,t-1}/\Pi_{C,t}\right)^{-1} \neq \Lambda_{t-1,t}$. The discount factors differ due to the wealth in the utility, $\psi_A \neq 0$.

$\Gamma_{S,t}^{k,e}$ is determined as discussed in Section 3.5.

Evaluated in period t and expressing the objective as an infinite sum rather than a value function, the optimization problem of firm i in sector k and regime e reads

$$\max_{\left\{\begin{array}{l}\{Z_{j,s}^{k,e,i}\}_{j\in\{HHS,VAS,FV,IMS\}},\{Z_{IV,Pj,s}^{k,e,i}\}_{j\in\{e,f,g\}},\\ \{K_{Pj,s}^{k,e,i}\}_{j\in\{e,f,g\}},u_s^{k,e,i},x_s^{k,e,i},L_{d,BK,s}^{k,e,i},L_{f,RW,s}^{k,e,i}\end{array}\right\}_{s=0}^{\infty}} \mathrm{E}_t \sum_{s=t}^{\infty} \frac{\prod_{r=t}^{s} \Lambda_{r-1,r}\left(1-\Gamma_{S,r}^{k,e}\right)}{\Lambda_{t-1,t}} \frac{J_s^{k,e,i}}{P_{C,s}}$$

subject to the firm's budget constraint which equates uses of funds (production input expenses, investment expenses including a finance premium, corporate income taxes, ad-valorem input taxes, unit input taxes, emission taxes, carbon credit and offset expenses, capital utilization costs, input adjustment costs, emission abatement costs, domestic and foreign debt obligations, and dividends) and sources of funds (sales revenues, unit input subsidies, grandfathered carbon credits, carbon offset revenues, domestic and foreign borrowing, lump-sum rebate of adjustment, finance and abatement costs) as

$$\begin{aligned}
&\sum_{j\in\{HHS,VAS,FV,IMS\}} P_{Y,s}^{j} Z_{j,s}^{k,e,i} + R_{Fd,s} P_{Y,s}^{IV} Z_{IV,s}^{k,e,i} + \\
&+T_{P,s}^{k,e,i} + T_{V,s}^{k,e,i} + T_{U,s}^{k,e,i} + T_{G,s}^{k,e,i} + \\
&+d^e\left(\omega_{COx} P_{COx,s} + \omega_{COs} P_{COs,s} + \left(1-\omega_{COx}-\omega_{COs}\right) P_{CC,s}\right) G_s^{k,e,i} + \\
&+\Psi_{u,s}^{k,e,i} + \sum_{j\in\{VAS,FV,IMS\}} \Psi_{Z,j,s}^{k,e,i} + \Psi_{x,s}^{k,e,i} + \\
&+R_{Ld,s-1} L_{d,s-1}^{k,e,i} + R_{Lf,s-1}\mathcal{S}_s L_{f,s-1}^{k,e,i} + J_s^{k,e,i} = P_{Y,s}^{k,e,i} Y_s^{k,e,i} + S_{U,s}^{k,e,i} + \\
&\qquad + d^e\left(1-\omega_{COx}-\omega_{COs}\right)\omega_{CCg} P_{CC,s} G_s^{k,e} + \\
&\qquad + P_{COx,s} CO_{x,s}^{k,e,i} + P_{COs,s} CO_{s,s}^{k,e,i} + \\
&\qquad + L_{d,s}^{k,e,i} + \mathcal{S}_t L_{f,s}^{k,e,i} + \Psi_s^k
\end{aligned}$$

with

$$Z_{IV,s}^{k,e,i} = \sum_{j\in\{e,f,g\}} Z_{IV,Pj,s}^{k,e,i},$$

corporate income taxes, ad-valorem input taxes, unit input taxes, and unit input subsidies,

$$T_{P,s}^{k,e,i} = t_P \left[\begin{array}{c}
P_{Y,s}^{k,e,i} Y_s^{k,e,i} - \sum_{j\in\{HHS,VAS,FV,IMS\}} P_{Y,s}^{j} Z_{j,s}^{k,e,i} - \\
-T_{V,s}^{k,e,i} - T_{U,s}^{k,e,i} - T_{G,s}^{k,e,i} + S_{U,s}^{k,e,i} - \\
-d^e\left(\omega_{COx} P_{COx,s} + \omega_{COs} P_{COs,s} + \left(1-\omega_{COx}-\omega_{COs}\right) P_{CC,s}\right) G_s^{k,e,i} \\
+d^e\left(1-\omega_{COx}-\omega_{COs}\right)\omega_{CCg} P_{CC,s} G_s^{k,e} + \\
+P_{COx,s} CO_{x,s}^{k,e,i} + P_{COs,s} CO_{s,s}^{k,e,i} - \Psi_{u,s}^{k,e,i} - \sum_{j\in\{VAS,FV,IMS\}} \Psi_{Z,j,s}^{k,e,i} - \Psi_{x,s}^{k,e,i} + \Psi_s^{k,e} - \\
-P_{C,s}\delta \sum_{j\in\{e,f,g\}} Q_{j,s}^{k,e} K_{Pj,s-1}^{k,e,i} - P_{C,s}\varepsilon_{K,s}^{k} \frac{\tau_I}{2}\left(\frac{Z_{IV,s}^{k,e,i}}{g_{s-1} Z_{IV,s-1}^{k,e,i}} - 1\right)^2 \sum_{j\in\{e,f,g\}} Q_{j,s}^{k,e} Z_{IV,Pj,s}^{k,e,i}
\end{array}\right]$$

$$T_{V,s}^{k,e,i} = \sum_{j\in\{HHS,VAS,FV,IMS\}} t_{V,j} P_{Y,s}^{j} Z_{j,s}^{k,e,i}$$

$$T_{U,s}^{k,e,i} = P_{C,s} \sum_{j\in\{HHS,VAS,FV,IMS\}} t_{U,j} Z_{j,s}^{k,e,i}$$

$$T_{G,s}^{k,e,i} = P_{C,s} t_G G_s^{k,e,i}$$

$$S_{U,s}^{k,e,i} = P_{C,s} \sum_{j \in \{HHS, VAS, FV, IMS\}} s_{U,j} Z_{j,s}^{k,e,i}$$

general capital utilization and input adjustment costs,

$$\Psi_{u,s}^{k,e,i} = P_{C,s} \left(\tau_{u1}^k \left(u_s^{k,e,i} - 1 \right) + \frac{\tau_{u2}^k}{2} \left(u_s^{k,e,i} - 1 \right)^2 \right) K_{g,s}^{k,e}$$

$$\Psi_{Z,j,s}^{k,e,i} = P_{C,s} \frac{\tau_{Z,j}}{2} \left(\frac{Z_{j,s}^{k,e,i}}{g_{s-1} Z_{j,s-1}^{k,e,i}} - 1 \right)^2 Z_{j,s-1}^{k,e}$$

$$\forall j \in \{VAS, FV, IMS\},$$

abatement costs,

$$\Psi_{x,s}^{k,e,i} = P_{C,s} \frac{\tau_x^k}{\phi_x} \left(x_s^{k,e,i} \right)^{\phi_x} Y_s^{k,e},$$

the nested CES production structure,

$$Y_s^{k,e,i} = \epsilon_{Y,s}^k \psi_Y^k \left(\left(\alpha_Y^k \right)^{\frac{1}{\sigma_Y^k}} \left(T_s^{k,e,i} \right)^{\frac{\sigma_Y^k - 1}{\sigma_Y^k}} + \left(1 - \alpha_Y^k \right)^{\frac{1}{\sigma_Y^k}} \left(Z_{CB,s}^{k,e,i} \right)^{\frac{\sigma_Y^k - 1}{\sigma_Y^k}} \right)^{\frac{\sigma_Y^k}{\sigma_Y^k - 1}} - \chi_Y^{k,e} H_{t-1}$$

$$T_s^{k,e,i} = \left(\left(\alpha_T^k \right)^{\frac{1}{\sigma_T^k}} \left(U_s^{k,e,i} \right)^{\frac{\sigma_T^k - 1}{\sigma_T^k}} + \left(1 - \alpha_T^k \right)^{\frac{1}{\sigma_T^k}} \left(Z_{LD,s}^{k,e,i} \right)^{\frac{\sigma_T^k - 1}{\sigma_T^k}} \right)^{\frac{\sigma_T^k}{\sigma_T^k - 1}}$$

$$U_s^{k,e,i} = \left(\left(\alpha_U^k \right)^{\frac{1}{\sigma_U^k}} \left(X_s^{k,e,i} \right)^{\frac{\sigma_U^k - 1}{\sigma_U^k}} + \left(1 - \alpha_U^k \right)^{\frac{1}{\sigma_U^k}} \left(S_s^{k,e,i} \right)^{\frac{\sigma_U^k - 1}{\sigma_U^k}} \right)^{\frac{\sigma_U^k}{\sigma_U^k - 1}}$$

$$X_s^{k,e,i} = \left(\sum_{j \in \{AG, MI, MA, CN, TR, SV\}} \left(\alpha_{X,j}^k \right)^{\frac{1}{\sigma_X^k}} \left(W_{j,s}^{k,e,i} \right)^{\frac{\sigma_X^k - 1}{\sigma_X^k}} \right)^{\frac{\sigma_X^k}{\sigma_X^k - 1}}$$

$$S_s^{k,e,i} = \left(\left(\alpha_S^k \right)^{\frac{1}{\sigma_S^k}} \left(V_s^{k,e,i} \right)^{\frac{\sigma_S^k - 1}{\sigma_S^k}} + \left(1 - \alpha_S^k \right)^{\frac{1}{\sigma_S^k}} \left(N_s^{k,e,i} \right)^{\frac{\sigma_S^k - 1}{\sigma_S^k}} \right)^{\frac{\sigma_S^k}{\sigma_S^k - 1}}$$

$$V_s^{k,e,i} = \left(\left(\alpha_V^k \right)^{\frac{1}{\sigma_V^k}} \left(u_s^{k,e,i} O_s^{k,e,i} \right)^{\frac{\sigma_V^k - 1}{\sigma_V^k}} + \left(1 - \alpha_V^k \right)^{\frac{1}{\sigma_V^k}} \left(H_{s-1} L_s^{k,e,i} \right)^{\frac{\sigma_V^k - 1}{\sigma_V^k}} \right)^{\frac{\sigma_V^k}{\sigma_V^k - 1}}$$

$$O_s^{k,e,i} = \psi_O^k \left(\left(\alpha_O^k\right)^{\frac{1}{\sigma_O^k}} \left(K_{g,s}^{k,e,i}\right)^{\frac{\sigma_O^k - 1}{\sigma_O^k}} + \left(1-\alpha_O^k\right)^{\frac{1}{\sigma_O^k}} \left(Z_{FV,s}^{k,e,i}\right)^{\frac{\sigma_O^k - 1}{\sigma_O^k}} \right)^{\frac{\sigma_O^k}{\sigma_O^k - 1}}$$

$$L_s^{k,e,i} = \psi_L^k \left(\left(\alpha_L^k\right)^{\frac{1}{\sigma_L^k}} \left(Z_{HS,s}^{k,e,i}\right)^{\frac{\sigma_L^k - 1}{\sigma_L^k}} + \left(1-\alpha_L^k\right)^{\frac{1}{\sigma_L^k}} \left(Z_{LS,s}^{k,e,i}\right)^{\frac{\sigma_L^k - 1}{\sigma_L^k}} \right)^{\frac{\sigma_L^k}{\sigma_L^k - 1}}$$

$$N_s^{k,e,i} = \left(\left(\alpha_N^k\right)^{\frac{1}{\sigma_N^k}} \left(EK_s^{k,e,i}\right)^{\frac{\sigma_N^k - 1}{\sigma_N^k}} + \left(1-\alpha_N^k\right)^{\frac{1}{\sigma_N^k}} \left(FK_s^{k,e,i}\right)^{\frac{\sigma_N^k - 1}{\sigma_N^k}} \right)^{\frac{\sigma_N^k}{\sigma_N^k - 1}}$$

$$EK_s^{k,e,i} = \psi_{EK}^k \left(\left(\alpha_{EK}^k\right)^{\frac{1}{\sigma_{EK}^k}} \left(E_s^{k,e,i}\right)^{\frac{\sigma_{EK}^k - 1}{\sigma_{EK}^k}} + \left(1-\alpha_E^k\right)^{\frac{1}{\sigma_{EK}^k}} \left(K_{e,s}^{k,e,i}\right)^{\frac{\sigma_{EK}^k - 1}{\sigma_{EK}^k}} \right)^{\frac{\sigma_{EK}^k}{\sigma_{EK}^k - 1}}$$

$$FK_s^{k,e,i} = \psi_{FK}^k \left(\left(\alpha_{FK}^k\right)^{\frac{1}{\sigma_{FK}^k}} \left(F_s^{k,e,i}\right)^{\frac{\sigma_{FK}^k - 1}{\sigma_{FK}^k}} + \left(1-\alpha_F^k\right)^{\frac{1}{\sigma_{FK}^k}} \left(K_{f,s}^{k,e,i}\right)^{\frac{\sigma_{FK}^k - 1}{\sigma_{FK}^k}} \right)^{\frac{\sigma_{FK}^k}{\sigma_{FK}^k - 1}}$$

$$E_s^{k,e,i} = \left(\left(\alpha_E^k\right)^{\frac{1}{\sigma_E^k}} \left(W_{RY,s}^{k,e,i}\right)^{\frac{\sigma_E^k - 1}{\sigma_E^k}} + \left(1-\alpha_E^k\right)^{\frac{1}{\sigma_E^k}} \left(W_{FY,s}^{k,e,i}\right)^{\frac{\sigma_E^k - 1}{\sigma_E^k}} \right)^{\frac{\sigma_E^k}{\sigma_E^k - 1}}$$

$$F_s^{k,e,i} = \left(\left(\alpha_F^k\right)^{\frac{1}{\sigma_F^k}} \left(W_{RF,s}^{k,e,i}\right)^{\frac{\sigma_F^k - 1}{\sigma_F^k}} + \left(1-\alpha_F^k\right)^{\frac{1}{\sigma_F^k}} \left(W_{FF,s}^{k,e,i}\right)^{\frac{\sigma_F^k - 1}{\sigma_F^k}} \right)^{\frac{\sigma_F^k}{\sigma_F^k - 1}}$$

$$W_{j,s}^{k,e,i} = \psi_{W,j}^k \left(\left(\alpha_{W,j}^k\right)^{\frac{1}{\sigma_{W,j}}} \left(Z_{j,s}^{k,e,i}\right)^{\frac{\sigma_{W,j} - 1}{\sigma_{W,j}}} + \left(1-\alpha_{W,j}^k\right)^{\frac{1}{\sigma_{W,j}}} \left(Z_{jM,s}^{k,e,i}\right)^{\frac{\sigma_{W,j} - 1}{\sigma_{W,j}}} \right)^{\frac{\sigma_{W,j}}{\sigma_{W,j} - 1}}$$
$$\forall j \in VAS$$

$$K_{j,s}^{k,e,i} = \psi_{Kj}^k \sum_{c \in PIC} \left(\frac{K_{Ic,s-1}^{GV}}{K_{Ic}^{GV}/g_{t-1}} \right)^{\phi_{Kj,c}^k} \left(K_{Pj,s-1}^{k,e,i} + K_{Pj,k,e,s-1}^{GV} \right)$$
$$\forall j \in \{e, f, g\},$$

the lump-sum rebate of adjustment, utilization, and abatement costs as well as the finance premium

$$\Psi_s^{k,e} = \sum_{j \in \{VAS, FV, IMS\}} \Psi_{Z,j,s}^{k,e} + \Psi_{u,s}^{k,e} + \Psi_{x,s}^{k,e} + R_{Fd,s} P_{Y,s}^{IV} Z_{IV,s}^{k,e},$$

the inter-period domestic and foreign borrowing constraints (expressed in domestic currency),

$$L_{d,s}^{k,e,i} \leq \lambda_d^k \mathrm{E}_t \frac{P_{C,s+1}}{R_{Ld,s}} (1-\delta) \sum_{j \in \{e,f,g\}} Q_{j,s+1}^{k,e,i} K_{Pj,s}^{k,e,i},$$

$$\mathcal{S}_s L^{k,e,i}_{f,s} \leq \lambda^k_f \mathrm{E}_t \frac{\mathcal{S}_s}{\mathcal{S}_{s+1}} \frac{P_{C,s+1}}{R_{Lf,s}} (1-\delta) \sum_{j\in\{e,f,g\}} Q^{k,e,i}_{j,s+1} K^{k,e,i}_{Pj,s},$$

the law of motion of capital of use type $j \in \{e,f,g\}$,

$$K^{k,e,i}_{Pj,s} = \varepsilon^k_{K,s} \frac{\tau_I}{2} \left(\frac{Z^{k,e,i}_{IV,Pj,s}}{g_{s-1} Z^{k,e,i}_{IV,Pj,s-1}} - 1 \right)^2 Z^{k,e,i}_{IV,Pj,s} + (1-\delta) K^{k,e,i}_{Pj,s-1},$$

the greenhouse gas (GHG) emission function,

$$G^{k,e,i}_s = \sum_{j\in\{VAS,IMS\}} \psi^k_{Gj} Z^{k,e,i}_{j,s} + \left(1 - x^{k,e,i}_s\right) \sum_{m\in GHG} \psi^k_{GY,m} Y^{k,e,i}_s,$$

and the supply of abatement-backed carbon offsets,

$$CO^{k,e,i}_{x,s} = (1-d^e)\, x^{k,e,i}_s \sum_{m\in GHG} \psi^k_{GY,m} Y^{k,e,i}_s$$

where

$$d^e = \begin{cases} 1 & \text{for } e = c, \\ 0 & \text{for } e = v, \end{cases} \tag{44}$$

because only unregulated firms can supply carbon offsets on the voluntary carbon market. Investment adjustment costs are a form of depreciation. They are technological in the sense that they affect the way investment translates into capital. In contrast to this, input adjustment costs, capital utilization costs, and abatement costs are monetary costs. They reduce the firm's cash flow. We assume these costs to be refunded to the firms in a lump sum manner such that they do not affect the flow of funds between aggregate sectors. This assumption eliminates the income effect, but maintains the respective incentives on firm behavior.

The sector k and ETS-compliance regime e's FOCs w.r.t. domestic and imported intermediate-good inputs plus carbon, $Z^{k,e,i}_{j,s}$ for $j \in \{VAS, IMS\}$, and value-added from foreign-owned firms, $Z^{k,e,i}_{FV,s}$, imply

$$(1-t_P) \begin{bmatrix} (1+t_{V,j}) p^j_{Y,t} + t_{U,j} - s_{U,j} + \\ + \left(t_G + d^e \left(\omega_{COx} p_{COx,t} + \omega_{COs} p_{COs,t} + (1-\omega_{COx} - \omega_{COs})\, p_{CC,t}\right)\right) \psi^k_{Gj} + \\ + \tau_{Z,j} \left(\frac{\tilde{Z}^{k,e}_{j,t}}{\tilde{Z}^{k,e}_{j,t-1}} - 1 \right) \frac{1}{g_{t-1}} - \mathrm{E}_t \Lambda_{t,t+1} \left(1 - \Gamma^{k,e}_{S,t+1}\right) \tau_{Z,j} \left(\frac{\tilde{Z}^{k,e}_{j,t+1}}{\tilde{Z}^{k,e}_{j,t}} - 1 \right) \frac{\tilde{Z}^{k,e}_{j,t+1}}{\tilde{Z}^{k,e}_{j,t}} \end{bmatrix} = \varphi^{k,e}_t \frac{\partial Y^{k,e}_t}{\partial Z^{k,e}_{j,t}} \tag{45}$$

$$\forall k \in \{FGS, VAS\} \text{ and } \forall e \in \{c,v\} \text{ and } \forall j \in \{VAS, FV, IMS\}$$

where we leave the computation of the marginal products to the reader; that is, to take the derivatives of the sector outputs $Y^{k,e}_t$ w.r.t. the respective inputs $Z^{k,e}_{j,t}$ using the firm's CES production structure.[8] The

[8] For illustrative purposes, consider a CES production function with fixed costs,

$$Y = \psi \left(\alpha^{\frac{1}{\sigma}} Z_1^{\frac{\sigma-1}{\sigma}} + (1-\alpha)^{\frac{1}{\sigma}} Z_2^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}} - \chi.$$

Then,

$$\begin{aligned} \frac{\partial Y}{\partial Z_1} &= \psi \left(\alpha^{\frac{1}{\sigma}} Z_1^{\frac{\sigma-1}{\sigma}} + (1-\alpha)^{\frac{1}{\sigma}} Z_2^{\frac{\sigma-1}{\sigma}} \right)^{\frac{1}{\sigma-1}} \alpha^{\frac{1}{\sigma}} Z_1^{-\frac{1}{\sigma}} \\ &= \psi^{1-\frac{1}{\sigma}} \psi^{\frac{1}{\sigma}} \left(\alpha^{\frac{1}{\sigma}} Z_1^{\frac{\sigma-1}{\sigma}} + (1-\alpha)^{\frac{1}{\sigma}} Z_2^{\frac{\sigma-1}{\sigma}} \right)^{\frac{1}{\sigma-1}} \alpha^{\frac{1}{\sigma}} Z_1^{-\frac{1}{\sigma}} \\ &= \psi^{\frac{\sigma-1}{\sigma}} \alpha^{\frac{1}{\sigma}} \left(\frac{Y+\chi}{Z_1} \right)^{\frac{1}{\sigma}}. \end{aligned}$$

representative firm in sector k and compliance regime e employs an input up to the point where the value generated by the last unit of input (right-hand side) equals the cost of that unit of input (left-hand side). The costs consist of the input price plus ad-valorem taxes, net unit taxes, the process emission payments associated with the input, and input adjustment costs. Note that all of these costs reduce the corporate income tax base.

The FOC w.r.t. land input $Z_{LD,s}^{k,e,i}$ implies

$$(1-t_P)\left[\begin{array}{c} r_{LD,t}+ \\ +\tau_{Z,LD}\left(\frac{\tilde{Z}_{LD,t}^{k,e}}{\tilde{Z}_{LD,t-1}^{k,}}-1\right)\frac{1}{g_{t-1}} - \mathrm{E}_t\Lambda_{t,t+1}^{k,e}\tau_{Z,LD}\left(\frac{\tilde{Z}_{LD,t+1}^{k,e}}{\tilde{Z}_{LD,t}^{k,e}}-1\right)\frac{\tilde{Z}_{LD,t+1}^{k,e}}{\tilde{Z}_{LD,t}^{k,e}}\end{array}\right] = \varphi_t^{k,e}\frac{\partial Y_t^{k,e}}{\partial Z_{LD,t}^{k,e}} \tag{46}$$

$$\forall k \in \{FGS, VAS\} \text{ and } \forall e \in ETS$$

where we leave it to the reader to compute the marginal products.

The FOCs w.r.t. labor input, $Z_{j,s}^{k,i}$ for $j \in HHS$, imply

$$(1-t_P)(1+t_{V,j})\tilde{p}_{Y,t}^{j} = \varphi_t^{k,e}\frac{\partial Y_t^{k,e}}{\partial Z_{j,t}^{k,e}} \tag{47}$$

$$\forall k \in \{FGS, VAS\} \text{ and } \forall e \in \{c,v\} \text{ and } \forall j \in HHS$$

where we leave it to the reader to compute the marginal products.

The FOCs w.r.t. end-of-period domestic loans, $L_{d,BK,s}^{k,e,i}$, and end-of-period foreign loans $L_{f,RW,s}^{k,e,i}$ imply

$$\mu_{d,t}^{k,e} = 1 - \mathrm{E}_t\Lambda_{t,t+1}\left(1-\Gamma_{S,t+1}^{k,e}\right)\frac{R_{Ld,t}}{\Pi_{C,t+1}}$$

$$\forall k \in \{FGS, VAS\} \text{ and } \forall e \in \{c,v\}$$

and

$$\mu_{f,t}^{k,e} = 1 - \mathrm{E}_t\Lambda_{t,t+1}\left(1-\Gamma_{S,t+1}^{k,e}\right)\frac{R_{Lf,t}}{\Pi_{C,t+1}}\frac{\mathcal{S}_{t+1}}{\mathcal{S}_t}$$

$$\forall k \in \{FGS, VAS\} \text{ and } \forall e \in \{c,v\},$$

respectively, which characterize the firm's preference for external finance.

The demand functions for end-of-period domestic and foreign loans in sector k and regime e satisfy

$$R_{Ld,t}\hat{L}_{d,t}^{k,e} = \lambda_d^k \mathrm{E}_t\Pi_{C,t+1}(1-\delta)\sum_{j\in\{e,f,g\}} Q_{j,t+1}^{k,e}\tilde{K}_{Pj,t}^{k,e} \tag{48}$$

$$\forall k \in \{FGS, VAS\} \text{ and } \forall e \in \{c,v\}$$

and

$$R_{Lf,t}\mathcal{E}_t\hat{L}_{f,t}^{k,e} = \lambda_f^k \mathrm{E}_t\frac{\Pi_{C,t+1}}{\Delta_{\mathcal{S},t+1}}(1-\delta)\sum_{j\in\{e,f,g\}} Q_{j,t+1}^{k,e}\tilde{K}_{Pj,t}^{k,e} \tag{49}$$

$$\forall k \in \{FGS, VAS\} \text{ and } \forall e \in \{c,v\}$$

respectively.

The FOCs w.r.t. investment demand, $Z^{k,e,i}_{IV,Pj,s}$, imply

$$
\begin{aligned}
R_{Fd,t}p^{IV}_{Y,t} = {} & Q^{k,e}_{j,t}\varepsilon^k_{K,t} - \\
& -(1-t_P)\,Q^{k,e}_{j,t}\varepsilon^k_{K,t}\left(\frac{\tau_I}{2}\left(\frac{\tilde{Z}^{k,e}_{IV,Pj,t}}{\tilde{Z}^{k,e}_{IV,Pj,t-1}}-1\right)^2+\tau_I\left(\frac{\tilde{Z}^{k,e}_{IV,Pj,t}}{\tilde{Z}^{k,e}_{IV,Pj,t-1}}-1\right)\frac{\tilde{Z}^{k,e}_{IV,Pj,t}}{\tilde{Z}^{k,e}_{IV,Pj,t-1}}\right) \\
& +\mathrm{E}_t\Lambda_{t,t+1}\left(1-\Gamma^{k,e}_{S,t+1}\right)(1-t_P)\,Q^{k,e}_{j,t+1}\varepsilon^k_{K,t+1}\tau_I\left(\frac{\tilde{Z}^{k,e}_{IV,Pj,t+1}}{\tilde{Z}^{k,e}_{IV,Pj,t}}-1\right)\frac{\tilde{Z}^{k,e}_{IV,Pj,t+1}}{\left(\tilde{Z}^{k,e}_{IV,Pj,t}\right)^2}\tilde{Z}^{k,e}_{IV,Pj,t+1}g_t \\
& \forall k\in\{FGS,VAS\} \text{ and } \forall e\in\{c,v\} \text{ and } \forall j\in\{e,f,g\}.
\end{aligned}
\tag{50}
$$

Eq. (50) relates investment to the value of capital, i.e. the shadow price of capital, $Q^{k,e}_t$. The firm chooses investment such that the marginal revenue of an additional unit of investment (which is the value of capital, Tobin's Q, since one unit of investment increases capital by one unit) equals its marginal cost (which is the price of the investment good plus the adjustment costs).

The FOCs w.r.t. capital, $K^{k,e,i}_{Pj,s}$, imply

$$
\begin{aligned}
Q^{k,e}_{j,t} = {} & \mathrm{E}_t\Lambda_{t,t+1}\left(1-\Gamma^{k,e}_{S,t+1}\right)\left(\varphi^{k,e}_{t+1}\frac{\partial Y^{k,e}_{t+1}}{\partial K^{k,e}_{Pj,t}}+(1-(1-t_P)\delta)\,Q^{k,e}_{j,t+1}\right)+ \\
& +\lambda^k_d(1-\delta)\mathrm{E}_t\left(\frac{\Pi_{C,t+1}}{R_{Ld,t}}-\Lambda_{t,t+1}\left(1-\Gamma^{k,e}_{S,t+1}\right)\right)Q^{k,e}_{j,t+1} \\
& +\lambda^k_f(1-\delta)\mathrm{E}_t\left(\frac{\Pi_{C,t+1}}{R_{Lf,t}}\frac{1}{\Delta_{\mathcal{S},t+1}}-\Lambda_{t,t+1}\left(1-\Gamma^{k,e}_{S,t+1}\right)\right)Q^{k,e}_{j,t+1} \\
& \forall k\in\{FGS,VAS\} \text{ and } \forall e\in\{c,v\} \text{ and } \forall j\in\{e,f,g\}
\end{aligned}
\tag{51}
$$

where we used the FOCs w.r.t. $L^{k,e,i}_{d,BK,s}$ and $L^{k,e,i}_{f,RW,s}$. We leave it to the reader to compute the marginal products. The FOC links $Q^{k,e}_t$ to the desired capital stock. The firm chooses capital to equate marginal revenues and marginal costs. The marginal cost of capital is simply the cost of investment, which by eq. (50) is Tobin's Q because one unit of investment is required to increase capital by one unit. The marginal revenues include the following components: the value of the marginal product of the additional unit of capital; savings in the corporate income tax because of capital depreciation; the value of the non-depreciated part of the capital unit in the next period; and, finally, the value of higher collateral in the next period which alleviates domestic and foreign end-of-period borrowing constraints.

The FOC w.r.t. capital utilization, $u^{k,e,i}_s$, implies

$$
\begin{aligned}
(1-t_P)\left(\tau^k_{u1}+\tau^k_{u2}\left(u^{k,e}_t-1\right)\right)\tilde{K}^{k,e}_{g,t} &= \varphi^{k,e}_t\frac{\partial\tilde{Y}^{k,e}_t}{\partial u^{k,e}_t} \\
& \forall k\in\{FGS,VAS\} \text{ and } \forall e\in\{c,v\}
\end{aligned}
\tag{52}
$$

which determines the optimal rate of capital utilization.

The FOC w.r.t. the abatement effort, $x^{k,e,i}_t$, states that the marginal revenue of increasing the abatement effort by one unit (lower payments for process emissions at a given emission tax and price) is equal to the marginal abatement cost. This implies

$$
\begin{aligned}
\left(t_G+d^e\left(\omega_{COx}p_{COx,t}+\omega_{COs}p_{COs,t}+(1-\omega_{COx}-\omega_{COs})\,p_{CC,t}\right)+(1-d^e)\,p_{COx,t}\right)\sum_{m\in GHG}\psi^k_{GY,m} &= \tau^k_x\left(x^{k,e}_t\right)^{\phi^k_x-1} \\
& \forall k\in\{FGS,VAS\} \text{ and } \forall e\in\{c,v\}.
\end{aligned}
\tag{53}
$$

The production structure can be aggregated $\forall k \in \{FGS, VAS\}$ and $\forall e \in \{c, v\}$ as

$$\tilde{Y}_t^{k,e} = \epsilon_{Y,t}^k \psi_Y^k \left(\left(\alpha_Y^k\right)^{\frac{1}{\sigma_Y^k}} \left(\tilde{T}_t^{k,e}\right)^{\frac{\sigma_Y^k - 1}{\sigma_Y^k}} + \left(1 - \alpha_Y^k\right)^{\frac{1}{\sigma_Y^k}} \left(\tilde{Z}_{CB,t}^{k,e}\right)^{\frac{\sigma_Y^k - 1}{\sigma_Y^k}} \right)^{\frac{\sigma_Y^k}{\sigma_Y^k - 1}} - \chi_Y^{k,e} \tag{54}$$

$$\tilde{T}_s^{k,e} = \left(\left(\alpha_T^k\right)^{\frac{1}{\sigma_T^k}} \left(\tilde{U}_s^{k,e}\right)^{\frac{\sigma_T^k - 1}{\sigma_T^k}} + \left(1 - \alpha_T^k\right)^{\frac{1}{\sigma_T^k}} \left(Z_{LD,s}^{k,e}\right)^{\frac{\sigma_T^k - 1}{\sigma_T^k}} \right)^{\frac{\sigma_T^k}{\sigma_T^k - 1}} \tag{55}$$

$$\tilde{U}_t^{k,e} = \left(\left(\alpha_U^k\right)^{\frac{1}{\sigma_U^k}} \left(\tilde{X}_t^{k,e}\right)^{\frac{\sigma_U^k - 1}{\sigma_U^k}} + \left(1 - \alpha_U^k\right)^{\frac{1}{\sigma_U^k}} \left(\tilde{S}_t^{k,e}\right)^{\frac{\sigma_U^k - 1}{\sigma_U^k}} \right)^{\frac{\sigma_U^k}{\sigma_U^k - 1}} \tag{56}$$

$$\tilde{X}_t^{k,e} = \left(\sum_{j \in \{AG, MI, MA, CN, TR, SV\}} \left(\alpha_{X,j}^k\right)^{\frac{1}{\sigma_X^k}} \left(\tilde{W}_{j,t}^{k,e}\right)^{\frac{\sigma_X^k - 1}{\sigma_X^k}} \right)^{\frac{\sigma_X^k}{\sigma_X^k - 1}} \tag{57}$$

$$\tilde{S}_t^{k,e} = \left(\left(\alpha_S^k\right)^{\frac{1}{\sigma_S^k}} \left(\tilde{V}_t^{k,e}\right)^{\frac{\sigma_S^k - 1}{\sigma_S^k}} + \left(1 - \alpha_S^k\right)^{\frac{1}{\sigma_S^k}} \left(\tilde{N}_t^{k,e}\right)^{\frac{\sigma_S^k - 1}{\sigma_S^k}} \right)^{\frac{\sigma_S^k}{\sigma_S^k - 1}} \tag{58}$$

$$\tilde{V}_t^{k,e} = \left(\left(\alpha_V^k\right)^{\frac{1}{\sigma_V^k}} \left(u_t^{k,e} \tilde{O}_t^{k,e}\right)^{\frac{\sigma_V^k - 1}{\sigma_V^k}} + \left(1 - \alpha_V^k\right)^{\frac{1}{\sigma_V^k}} \left(L_t^{k,e}\right)^{\frac{\sigma_V^k - 1}{\sigma_V^k}} \right)^{\frac{\sigma_V^k}{\sigma_V^k - 1}} \tag{59}$$

$$\tilde{O}_t^{k,e} = \psi_O^k \left(\left(\alpha_O^k\right)^{\frac{1}{\sigma_O^k}} \left(\tilde{K}_{g,t}^{k,e}\right)^{\frac{\sigma_O^k - 1}{\sigma_O^k}} + \left(1 - \alpha_O^k\right)^{\frac{1}{\sigma_O^k}} \left(\tilde{Z}_{FV,t}^{k,e}\right)^{\frac{\sigma_O^k - 1}{\sigma_O^k}} \right)^{\frac{\sigma_O^k}{\sigma_O^k - 1}} \tag{60}$$

$$L_t^{k,e} = \psi_L^k \left(\left(\alpha_L^k\right)^{\frac{1}{\sigma_L^k}} \left(Z_{HS,t}^{k,e}\right)^{\frac{\sigma_L^k - 1}{\sigma_L^k}} + \left(1 - \alpha_L^k\right)^{\frac{1}{\sigma_L^k}} \left(Z_{LS,t}^{k,e}\right)^{\frac{\sigma_L^k - 1}{\sigma_L^k}} \right)^{\frac{\sigma_L^k}{\sigma_L^k - 1}} \tag{61}$$

$$\tilde{N}_t^{k,e} = \left(\left(\alpha_N^k\right)^{\frac{1}{\sigma_N^k}} \left(\tilde{EK}_t^{k,e}\right)^{\frac{\sigma_N^k - 1}{\sigma_N^k}} + \left(1 - \alpha_N^k\right)^{\frac{1}{\sigma_N^k}} \left(\tilde{FK}_t^{k,e}\right)^{\frac{\sigma_N^k - 1}{\sigma_N^k}} \right)^{\frac{\sigma_N^k}{\sigma_N^k - 1}} \tag{62}$$

$$\tilde{EK}_t^{k,e} = \psi_{EK}^k \left(\left(\alpha_{EK}^k\right)^{\frac{1}{\sigma_{EK}^k}} \left(\tilde{E}_t^{k,e}\right)^{\frac{\sigma_{EK}^k - 1}{\sigma_{EK}^k}} + \left(1 - \alpha_E^k\right)^{\frac{1}{\sigma_{EK}^k}} \left(\tilde{K}_{e,t}^{k,e}\right)^{\frac{\sigma_{EK}^k - 1}{\sigma_{EK}^k}} \right)^{\frac{\sigma_{EK}^k}{\sigma_{EK}^k - 1}} \tag{63}$$

$$\tilde{FK}_t^{k,e} = \psi_{FK}^k \left(\left(\alpha_{FK}^k\right)^{\frac{1}{\sigma_{FK}^k}} \left(\tilde{F}_t^{k,e}\right)^{\frac{\sigma_{FK}^k-1}{\sigma_{FK}^k}} + \left(1-\alpha_F^k\right)^{\frac{1}{\sigma_{FK}^k}} \left(\tilde{K}_{f,t}^{k,e}\right)^{\frac{\sigma_{FK}^k-1}{\sigma_{FK}^k}} \right)^{\frac{\sigma_{FK}^k}{\sigma_{FK}^k-1}} \tag{64}$$

$$\tilde{E}_t^{k,e} = \left(\left(\alpha_E^k\right)^{\frac{1}{\sigma_E^k}} \left(\tilde{W}_{RY,t}^k\right)^{\frac{\sigma_E^k-1}{\sigma_E^k}} + \left(1-\alpha_E^k\right)^{\frac{1}{\sigma_E^k}} \left(\tilde{W}_{FY,t}^k\right)^{\frac{\sigma_E^k-1}{\sigma_E^k}} \right)^{\frac{\sigma_E^k}{\sigma_E^k-1}} \tag{65}$$

$$\tilde{F}_t^{k,e} = \left(\left(\alpha_F^k\right)^{\frac{1}{\sigma_F^k}} \left(\tilde{W}_{RF,t}^k\right)^{\frac{\sigma_F^k-1}{\sigma_F^k}} + \left(1-\alpha_F^k\right)^{\frac{1}{\sigma_F^k}} \left(\tilde{W}_{FF,t}^k\right)^{\frac{\sigma_F^k-1}{\sigma_F^k}} \right)^{\frac{\sigma_F^k}{\sigma_F^k-1}} \tag{66}$$

$$\tilde{W}_{j,t}^k = \psi_{W,j}^k \left(\left(\alpha_{W,j}^k\right)^{\frac{1}{\sigma_{W,j}}} \left(\tilde{Z}_{j,t}^{k,e}\right)^{\frac{\sigma_{W,j}-1}{\sigma_{W,j}}} + \left(1-\alpha_{W,j}^k\right)^{\frac{1}{\sigma_{W,j}}} \left(\tilde{Z}_{jM,t}^{k,e}\right)^{\frac{\sigma_{W,j}-1}{\sigma_{W,j}}} \right)^{\frac{\sigma_{W,j}}{\sigma_{W,j}-1}} \tag{67}$$
$$\forall j \in VAS$$

$$\tilde{K}_{j,t}^{k,e} = \psi_{Kj}^k \sum_{c\in PIC} \left(\frac{\tilde{K}_{Ic,t-1}^{GV}}{\tilde{K}_{Ic}^{GV}}\right)^{\phi_{Kj,c}^k} \left(\tilde{K}_{Pj,t-1}^{k,e} + \tilde{K}_{Pj,k,e,t-1}^{GV}\right)/g_{t-1} \tag{68}$$
$$\forall j \in \{e,f,g\}.$$

The aggregate laws of motion of the capital stock normalized by trend growth are

$$\tilde{K}_{Pj,t}^{k,e} = \varepsilon_{K,t}^k \left(1 - \frac{\tau_I}{2}\left(\frac{\tilde{Z}_{IV,Pj,t}^{k,e}}{\tilde{Z}_{IV,Pj,t-1}^{k,e}} - 1\right)^2\right) \tilde{Z}_{IV,Pj,t}^{k,e} + (1-\delta)\tilde{K}_{Pj,t-1}^{k,e}/g_{t-1} \tag{69}$$
$$\forall k \in \{FGS, VAS\} \text{ and } \forall e \in \{c,v\} \text{ and } \forall j \in \{e,f,g\}.$$

The aggregated detrended real distributed profits are

$$\begin{aligned}\tilde{J}_t^{k,e} &= p_{Y,t}^{k,e}\tilde{Y}_t^{k,e} - \sum_{j\in\{VAS,FV,IMS\}} p_{Y,t}^j \tilde{Z}_{j,t}^{k,e} - \sum_{j\in HHS} \tilde{p}_{Y,t}^j Z_{j,t}^{k,e} - \hat{T}_{P,t}^{k,e} - \hat{T}_{V,t}^{k,e} - \hat{T}_{U,t}^{k,e} - \hat{T}_{G,t}^{k,e} + \hat{S}_{U,t}^{k,e} \\ &\quad - d^e\left(\omega_{COx}p_{COx,t} + \omega_{COs}p_{COs,t} + (1-\omega_{COx}-\omega_{COs})(1-\omega_{CCg})\,p_{CC,t}\right)G_t^{k,e} + p_{COx,t}CO_{x,t}^{k,e} \\ &\quad + p_{COs,t}CO_{s,t}^{k,e} - p_{Y,t}^{IV}\tilde{Z}_{IV,t}^{k,e} + \hat{L}_{d,t}^{k,e} + \mathcal{E}_t\hat{L}_{f,t}^{k,e} - \frac{R_{Ld,t-1}}{\Pi_{C,t}}\hat{L}_{d,t-1}^{k,e}/g_{t-1} - \frac{R_{Lf,t-1}}{\Pi_{C,t}^{RW}}\mathcal{E}_t\hat{L}_{f,t-1}^{k,e}/g_{t-1} \\ &\quad \forall k \in \{FGS, VAS\}.\end{aligned} \tag{70}$$

where

$$\tilde{Z}_{IV,t}^{k,e} = \sum_{j\in\{e,f,g\}} \tilde{Z}_{IV,Pj,t}^{k,e} \tag{71}$$

and where $\hat{T}_{P,t}^{k,e}$, $\hat{T}_{V,t}^{k,e}$, $\hat{T}_{U,t}^{k,e}$, $\hat{T}_{G,t}^{k,e}$, and $\hat{S}_{U,t}^{k,e}$ are corporate income taxes, ad valorem taxes on inputs, unit taxes on inputs, emission taxes, as well as unit subsidies. They are defined in section 5.

Green house gas emissions in sector k and regime e can be aggregated to

$$G_t^{k,e} = \sum_{j\in\{VAS,IMS\}} \psi_{Gj}^k Z_{j,t}^{k,e} + \left(1 - x_t^{k,e}\right) \sum_{m\in GHG} \psi_{GY,m}^k Y_t^{k,e} \tag{72}$$
$$\forall k \in \{FGS, VAS\}.$$

The total supply of abatement-backed carbon offsets in sector k and regime e is

$$CO_{x,t}^{k,e} = (1 - d^e)\, x_t^{k,e} \sum_{m \in GHG} \psi_{GY,m}^{k} Y_t^{k,e} \tag{73}$$

where $CO_{x,t}^{k,c} = 0$ as $d^c = 1$ per (44). Only firms that are not subject to ETS compliance can supply carbon offsets to voluntary carbon markets.

Price setting. Taking as given the price of each intermediate good variety, $P_{Y,t}^{k,e,i}$, and the price of the homogeneous good, $P_{Y,t}^{k,e}$, the representative aggregator in sector k and regime e chooses the optimal demand for input variety $Z_{i,t}^{k,e}$ in order to minimize the costs of producing $Y_t^{k,e}$ units of a homogeneous k-good. The cost minimization problem of the aggregator in sector k and regime e reads:

$$\min_{Z_{i,t}^{k,e}} \int_0^1 P_{Y,t}^{k,e,i} Z_{i,t}^{k,e} di$$

subject to a Dixit-Stiglitz aggregator

$$Y_t^{k,e} = \left(\int_0^1 \left(Z_{i,t}^{k,e} \right)^{\frac{\sigma_{P,t}^k - 1}{\sigma_{P,t}^k}} di \right)^{\frac{\sigma_{P,t}^k}{\sigma_{P,t}^k - 1}},$$

where $\sigma_{P,t}^k > 1$ is the elasticity of substitution between the intermediate good varieties in sector k. Noting that the Lagrangian multiplier of the constraint is equal to the aggregate price index, $P_{Y,t}^{k,e}$, the FOC implies

$$Z_{i,t}^{k,e} = \left(\frac{P_{Y,t}^{k,e,i}}{P_{Y,t}^{k,e}} \right)^{-\sigma_{P,t}^k} Y_t^{k,e}.$$

The demand function for variety i in sector k imposes a constraint to the intermediate-good firm's problem of choosing the optimal price for its variety $P_{Y,t}^{k,e,i}$ such that the firm's supply of its variety matches the aggregator's demand, $Y_t^{k,e,i} = Z_{i,t}^{k,e}$.

Only a fraction $1 - \xi_P^k$ of firms in sector k and regime e can reset their price $P_{Y,t}^{\#k,e,i}$ in any given period t. The remaining firms index their price according to the rule

$$P_{Y,t}^{k,e,i} = \Theta_{t-1}^k P_{Y,t-1}^{k,e,i}$$

where

$$\Theta_t^k = \left(\Pi_{Y,t}^k \right)^{\omega^k} \bar{\Pi}_C^{1-\omega^k} \qquad \forall k \in \{FGS, VAS\}. \tag{74}$$

Taking as given the demand schedule for its intermediate good variety and the price indexation rule, a firm i in sector k and regime e which is able to reset its price $P_{Y,t}^{\#k,e,i}$ solves the following optimization problem:

$$\max_{P_{Y,t}^{\#k,e,i}} \mathrm{E}_t \sum_{s=t}^{\infty} \frac{\prod_{r=t}^{s} \Lambda_{r-1,r} \left(1 - \Gamma_{S,r}^{k,e} \right)}{\Lambda_{t-1,t}} \left(\xi_P^k \right)^{s-t} \frac{J_s^{k,e,i}}{P_{C,s}}$$

where

$$J_s^{k,e,i} = (1-t_P) \prod_{r=t+1}^{s} \Theta_{r-1}^{k} P_{Y,t}^{\#k,e,i} \left(\frac{\prod_{r=t+1}^{s} \Theta_{r-1}^{k} P_{Y,t}^{\#k,e,i}}{P_{Y,s}^{k,e}} \right)^{-\sigma_{P,s}^{k}} Y_s^{k,e} - $$
$$- MC_s^{k,e,i} \left(\frac{\prod_{r=t+1}^{s} \Theta_{r-1}^{k} P_{Y,t}^{\#k,e,i}}{P_{Y,s}^{k,e}} \right)^{-\sigma_{P,s}^{k}} Y_s^{k,e}$$

The FOC w.r.t. $P_{Y,t}^{\#k,e,i}$ implies

$$p_{Y,t}^{\#k,e} = \frac{\tilde{A}_{1,t}^{k,e}}{\tilde{A}_{2,t}^{k,e}} \tag{75}$$
$$\forall k \in \{FGS, VAS\} \text{ and } \forall e \in \{c, v\}$$

where

$$\tilde{A}_{1,t}^{k,e} = \sigma_{P,t}^{k} mc_t^{k,e} \left(p_{Y,t}^{k,e}\right)^{\sigma_{P,t}^{k}} \tilde{Y}_t^{k,e} + +\xi_P^k \mathrm{E}_t \Lambda_{t,t+1} \left(1 - \Gamma_{S,t+1}^{k,e}\right) \left(\frac{\Pi_{C,t+1}}{\Theta_t^k}\right)^{\sigma_{P,t+1}^{k}} \tilde{A}_{1,t+1}^{k,e} g_t \tag{76}$$

$$\tilde{A}_{2,t}^{k,e} = \left(\sigma_{P,t}^{k} - 1\right)(1-t_P) \left(p_{Y,t}^{k,e}\right)^{\sigma_{P,t}^{k}} \tilde{Y}_t^{k,e} + \xi_P^k \mathrm{E}_t \Lambda_{t,t+1} \left(1 - \Gamma_{S,t+1}^{k,e}\right) \left(\frac{\Pi_{C,t+1}}{\Theta_t^k}\right)^{\sigma_{P,t+1}^{k}-1} \tilde{A}_{2,t+1}^{k,e} g_t \tag{77}$$

with

$$mc_t^{k,e} = \varphi_t^{k,e} + \tag{78}$$
$$+ (1-t_P) \left[\begin{array}{c} \left(t_G + d^e \left(\omega_{COx} p_{COx,t} + \omega_{COs} p_{COs,t} + (1 - \omega_{COx} - \omega_{COs}) p_{CC,t}\right)\right) \left(1 - x_t^{k,e}\right) - \\ - (1-d^e) p_{COx,t} x_t^{k,e} \end{array} \right] \sum_{m \in GHG} \psi_{GY,m}^{k}.$$

We used the fact that all firms able to do so set the same price and we approximated $\Pi_{C,t}^{1+\sigma_{P,t}^k} \simeq P_{C,t}^{1+\sigma_{P,t}^k} / P_{C,t-1}^{1+\sigma_{P,t-1}^k}$ and $\Pi_{C,t}^{\sigma_{P,t}^k} \simeq P_{C,t}^{\sigma_{P,t}^k} / P_{C,t-1}^{\sigma_{P,t-1}^k}$.

The resulting price index of optimal and indexed prices in sector k and regime e is

$$p_{Y,t}^{k,e} = \left(\xi_P^k \left(\Theta_{t-1}^k \frac{p_{Y,t-1}^{k,e}}{\Pi_{C,t}} \right)^{1-\sigma_{P,t}^k} + \left(1 - \xi_P^k\right) \left(p_{Y,t}^{\#k,e}\right)^{1-\sigma_{P,t}^k} \right)^{\frac{1}{1-\sigma_{P,t}^k}} \tag{79}$$
$$\forall k \in \{FGS, VAS\} \text{ and } \forall e \in \{c, v\}.$$

3.2 Emission-trading-system firm (ETS)

The representative ETS-firm in sector $k \in \{FGS, VAS\}$ produces a homogeneous good Y_t^k combining the output of firms regulated by the ETS $Z_{c,t}^k$ and the output of firms unregulated by the ETS $Z_{v,t}^k$.[9] It uses a CES production function and operates under perfect competition. The cost minimization problem of the ETS-firm in sector k reads

$$\min_{Z_{c,t}^k, Z_{v,t}^k} P_{Y,t}^{k,c} Z_{c,t}^k + P_{Y,t}^{k,v} Z_{v,t}^k$$

[9] Note that, after the zero-restrictions are in place, ETS-firms will only be relevant in VAS sectors.

subject to a CES production structure,

$$Y_t^k = \left(\left(\alpha_G^k\right)^{\frac{1}{\sigma_G}} \left(Z_{c,t}^k\right)^{\frac{\sigma_G-1}{\sigma_G}} + (1-\alpha_G^k)^{\frac{1}{\sigma_G}} \left(Z_{v,t}^k\right)^{\frac{\sigma_G-1}{\sigma_G}} \right)^{\frac{\sigma_G}{\sigma_G-1}}.$$

After detrending, the FOCs of this problem imply

$$p_{Y,t}^{k,c} = p_{Y,t}^k \left(\alpha_G^k\right)^{\frac{1}{\sigma_G}} \frac{\tilde{Y}_t^k}{\tilde{Z}_{c,t}^k} \quad \forall k \in \{FGS, VAS\} \tag{80}$$

$$p_{Y,t}^{k,v} = p_{Y,t}^k \left(1-\alpha_G^k\right)^{\frac{1}{\sigma_G}} \frac{\tilde{Y}_t^k}{\tilde{Z}_{v,t}^k} \quad \forall k \in \{FGS, VAS\} \tag{81}$$

$$p_{Y,t}^k \tilde{Y}_t^k = p_{Y,t}^{k,c} \tilde{Z}_{c,t}^k + p_{Y,t}^{k,v} \tilde{Z}_{v,t}^k \quad \forall k \in \{FGS, VAS\} \tag{82}$$

where $p_{Y,t}^k \equiv P_{Y,t}^k / P_{C,t}$, $p_{Y,t}^{k,c} \equiv P_{Y,t}^{k,c} / P_{C,t}$, and $p_{Y,t}^{k,v} \equiv P_{Y,t}^{k,v} / P_{C,t}$. Eqs. (80) and (81) are the ETS-firm's demand functions for output by compliant firms and unregulated firms, respectively. Note that σ_G controls the elasticity of substitution between complient and unregulated output and, hence, captures an important form of carbon leakage. Eq. (82) is the ETS-firm's budget constraint.

3.3 Banking sector

A representative domestic bank determines the end-of-period debt constraint of FGS and VAS firms, the finance premium on firms' investment expenses, and the lending rate. The bank operates under perfect competition and faces an exogenous share of non-performing loans, ω_L.

As mentioned in section 3.1, the bank imposes a constraint to firms' end-of-period debt: The return on debt in sector k cannot exceed a given share λ_d^k of the expected value of the collateral. We assume the bank chooses λ_d^k in an ad-hoc manner $\forall k \in \{FGS, VAS\}$.

The investment finance premium is related to deposit rate as

$$R_{Fd,t} \left(1 - \Gamma_{Fd}\right) = R_{Dd,t}. \tag{83}$$

The bank's budget constraint equates the uses of funds (interest on deposits and new loans) to the sources of funds (new deposits, interest on performing loans, and investment finance premium). It determines the lending rate and implies

$$\begin{aligned} & R_{Dd,t-1} \int_0^1 D_{d,t-1}^{HS,i} di + \\ & + \sum_{k\in\{FGS,VAS\}} \sum_{e\in\{c,v\}} \int_0^1 L_{d,t}^{k,e,i} di = \int_0^1 D_{d,t}^{HS,i} di \\ & \quad + R_{Ld,t-1} \sum_{k\in\{FGS,VAS\}} \sum_{e\in\{c,v\}} \int_{\omega_L}^1 L_{d,t-1}^{k,e,i} di + \\ & \quad + (R_{Fd,t} - 1) \sum_{k\in\{FGS,VAS\}} \sum_{e\in\{c,v\}} \int_0^1 P_{Y,t}^{IV} Z_{IV,t}^{k,e,i} di \end{aligned}$$

$$
\begin{aligned}
&R_{Dd,t-1}\hat{D}^{HS}_{d,t-1}/g_{t-1}+\\
&+\sum_{k\in\{FGS,VAS\}}\sum_{e\in\{c,v\}}\hat{L}^{k,e}_{d,t}=\hat{D}^{HS}_{d,t}\\
&\qquad\qquad+R_{Ld,t-1}\omega_L\sum_{k\in\{FGS,VAS\}}\sum_{e\in\{c,v\}}\hat{L}^{k,e}_{d,t-1}/g_{t-1}+\\
&\qquad\qquad+\left(R_{Fd,t}-1\right)\sum_{k\in\{FGS,VAS\}}\sum_{e\in\{c,v\}}p^{IV}_{Y,t}\tilde{Z}^{k,e}_{IV,t}
\end{aligned}
\tag{84}
$$

In the OMEGA model for Brazil, we assume that the lending rate $R_{Ld,t}$ is a policy variable specified below. Hence, the budget constraint is not necessarily satisfied unless the share of non-performing loans is endogenous, which we assume here.

3.4 Employment firm

Since, employment and unemployment are both more sluggish than hours worked, we follow Smets and Wouters (2003) and the subsequent literature and establish an auxiliary equation which relates employment and hours. In particular, we derive a relationship between the low-skilled employment rate on the extensive margin, $e_{e,t}$, and the ratio of low-skilled hours worked and hours supplied, $e_t \equiv \left(\sum_{k\in VAS}\sum_{k\in\{c,v\}} Z^{k,e}_{LS,t} + Z^{FV}_{LS,t}\right)/Y^{LS}_t$.

A representative low-skilled employment firm observes the employment rate in terms of hours and chooses the employment rate on the extensive margin, $e^{\#}_{e,t}$, to minimize the quadratic difference between the employment rate in persons and in hours. In every period, the EM-firm can reset $e^{\#}_{e,t}$ only with a probability $1-\xi_E\varepsilon_{\xi,t}$ where $\varepsilon_{\xi,t}$ is an i.i.d. shock. The problem reads

$$
\min_{e^{\#}_{e,t}} \mathrm{E}_t \sum_{s=t}^{\infty}\left(\beta\xi_E\varepsilon_{\xi,s}\right)^{s-t}\frac{1}{2}\left(e_{e,s}-e^{\#}_{e,t}\right)^2 .
$$

The FOC implies

$$
\varepsilon_{\xi,t}e^{\#}_{e,t}=\left(1-\beta\xi_E\right)\sum_{s=t}^{\infty}\left(\beta\xi_E\varepsilon_{\xi,s}\right)^{s-t}e_{e,s}
$$

The aggregate extensive employment rate follows

$$
e_{e,t}=\xi_E\varepsilon_{\xi,t}e_{e,t-1}+\left(1-\xi_E\varepsilon_{\xi,t}\right)e^{\#}_{e,t}.
$$

Therefore, the relationship between the extensive employment rate and the employment rate in hours is

$$
e_{e,t}-e_{e,t-1}=\beta\left(\mathrm{E}_te_{e,t+1}-e_{e,t}\right)+\frac{\left(1-\xi_E\right)\left(1-\beta\xi_E\right)}{\xi_E}\left(\frac{\sum_{k\in VAS}\sum_{e\in\{c,v\}}Z^{k,e}_{LS,t}+Z^{FV}_{LS,t}}{Y^{LS}_t}-e_{e,t}\right). \tag{85}
$$

The employment rate on the intensive margin is

$$
e_{i,t}=\frac{1}{e_{e,t}}\frac{\sum_{k\in VAS}\sum_{e\in\{c,v\}}Z^{k,e}_{LS,t}+Z^{FV}_{LS,t}}{Y^{LS}_t}. \tag{86}
$$

3.5 Expropriators

We follow Germaschewski et al. (2021) and assume each firm i in sector $k\in VAS$ and ETS-compliance regime $e\in\{c,v\}$ faces an expropriator i which has the power to expropriate some of firm i's value. Foreign-owned value-added (FV-)firms (introduced below in section 3.8) are also subject to expropriation. Expropriation risk allows the model to capture a weak rule of law which poses a dis-incentive for long-term capital investment and foreign direct investments, respectively.

In each period t, the expropriator i expropriates a share $\Gamma_{S,t}^{k,e,i}$ of the corresponding firm i's end-of-period real value. The expropriated share of the firm value is then distributed back to the household sector in a lump-sum manner. $\Gamma_{S,t}^{FV,i}$ is the expropriated share of FV-firms which is transferred to the RW.

Expropriation is subject to convex expropriation costs. These expropriation costs are best understood as non-monetary costs such as the risk of getting prosecuted. To eliminate flows of funds associated with expropriation costs, We assume the prosecution risk for expropriator i to depend on the behavior of the other expropriators $-i$. This setup is equivalent to assuming that expropriation costs are monetary costs which are rebated to the expropriators in a lump-sum way – similar to the VAS firms' emission abatement costs. In the aggregate, the net expropriation cost is zero as all expropriators in sector k and regime e choose the same $\Gamma_{S,t}^{k,e}$.

In each period t, the expropriator i in sector $k \in VAS$ chooses $\Gamma_{S,t}^{k,e,i}$ to solve

$$\max_{\Gamma_{S,t}^{k,e,i}} \quad \Gamma_{S,t}^{k,e,i}\left(\frac{J_t^{k,e,i}}{P_{C,t}}+\mathcal{V}_{S,t}^{k,e,i}\right)-\varepsilon_{\Gamma S,t}^{k}\frac{\tau_S^k}{\phi_S}\left(\Gamma_{S,t}^{k,e,i}\right)^{\phi_S}H_{t-1}+\varepsilon_{\Gamma S,t}^{k}\frac{\tau_S^k}{\phi_S}\left(\Gamma_{S,t}^{k,e}\right)^{\phi_S}H_{t-1}.$$

The FOC w.r.t. $\Gamma_{S,t}^{k,e,i}$ equates the marginal revenue from expropriation to its marginal cost which, in the aggregate, implies

$$\tilde{J}_t^{k,e}+\tilde{\mathcal{V}}_{S,t}^{k,e}=\varepsilon_{\Gamma S,t}\tau_S^k\left(\Gamma_{S,t}^{k,e}\right)^{\phi_S-1} \qquad \forall k \in VAS \tag{87}$$

where

$$\tilde{\mathcal{V}}_t^{k,e}=\mathrm{E}_t\Lambda_{t,t+1}\left(1-\Gamma_{S,t+1}^{k,e}\right)\left(\tilde{J}_{t+1}^{k,e}+\tilde{\mathcal{V}}_{t+1}^{k,e}\right)g_t \qquad \forall k \in VAS. \tag{88}$$

The optimal expropriation rates $\Gamma_{S,t}^{k,e}$ and, hence the risk premium on equity finance in a sector, increases with the value of the firms in that sector.

To be consistent with the objective function of the FV-firms (introduced below), we assume their firm value is expropriated in foreign currency. The expropriator i in sector FV chooses $\Gamma_{S,t}^{FV,i}$ to solve

$$\max_{\Gamma_{S,t}^{FV,i}} \quad \Gamma_{S,t}^{FV,i}\left(\frac{J_t^{FV,i}}{P_{C,t}}\frac{1}{\mathcal{E}_t}+\mathcal{V}_{S,t}^{FV,i}\right)-\varepsilon_{\Gamma S,t}^{FV}\frac{\tau_S^{FV}}{\phi_S}\left(\Gamma_{S,t}^{FV,i}\right)^{\phi_S}H_{t-1}+\varepsilon_{\Gamma S,t}^{FV}\frac{\tau_S^{FV}}{\phi_S}\left(\Gamma_{S,t}^{FV}\right)^{\phi_S}H_{t-1}.$$

The FOC w.r.t. $\Gamma_{S,t}^{FV,i}$ implies

$$\tilde{J}_t^{FV}\frac{1}{\mathcal{E}_t}+\tilde{\mathcal{V}}_{S,t}^{FV}=\varepsilon_{\Gamma S,t}\tau_S^{FV}\left(\Gamma_{S,t}^{FV}\right)^{\phi_S-1} \tag{89}$$

where

$$\tilde{\mathcal{V}}_t^{FV}=\mathrm{E}_t\Lambda_{t,t+1}^{RW}\left(1-\Gamma_{S,t+1}^{FV}\right)\left(\frac{\tilde{J}_{t+1}^{FV}}{\mathcal{E}_{t+1}}+\tilde{\mathcal{V}}_{t+1}^{FV}\right)g_t \tag{90}$$

and where $\Lambda_{t,t+1}^{RW}$ is the stochastic discount factor of the RW defined below in section 3.8.

3.6 Carbon-credit firm

A carbon-credit firm certifies the carbon sequestration by LS households and sells tradable carbon credits on the voluntary carbon market.

3.7 Importers

To keep the flow of funds simple, we assume importers (as seen from the domestic country) to be located in the RW. In that case, profits from import activities are distributed to RW-households and not domestic households.

In each sector $k \in \{VAS, CB\}$, there is a continuum of importers indexed by $i \in [0, 1]$. Each importer purchases an *imported good variety* $Z^{kM,i}_{kW,t}$ from the world market k at a given price $P^{kW}_{Y,t}$ denominated in foreign currency and equal across varieties and resells it in the domestic country as an *import good variety* $Y^{kM,i}_{t}$ to a sector-specific aggregator which transforms the varieties into a homogeneous import good Y^{kM}_{t}. Importers operate under monopolistic competition and, therefore, have price setting power.

We discuss the representative aggregator first and the importers after. Taking as given the price of each import good variety, $P^{kM,i}_{Y,t}$, and the price of the homogeneous import good, $P^{kM}_{Y,t}$, the aggregator in sector k chooses the optimal demand for the input composition variety $Z^{kM}_{i,t}$ in order to minimize the costs of producing Y^{kM}_{t} units of a homogeneous import good for sector k. The cost minimization problem of the aggregator in sector k reads:

$$\min_{Z^{kM}_{i,t}} \int_0^1 P^{kM,i}_{Y,t} Z^{kM}_{i,t} \, di$$

subject to a Dixit-Stiglitz aggregator

$$Y^{kM}_{t} = \left(\int_0^1 \left(Z^{kM}_{i,t} \right)^{\frac{\sigma^{kM}_{P,t}-1}{\sigma^{kM}_{P,t}}} di \right)^{\frac{\sigma^{kM}_{P,t}}{\sigma^{kM}_{P,t}-1}},$$

where $\sigma^{kM}_{P,t} > 1$ is the elasticity of substitution between the import good varieties in sector k. Noting that the Lagrangian multiplier of the constraint is equal to the aggregate price index, $P^{kM}_{Y,t}$, the FOC implies

$$Z^{kM}_{i,t} = \left(\frac{P^{kM,i}_{Y,t}}{P^{kM}_{Y,t}} \right)^{-\sigma^{kM}_{P,t}} Y^{kM}_{t}.$$

The demand function for import good variety i in sector k imposes a constraint to the importer's problem of choosing the optimal price for its variety $P^{kM,i}_{Y,t}$ such that the firm's supply of its variety matches the aggregator's demand, $Y^{kM,i}_{t} = Z^{kM}_{i,t}$.

Let us turn to the importers. Only a fraction $1-\xi^{kM}_{P}$ of importers in sector k can reset their price $P^{\#kM,i}_{Y,t}$ in any given period t. The remaining firms index their price according to the rule

$$P^{kM,i}_{Y,t} = \Theta^{kM}_{t-1} P^{kM,i}_{Y,t-1}$$

where

$$\Theta^{kM}_{t} = \left(\Pi^{kM}_{Y,t} \right)^{\omega^{kM}} \bar{\Pi}_{C}^{1-\omega^{kM}} \qquad \forall k \in \{VAS, CB\}. \tag{91}$$

Taking as given the demand schedule for its import good variety, the price indexation rule, the import tariffs, and the simple import-good production technology, $Y_t^{kM,i} = \psi_Y^{kM} Z_{kW,t}^{kM,i}$, an importer i in sector k which is able to reset its price $P_{Y,t}^{\#kM,i}$ solves the following optimization problem:

$$\max_{P_{Y,t}^{\#kM,i}} \mathrm{E}_t \sum_{s=t}^{\infty} \frac{\prod_{r=t}^{s} \Lambda_{r-1,r}^{RW}}{\Lambda_{t-1,t}^{RW}} \left(\xi_P^{kM}\right)^{s-t} \frac{J_s^{kM,i}}{P_{C,s}} \frac{1}{\mathcal{E}_s}$$

where

$$J_s^{kM,i} = \prod_{r=t+1}^{s} \Theta_{r-1}^{kM} P_{Y,t}^{\#kM,i} \left(\frac{\prod_{r=t+1}^{s} \Theta_{r-1}^{kM} P_{Y,t}^{\#kM,i}}{P_{Y,s}^{kM}} \right)^{-\sigma_{P,s}^{kM}} Y_s^{kM} -$$
$$- \mathcal{S}_s P_{Y,s}^{kW,i} \left(\frac{\prod_{r=t+1}^{s} \Theta_{r-1}^{kM} P_{Y,t}^{\#kM,i}}{P_{Y,s}^{kM}} \right)^{-\sigma_{P,s}^{kM}} \frac{Y_s^{kM}}{\psi_Y^{kM}}$$

The FOC w.r.t. $P_{Y,t}^{\#kM,i}$ implies

$$p_{Y,t}^{\#kM} = \frac{\tilde{A}_{1,t}^{kM}}{\tilde{A}_{2,t}^{kM}} \qquad \forall k \in \{VAS, CB\} \tag{92}$$

where

$$\tilde{A}_{1,t}^{kM} = \sigma_{P,t}^{kM} p_{Y,t}^{kW} \left(p_{Y,t}^{kM}\right)^{\sigma_{P,t}^{kM}} \frac{\tilde{Y}_t^{kM}}{\psi_Y^{kM}} + \xi_P^{kM} \mathrm{E}_t \Lambda_{t,t+1}^{RW} \left(\frac{\Pi_{C,t+1}}{\Theta_t^{kM}} \right)^{\sigma_{P,t+1}^{kM}} \tilde{A}_{1,t+1}^{kM} g_t \tag{93}$$

$$\tilde{A}_{2,t}^{kM} = \left(\sigma_{P,t}^{kM} - 1\right) \frac{1}{\mathcal{E}_t} \left(p_{Y,t}^{kM}\right)^{\sigma_{P,t}^{kM}} \tilde{Y}_t^{kM} + \xi_P^{kM} \mathrm{E}_t \Lambda_{t,t+1}^{RW} \left(\frac{\Pi_{C,t+1}}{\Theta_t^{kM}} \right)^{\sigma_{P,t+1}^{kM} - 1} \tilde{A}_{2,t+1}^{kM} g_t. \tag{94}$$

We used the fact that all firms able to do so set the same price and where we approximated $\Pi_{C,t}^{1+\sigma_{P,t}^{kM}} \simeq P_{C,t}^{1+\sigma_{P,t}^{kM}} / P_{C,t-1}^{1+\sigma_{P,t-1}^{kM}}$ and $\Pi_{C,t}^{\sigma_{P,t}^{kM}} \simeq P_{C,t}^{\sigma_{P,t}^{kM}} / P_{C,t-1}^{\sigma_{P,t-1}^{kM}}$. Note that ψ_Y^{kM} is calibrated such that $p_{Y,t}^{kM} = p_{Y,t}^{kW}$ at the steady state despite the importers' mark-up.

The resulting price index of optimal and indexed prices in sector k is

$$p_{Y,t}^{kM} = \left(\xi_P^{kM} \left(\Theta_{t-1}^{kM} \frac{p_{Y,t-1}^{kM}}{\Pi_{C,t}} \right)^{1-\sigma_{P,t}^{kM}} + \left(1 - \xi_P^{kM}\right) \left(p_{Y,t}^{\#kM}\right)^{1-\sigma_{P,t}^{kM}} \right)^{\frac{1}{1-\sigma_{P,t}^{kM}}} \tag{95}$$
$$\forall k \in \{VAS, CB\}.$$

3.8 Foreign-owned capital-service sector (FV)

Using capital inputs, the FV sector produces a captial service and sells it to VAS firms. There is a continuum of FV-firms indexed by $i \in [0,1]$ producing differentiated intermediate goods $Y_t^{FV,i}$. Each firm i operates under monopolistic competition and, therefore, has price setting power. The intermediate good $Y_t^{FV,i}$ is sold to a FV-specific aggregator which transforms the varieties into a homogeneous good, Y_t^{FV}. FV-firms do not generate emissions. FV-firms are owned by households in the RW. Dividends are paid in domestic currency but equity shares are traded in foreign currency. FV-firms are subject to firm-value expropriation as discussed above. For the sake of simplicity, we assume zero end-of-period debt.

Investment and capital. Firm i in sector FV chooses the paths of investment and the stock of general capital to maximize its non-expropriated beginning-of-period real value expressed in foreign currency units,

$$\left(1-\Gamma_{S,t}^{FV}\right)\left(\frac{J_t^{FV,i}}{P_{C,t}}\frac{1}{\mathcal{E}_t}+\mathcal{V}_{S,t}^{FV,i}\right)$$

with

$$\mathcal{V}_t^{FV,i}=\mathrm{E}_t\Lambda_{t,t+1}^{RW}\left(1-\Gamma_{S,t+1}^{FV}\right)\left(\frac{J_{t+1}^{FV,i}}{P_{C,t+1}}\frac{1}{\mathcal{E}_{t+1}}+\mathcal{V}_{t+1}^{FV,i}\right)$$

where

$$\Lambda_{t-1,t}^{RW}=\left(\frac{R_{Bf,t-1}^{RW}}{\Pi_{C,t}^{RW}}\right)^{-1} \tag{96}$$

is the stochastic discount factor of the RW applied to future real dividends denominated in foreign currency. Note that, for the sake of simplicity, we implicitly assume that households in the RW do not derive utility from holding wealth.[10]

Evaluated at period t and expressing the objective as an infinite sum rather than a value function, the optimization problem of firm i reads

$$\max_{\{Z_{IV,Pg,s}^{FV,i},K_{Pg,s}^{FV,i}\}_{s=0}^{\infty}}\mathrm{E}_t\sum_{s=t}^{\infty}\frac{\prod_{r=t}^{s}\Lambda_{r-1,r}^{RW}\left(1-\Gamma_{S,r}^{FV}\right)}{\Lambda_{t-1,t}^{RW}}\frac{J_s^{FV,i}}{P_{C,s}}\frac{1}{\mathcal{E}_{s+1}}$$

subject to the firm's budget constraint which equates uses of funds (investment good expenses, corporate income taxes, and dividends) and sources of funds (sales revenues) as

$$+P_{Y,s}^{IV}Z_{IV,Pg,s}^{FV,i}+T_{P,s}^{FV,i}+J_s^{FV,i}=P_{Y,s}^{FV,i}Y_s^{FV,i},$$

corporate income taxes,

$$T_{P,s}^{FV,i}=t_P\left[\begin{array}{c}P_{Y,s}^{FV,i}Y_s^{FV,i}-\\-P_{C,s}Q_{g,s}^{FV}\delta K_{Pg,s-1}^{FV,i}-P_{C,s}Q_{g,s}^{FV}\varepsilon_{K,s}^{FV}\frac{\tau_I}{2}\left(\frac{Z_{IV,Pg,s}^{FV,i}}{g_{s-1}Z_{IV,Pg,s-1}^{FV,i}}-1\right)^2Z_{IV,Pg,s}^{FV,i}\end{array}\right]$$

the nested CES production structure,

$$Y_s^{FV,i}=\epsilon_{Y,s}^{FV}\psi_Y^{FV}K_s^{FV,i}$$

$$K_{g,s}^{FV,i}=\psi_{Kg}^{FV}\sum_{c\in PIC}\left(\frac{K_{Ic,s-1}^{GV}}{K_{Ic}^{GV}/g_{s-1}}\right)^{\phi_{Kg,c}^{FV}}K_{Pg,s-1}^{FV,i},$$

and the law of motion of capital,

$$K_{Pg,s}^{FV,i}=\varepsilon_{K,s}^{FV}\frac{\tau_I}{2}\left(\frac{Z_{IV,Pg,s}^{FV,i}}{g_{s-1}Z_{IV,Pg,s-1}^{FV,i}}-1\right)^2Z_{IV,Pg,s}^{FV,i}+(1-\delta)K_{Pg,s-1}^{FV,i}.$$

[10] Per the definition of the stochastic discount factor, $\Lambda_{t,t+1}^{RW}$ equates the expected discounted utility of the RW-household from 1 unit of domestic income in $t+1$ and the utility from $\Lambda_{t,t+1}^{RW}$ units of income in t: $1\mathrm{E}_t\lambda_{t+1}^{RW}\beta^{RW}=\mathrm{E}_t\Lambda_{t,t+1}^{RW}\lambda_t^{RW}$. We then obtain (96) using the implicit RW-household's FOC w.r.t. foreign bonds, $\lambda_t^{RW}=\beta^{RW}\mathrm{E}_tR_{Bf,t}^{RW}/\Pi_{C,t+1}^{RW}\lambda_{t+1}^{RW}$.

The FOC w.r.t. investment demand, $Z^{FV,i}_{IV,Pg,s}$, implies

$$p^{IV}_{Y,t} = Q^{FV}_{g,t}\varepsilon^{FV}_{K,t} - (1-t_P)\,Q^{FV}_{g,t}\varepsilon^{FV}_{K,t}\left(\frac{\tau_I}{2}\left(\frac{\tilde{Z}^{FV}_{IV,Pg,t}}{\tilde{Z}^{FV}_{IV,Pg,t-1}}-1\right)^2 + \tau_I\left(\frac{\tilde{Z}^{FV}_{IV,Pg,t}}{\tilde{Z}^{FV}_{IV,Pg,t-1}}-1\right)\frac{\tilde{Z}^{FV}_{IV,Pg,t}}{\tilde{Z}^{FV}_{IV,Pg,t-1}}\right) \tag{97}$$

$$+\,\mathrm{E}_t\Lambda^{RW}_{t,t+1}\left(1-\Gamma^{FV}_{S,t+1}\right)\frac{\mathcal{E}_t}{\mathcal{E}_{t+1}}(1-t_P)\,Q^{FV}_{g,t+1}\varepsilon^{FV}_{K,t+1}\tau_I\left(\frac{\tilde{Z}^{FV}_{IV,Pg,t+1}}{\tilde{Z}^{FV}_{IV,Pg,t}}-1\right)\left(\frac{\tilde{Z}^{FV}_{IV,Pg,t+1}}{\tilde{Z}^{FV}_{IV,Pg,t}}\right)^2 g_t.$$

The FOC w.r.t. the general capital stock, $K^{FV,i}_{Pg,s}$, implies

$$Q^{FV}_{g,t} = \mathrm{E}_t\Lambda^{RW}_{t,t+1}\left(1-\Gamma^{FV}_{S,t+1}\right)\frac{\mathcal{E}_t}{\mathcal{E}_{t+1}}\left(\varphi^{FV}_{t+1}\frac{\partial Y^{FV}_{t+1}}{\partial K^{FV}_{Pg,t}} + (1-(1-t_P)\delta)\,Q^{FV}_{g,t+1}\right). \tag{98}$$

The CES production structure can be aggregated as

$$\tilde{Y}^{FV}_t = \epsilon^{FV}_{Y,t}\psi^{FV}_Y\tilde{K}^{FV}_t \tag{99}$$

$$\tilde{K}^{FV}_{g,t} = \psi^{FV}_{Kg}\sum_{c\in PIC}\left(\frac{\tilde{K}^{GV}_{Ic,t-1}}{\tilde{K}^{GV}_{Ic}}\right)^{\phi^{FV}_{Kg,c}}\tilde{K}^{FV}_{Pg,t-1}/g_{t-1}. \tag{100}$$

The aggregate laws of motion of the capital stock normalized by trend growth are

$$\tilde{K}^{FV}_{Pg,t} = \varepsilon^{FV}_{K,t}\left(1-\frac{\tau_I}{2}\left(\frac{\tilde{Z}^{FV}_{IV,Pg,t}}{\tilde{Z}^{FV}_{IV,Pg,t-1}}-1\right)^2\right)\tilde{Z}^{FV}_{IV,Pg,t} + (1-\delta)\tilde{K}^{FV}_{Pg,t-1}/g_{t-1}. \tag{101}$$

The aggregated detrended real distributed profits, expressed in domestic currency, are

$$\tilde{J}^{FV}_t = p^{FV}_{Y,t}\tilde{Y}^{FV}_t - \hat{T}^{FV}_{P,t} - p^{IV}_{Y,t}\tilde{Z}^{FV}_{IV,Pg,t}. \tag{102}$$

Price setting. Taking as given the price of each intermediate good variety, $P^{FV,i}_{Y,t}$, and the price of the homogeneous good, $P^{FV}_{Y,t}$, the representative aggregator in sector FV chooses the optimal demand for input variety $Z^{FV}_{i,t}$ in order to minimize the costs of producing Y^{FV}_t units of a homogeneous foreign-owned value-added. The cost minimization problem of the aggregator reads:

$$\min_{Z^{FV}_{i,t}}\int_0^1 P^{FV,i}_{Y,t}Z^{FV}_{i,t}\,di$$

subject to a Dixit-Stiglitz aggregator

$$Y^{FV}_t = \left(\int_0^1\left(Z^{FV}_{i,t}\right)^{\frac{\sigma^{FV}_{P,t}-1}{\sigma^{FV}_{P,t}}}di\right)^{\frac{\sigma^{FV}_{P,t}}{\sigma^{FV}_{P,t}-1}},$$

where $\sigma^{FV}_{P,t}>1$ is the elasticity of substitution between the intermediate good varieties in sector FV. Noting that the Lagrangian multiplier of the constraint is equal to the aggregate price index, $P^{FV}_{Y,t}$, the FOC implies

$$Y^{FV,i}_t = \left(\frac{P^{FV,i}_{Y,t}}{P^{FV}_{Y,t}}\right)^{-\sigma^{FV}_{P,t}}Y^{FV}_t.$$

The demand function for variety i in sector FV imposes a constraint to the intermediate-good firm's problem of choosing the optimal price for its variety $P_{Y,t}^{FV,i}$ such that the firm's supply of its variety matches the aggregator's demand, $Y_t^{FV,i} = Z_{i,t}^{FV}$.

Only a fraction $1-\xi_P^{FV}$ of firms in sector FV can reset their price $P_{Y,t}^{\#FV,i}$ in any given period t. The remaining firms index their price according to the rule

$$P_{Y,t}^{FV,i} = \Theta_{t-1}^{FV} P_{Y,t-1}^{FV,i}$$

where

$$\Theta_t^{FV} = \left(\Pi_{Y,t}^{FV}\right)^{\omega^{FV}} \bar{\Pi}_C^{1-\omega^{FV}}. \tag{103}$$

Taking as given the demand schedule for its intermediate good variety and the price indexation rule, a firm i in sector FV which is able to reset its price $P_{Y,t}^{\#FV,i}$ in period t solves the following optimization problem:

$$\max_{P_{Y,t}^{\#FV,i}} \mathrm{E}_t \sum_{s=t}^{\infty} \frac{\prod_{r=t}^{s} \Lambda_{r-1,r}^{RW}\left(1-\Gamma_{S,r}^{FV}\right)}{\Lambda_{t-1,t}^{RW}} \left(\xi_P^{FV}\right)^{s-t} \frac{J_s^{FV,i}}{P_{C,s}} \frac{1}{\mathcal{E}_s}$$

where

$$\begin{aligned} J_s^{FV,i} = {} & (1-t_P) \prod_{r=t+1}^{s} \Theta_{r-1}^{FV} P_{Y,t}^{\#FV,i} \left(\frac{\prod_{r=t+1}^{s}\Theta_{r-1}^{FV} P_{Y,t}^{\#FV,i}}{P_{Y,s}^{FV}}\right)^{-\sigma_{P,s}^{FV}} Y_s^{FV} - \\ & - MC_s^{FV,i} \left(\frac{\prod_{r=t+1}^{s}\Theta_{r-1}^{FV} P_{Y,t}^{\#FV,i}}{P_{Y,s}^{FV}}\right)^{-\sigma_{P,s}^{FV}} Y_s^{FV} \end{aligned}$$

The FOC w.r.t. $P_{Y,t}^{\#FV,i}$ implies

$$p_{Y,t}^{\#FV} = \frac{\tilde{A}_{1,t}^{FV}}{\tilde{A}_{2,t}^{FV}} \tag{104}$$

where

$$\begin{aligned} \tilde{A}_{1,t}^{FV} = {} & \sigma_{P,t}^{FV} \varphi_t^{FV} \left(p_{Y,t}^{FV}\right)^{\sigma_{P,t}^{FV}} \tilde{Y}_t^{FV} + \\ & + \xi_P^{FV} \mathrm{E}_t \Lambda_{t,t+1}^{RW} \left(1-\Gamma_{S,t+1}^{FV}\right) \frac{\mathcal{E}_t}{\mathcal{E}_{t+1}} \left(\frac{\Pi_{C,t+1}}{\Theta_t^{FV}}\right)^{\sigma_{P,t+1}^{FV}} \tilde{A}_{1,t+1}^{FV} g_t \end{aligned} \tag{105}$$

$$\tilde{A}_{2,t}^{FV} = \left(\sigma_{P,t}^{FV}-1\right)(1-t_P)\left(p_{Y,t}^{FV}\right)^{\sigma_{P,t}^{FV}} \tilde{Y}_t^{FV} + \tag{106}$$

$$+ \xi_P^{FV} \mathrm{E}_t \Lambda_{t,t+1}^{RW} \left(1-\Gamma_{S,t+1}^{FV}\right) \frac{\mathcal{E}_t}{\mathcal{E}_{t+1}} \left(\frac{\Pi_{C,t+1}}{\Theta_t^{FV}}\right)^{\sigma_{P,t+1}^{FV}-1} \tilde{A}_{2,t+1}^{FV} g_t. \tag{107}$$

We used the fact that all firms able to do so set the same price as well as the relation $MC_t^{FV}/P_{C,t} = \varphi_t^{FV}$, and we approximated $\Pi_{C,t}^{1+\sigma_{P,t}^{FV}} \simeq P_{C,t}^{1+\sigma_{P,t}^{FV}} / P_{C,t-1}^{1+\sigma_{P,t-1}^{FV}}$ and $\Pi_{C,t}^{\sigma_{P,t}^{FV}} \simeq P_{C,t}^{\sigma_{P,t}^{FV}} / P_{C,t-1}^{\sigma_{P,t-1}^{FV}}$.

The resulting price index of optimal and indexed prices is

$$p_{Y,t}^{FV} = \left(\xi_P^{FV}\left(\Theta_{t-1}^{FV}\frac{p_{Y,t-1}^{FV}}{\Pi_{C,t}}\right)^{1-\sigma_{P,t}^{FV}} + \left(1-\xi_P^{FV}\right)\left(p_{Y,t}^{\#FV}\right)^{1-\sigma_{P,t}^{FV}}\right)^{\frac{1}{1-\sigma_{P,t}^{FV}}}. \tag{108}$$

4 Rest of the World (RW)

Foreign export-demand firms located in the RW manage the exports from the domestic country to the RW (*aka* imports of the RW). The economic choices of the export-demand firms are derived from first principles. Yet, they take the following variables as exogenous which we model in an exogenous block: total foreign demand, foreign inflation, foreign interest rate, foreign price of carbon commodities as well as sector-specific foreign demand and sector-specific world-market prices.

4.1 Demand for domestic goods

For each value-added or carbon good k, there is a representative firm in the RW (called exporter as seen from the domestic perspective) that combines domestic goods (exports), $Z_{k,t}^{kW}$, and foreign goods, $Z_{RW,t}^{kW}$, to produce a final-good bundle

$$Y_t^{kW} = \left(\left(\alpha_Y^{kW}\right)^{\frac{1}{\sigma_Y^{kW}}} \left(Z_{k,t}^{kW}\right)^{\frac{\sigma_Y^{kW}-1}{\sigma_Y^{kW}}} + \left(1-\alpha_Y^{kW}\right)^{\frac{1}{\sigma_Y^{kW}}} \left(Z_{RW,t}^{kW}\right)^{\frac{\sigma_Y^{kW}-1}{\sigma_Y^{kW}}} \right)^{\frac{\sigma_Y^{kW}}{\sigma_Y^{kW}-1}}$$

$$\forall k \in \{VAS, CB\}$$

under perfect competition and facing quadratic input adjustment costs. Then, the RW's demand for the domestic good of sector k can be derived from the cost minimization problem as

$$\begin{aligned} &\frac{p_{Y,t}^{k}}{\mathcal{E}_t} + \tau_{Z,k}^{RW} \left(\frac{\tilde{Z}_{k,t}^{kW}}{\tilde{Z}_{k,t-1}^{kW}} - 1 \right) \frac{1}{g_{t-1}} - \\ -\mathrm{E}_t \Lambda_{t,t+1}^{RW} \tau_{Z,k}^{RW} &\left(\frac{\tilde{Z}_{k,t+1}^{kW}}{\tilde{Z}_{k,t}^{kW}} - 1 \right) \frac{\tilde{Z}_{k,t+1}^{kW}}{\tilde{Z}_{k,t}^{kW}} = p_{Y,t}^{kW} \left(\alpha_Y^{kW}\right)^{\frac{1}{\sigma_Y^{kW}}} \left(\frac{\tilde{Y}_t^{kW}}{\tilde{Z}_{k,t}^{kW}} \right)^{\frac{1}{\sigma_Y^{kW}}} \\ &\forall k \in \{VAS, CB\} \end{aligned} \tag{109}$$

where

$$p_{Y,t}^{CB} = p_{Y,t}^{CBW} \mathcal{E}_t \tag{110}$$

because domestic carbon is traded on the world market.

4.2 Exogenous block

The domestic economy is sufficiently small such that it does not directly affect overall foreign demand, foreign inflation, foreign interest rates, and commodity prices. These variables are modelled as an SVAR. We define

$$\Omega_t \equiv \left[\tilde{Z}_t^{RW} - \tilde{Z}^{RW} \quad \Pi_{C,t}^{RW} - \Pi_C^{RW} \quad R_{Bf,t}^{RW} - R_{Bf}^{RW} \quad p_{Y,t}^{CBW} - p_Y^{CBW} \right]'.$$

We assume the following Structural Vector-Autoregressive (SVAR) model:

$$A\Omega_t = B\Omega_{t-1} + C\varepsilon_t^{\Omega} \tag{111}$$

where A, B, and C are the structural parameter matrices and $\varepsilon_t^{\Omega} = [\varepsilon_{Z,t}^{RW}, \varepsilon_{\Pi,t}^{RW}, \varepsilon_{R,t}^{RW}, \varepsilon_{p,t}^{CBW}]$. We further assume

$$\tilde{Y}_t^{kW} = \left(\tilde{Z}_t^{RW} \right)^{\phi_{YZ}^{kW}} \tag{112}$$

$$\forall k \in \{VAS, CB\}.$$

Because of the assumption of a small domestic economy, domestic export prices do not affect world-market prices which we assume to be exogenous:

$$p_{Y,t}^{kW} = \left(p_{Y,t-1}^{kW}\right)^{\rho_p^{kW}} \left(p_{Y,t}^{CBW}\right)^{\left(1-\rho_p^{kW}\right)\phi_p^{kW}} \varepsilon_{p,t}^{kW} \qquad \forall k \in VAS. \tag{113}$$

We assume the interest rate on foreign loans to be equal to the return on foreign bonds,

$$R_{Lf,t} = R_{Bf,t}^{RW}. \tag{114}$$

To keep the RW simple, we assume households do not derive utility from holding wealth. Recall that equity shares of FV-firms are traded in foreign currency. Combining the RW household's FOCs w.r.t. foreign bonds,

$$\lambda_t^{RW} = \beta^{RW} \mathrm{E}_t \frac{R_{Bf,t}^{RW}}{\Pi_{C,t+1}^{RW}} \lambda_{t+1}^{RW},$$

and w.r.t. the equity shares of FV-firms,

$$\lambda_t^{RW} = \beta^{RW} \mathrm{E}_t \left(1 - \Gamma_{S,t+1}^{FV}\right) \frac{R_{S,t}^{FV}}{\Pi_{C,t+1}^{RW}} \lambda_{t+1}^{RW},$$

we get

$$R_{Bf,t}^{RW} = \mathrm{E}_t \left(1 - \Gamma_{S,t+1}^{FV}\right) R_{S,t}^{FV}. \tag{115}$$

4.3 Current account

The budget constraint of the RW equates uses of funds (domestic exports, domestic sales of reforestation-backed carbon credits to the RW, beginning-of-period foreign bonds held by domestic households plus interest, bonds issued by the domestic government in foreign currency, foreign loans to domestic firms, and equity shares of foreign-owned value-added firms) and sources of funds (domestic imports, foreign bonds held by domestic households, beginning-of-period foreign loans to domestic firms plus interest, beginning-of-period equity shares of foreign-owned value-added firms plus returns),

$$\begin{aligned}
\sum_{j \in VAS} \frac{p_{Y,t}^{j}}{\mathcal{E}_t} \tilde{Z}_{j,t}^{jW} + \frac{p_{COs,t}}{\mathcal{E}_t} \tilde{CO}_{s,t}^{RW} + \frac{R_{Bf,t-1}^{RW}}{\Pi_{C,t}^{RW}} \hat{B}_{f,t-1}^{RW}/g_{t-1} + \\
+ \hat{B}_{f,t}^{GV} + \sum_{k \in VAS} \sum_{e \in \{c,v\}} \hat{L}_{f,t}^{k,e} + \tilde{p}_{S,t}^{FV} = \sum_{k \in VAS} \frac{p_{Y,t}^{kM}}{\mathcal{E}_t} \tilde{Y}_t^{kM} + \\
+ \hat{B}_{f,t}^{RW} + \frac{R_{Bf,t-1}^{GV,e}}{\Pi_{C,t}^{RW}} \hat{B}_{f,t-1}^{GV}/g_{t-1} + \\
+ \frac{R_{Lf,t-1}}{\Pi_{C,t}^{RW}} \sum_{k \in VAS} \sum_{e \in \{c,v\}} \hat{L}_{f,t-1}^{k,e}/g_{t-1} \\
+ \frac{R_{S,t-1}^{FV}}{\Pi_{C,t}^{RW}} \tilde{p}_{S,t-1}^{FV}/g_{t-1}
\end{aligned} \tag{116}$$

where we assume that the expropriated value of each FV-firm i, $\Gamma_{S,t}^{FV,i}\left(J_t^{FV,i}/P_{C,t}/\mathcal{E}_t + \mathcal{V}_{S,t}^{FV,i}\right)$, is transferred to the RW. In this case, expropriation risk does not affect the flow of funds. Note that the price of FV-equity shares and the value of FV-firms in (90) are identical,

$$\tilde{p}_{S,t}^{FV} = \tilde{\mathcal{V}}_{S,t}^{FV}, \tag{117}$$

because we implicitly assume that RW-households do not derive utility from wealth and, hence, there is no wedge between the stochastic discount factor and the inverse return to equity.[11]

5 Government sector

The government sector consists of a fiscal authority (GV), a monetary authority (MT), and a low-skilled wage authority. The government sector's control variables are determined by discretionary choice or policy rules. They are not derived as solutions to optimization problems.

5.1 Fiscal authority (GV)

Similar to Adrian et al. (2022), government deficits are financed by issuing perpetual government bonds denominated in domestic and foreign currency. As discussed in section 2, coupon payments geometrically decay at rates $1-\zeta_{Bd}^{GV}$ and $1-\zeta_{Bf}^{GV}$, respectively. This implies debt amortization payments at the same rates. With $\zeta_{Bd}^{GV}=0$, for instance, domestic debt is fully amortized in the next period.

Government debt. The budget constraint equates uses (government consumption, public investment, household transfers, subsidies, domestic debt service payments, foreign debt service payments) and sources of funds (consumption taxes, wealth taxes, labor income taxes, capital income taxes, corporate income taxes, ad-valorem input taxes, unit input taxes, import tariffs, receipts from emission pricing, receipts from education services, carbon mining royalties, profit transfers from the monetary authority, as well as domestic and foreign borrowing). After normalizing by price and productivity trends, the budget constraint can be written as

$$\begin{aligned}
&p_{Y,t}^{CG}\tilde{Z}_{CG,t}^{GV}+p_{Y,t}^{IV}\sum_{c\in PIC}\tilde{Z}_{IV,c,t}^{GV}+\\
&+p_{Y,t}^{IV}\sum_{k\in VAS}\sum_{e\in\{c,v\}}\sum_{j\in\{e,f,g\}}\tilde{Z}_{IV,Pj,k,e,t}^{GV}+\\
&+\hat{\Psi}_{LS,t}^{GV}+\sum_{k\in\{FGS,VAS\}}\sum_{e\in\{c,v\}}\hat{S}_{U,t}^{k,e}+\\
&+\hat{S}_{d,t}^{GV}+\mathcal{E}_t\hat{S}_{f,t}^{GV}=\sum_{k\in HHS}\left(\hat{T}_{C,t}^{k}+\hat{T}_{A,t}^{k}+\hat{T}_{L,t}^{k}+\hat{T}_{R,t}^{k}\right)+\\
&\quad+\sum_{k\in\{FGS,VAS\}}\sum_{e\in\{c,v\}}\left(\hat{T}_{P,t}^{k,e}+\hat{T}_{V,t}^{k,e}+\hat{T}_{U,t}^{k,e}+\hat{T}_{G,t}^{k,e}\right)+\\
&\quad+\hat{T}_{P,t}^{FV}+\hat{T}_{V,t}^{FV}+\\
&\quad+(1-\omega_{CCg})\,p_{Y,t}^{CC,c}\tilde{C}C_t+\tilde{p}_{E,t}E_t+\hat{J}_t^{MT}+\\
&\quad+\omega_G^{CB}\mathcal{E}_t p_{Y,t}^{CBW}\tilde{Y}_t^{CB}+\\
&\quad+\hat{X}_{d,t}^{GV}+\mathcal{E}_t\hat{X}_{f,t}^{GV}
\end{aligned}\tag{118}$$

where debt service payments comprise the interest payments at an effective rate and amortization payments. They satisfy

$$\hat{S}_{d,t}^{GV}=\frac{R_{Bd,t-1}^{GV,e}-\zeta_{Bd}^{GV}}{\Pi_{C,t}}\hat{B}_{d,t-1}^{GV}/g_{t-1}\tag{119}$$

[11] Note that $R_{S,t}^{FV}$ satisfies $\hat{p}_{S,t}^{FV}=\mathrm{E}_t\left(R_{S,t}^{FV}/\Pi_{C,t+1}^{RW}\right)^{-1}\left(\hat{p}_{S,t+1}^{FV}+\hat{J}_{t+1}^{FV}/\mathcal{E}_{t+1}\right)g_t$. Using (96) and (115), this equation is identical to the FV-firm value equation (90) when $\tilde{p}_{S,t}^{FV}=\tilde{\mathcal{V}}_{S,t}^{FV}$.

$$\hat{S}^{GV}_{f,t} = \frac{R^{GV,e}_{Bf,t-1} - \zeta^{GV}_{Bf}}{\Pi^{RW}_{C,t}} \hat{B}^{GV}_{f,t-1}/g_{t-1} \tag{120}$$

where the effective interest rates evolve according to

$$R^{GV,e}_{Bd,t} = \left(1 - \frac{\hat{X}^{GV}_{d,t-1}}{\hat{B}^{GV}_{d,t-1}}\right) R^{GV,e}_{Bd,t-1} + \left(\frac{\hat{X}^{GV}_{d,t-1}}{\hat{B}^{GV}_{d,t-1}}\right) R^{GV}_{Bd,t} \tag{121}$$

$$R^{GV,e}_{Bf,t} = \left(1 - \frac{\hat{X}^{GV}_{f,t-1}}{\hat{B}^{GV}_{f,t-1}}\right) R^{GV,e}_{Bf,t-1} + \left(\frac{\hat{X}^{GV}_{f,t-1}}{\hat{B}^{GV}_{f,t-1}}\right) R^{GV}_{Bf,t} \tag{122}$$

with the laws of motion of perpetual domestic and foreign government debt satisfying

$$\hat{B}^{GV}_{d,t} = \frac{\zeta^{GV}_{Bd}}{\Pi_{C,t}} \hat{B}^{GV}_{d,t-1}/g_{t-1} + \hat{X}^{GV}_{d,t} \tag{123}$$

$$\hat{B}^{GV}_{f,t} = \frac{\zeta^{GV}_{Bf}}{\Pi^{RW}_{C,t}} \hat{B}^{GV}_{f,t-1}/g_{t-1} + \hat{X}^{GV}_{f,t}\,. \tag{124}$$

We assume that the government holds constant the share of domestic borrowing in total borrowing,

$$\frac{\hat{X}^{GV}_{d,t}}{\hat{X}^{GV}_{d,t} + \mathcal{E}_t \hat{X}^{GV}_{f,t}} = \omega^{GV}_{Xd}\,. \tag{125}$$

Government revenues. Tax revenues normalized by consumer prices and human capital amount to

$$\hat{T}^{HS}_{C,t} = t_C p^{CH}_{Y,t} \tilde{Z}^{HS}_{CH,t} \quad \text{and} \quad \hat{T}^{LS}_{C,t} = t_C p^{CL}_{Y,t} \tilde{Z}^{LS}_{CL,t} \tag{126}$$

$$\hat{T}_{A,t} = t_A \hat{A}_t \quad \text{and} \quad \hat{T}^{LS}_{A,t} = 0 \tag{127}$$

$$\hat{T}^{HS}_{L,t} = t^{HS}_L \tilde{p}^{HS}_{Y,t} Y^{HS}_t \quad \text{and} \quad \hat{T}^{LS}_{L,t} = t^{LS}_L e_{e,t} e_{i,t} \tilde{p}^{LS}_{Y,t} Y^{LS}_t \tag{128}$$

$$\hat{T}_{R,t} = t_R \begin{bmatrix} \frac{R_{Dd,t-1}-1}{\Pi_{C,t}} \hat{D}^{HS}_{d,t-1}/g_{t-1} + \\ + \frac{R^{GV,e}_{Bd,t-1}-1}{\Pi_{C,t}} \hat{B}^{GV}_{d,t-1}/g_{t-1} + \\ + \frac{R^{RW}_{Bf,t-1}-1}{\Pi^{RW}_{C,t}} \mathcal{E}_t \hat{B}^{RW}_{f,t-1}/g_{t-1} + \\ + \sum_{j \in VAS} \sum_{e \in \{c,v\}} \frac{R^{j,e}_{S,t-1}}{\Pi_{C,t}} \tilde{p}^{j,e}_{S,t-1}/g_{t-1} + \\ + \left(1 - \omega^{CB}_G\right) \mathcal{E}_t p^{CBW}_{Y,t} \tilde{Y}^{CB}_t + \sum_{j \in FGS} \hat{J}^j_t \end{bmatrix} \quad \text{and} \quad \hat{T}^{LS}_{R,t} = 0 \tag{129}$$

$$\hat{T}^{k,e}_{P,t} = t_P \begin{bmatrix} p^k_{Y,t} \tilde{Y}^{k,e}_t - \sum_{j \in \{HHS,VAS,FV,IMS\}} p^j_{Y,t} \tilde{Z}^{k,e}_{j,t} - \hat{T}^{k,e}_{V,t} - \hat{T}^{k,e}_{U,t} - \hat{T}^{k,e}_{G,t} + \hat{S}^{k,e}_{U,t} - \\ -d^e \left(\omega_{COx} p_{COx,t} + \omega_{COs} p_{COs,t} + \left(1 - \omega_{COx} - \omega_{COs}\right)\left(1 - \omega_{CCg}\right) p_{CC,t}\right) G^{k,e}_t + p_{COx,t} \tilde{CO}^{k,e}_{x,t} + p_{COs,t} \tilde{CO}^{k,e}_{s,t} - \\ -\delta \sum_{j \in \{e,f,g\}} Q^{k,e}_{j,t} \tilde{K}^{k,e}_{Pj,t-1}/g_{t-1} - \varepsilon^k_{K,t} \sum_{j \in \{e,f,g\}} Q^{k,e}_{j,t} \frac{\tau_I}{2} \left(\frac{\tilde{Z}^{k,e}_{IV,Pj,t}}{\tilde{Z}^{k,e}_{IV,Pj,t-1}} - 1\right)^2 \tilde{Z}^k_{IV,Pj,t} \end{bmatrix} \tag{130}$$

$\forall k \in k \in \{FGS, VAS\}$ and $\forall e \in \{c, v\}$

$$\hat{T}^{FV}_{P,t} = t_P \begin{bmatrix} p^{FV}_{Y,t} \tilde{Y}^{FV}_t - \sum_{j \in HHS} p^j_{Y,t} \tilde{Z}^{FV}_{j,t} - \hat{T}^{FV}_{V,t} - \\ -\delta Q^{FV}_{g,t} \tilde{K}^{FV}_{Pg,t-1}/g_{t-1} - \varepsilon^k_{K,t} Q^{FV}_{g,t} \frac{\tau_I}{2} \left(\frac{\tilde{Z}^{FV}_{IV,Pg,t}}{\tilde{Z}^{FV}_{IV,Pg,t-1}} - 1\right)^2 \tilde{Z}^{FV}_{IV,Pg,t} \end{bmatrix} \tag{131}$$

$$\hat{T}^{k,e}_{V,t} = \sum_{j\in\{HHS,VAS,FV,IMS\}} t_{V,j} p^{j}_{Y,t} \tilde{Z}^{k,e}_{j,t} \tag{132}$$
$$\forall k \in \{FGS, VAS, FV\} \text{ and } \forall e \in \{c, v\}$$

$$\hat{T}^{FV}_{V,t} = \sum_{j\in HHS} t_{V,j} p^{j}_{Y,t} \tilde{Z}^{FV}_{j,t} \tag{133}$$

$$\hat{T}^{k,e}_{U,t} = \sum_{j\in\{HHS,VAS,FV,IMS\}} t_{U,j} \tilde{Z}^{k,e}_{j,t} \tag{134}$$
$$\forall k \in \{FGS, VAS\} \text{ and } \forall e \in \{c, v\}$$

$$\hat{T}^{k,e}_{G,t} = t_G \tilde{G}^{k,e}_{t} \tag{135}$$
$$\forall k \in \{FGS, VAS\} \text{ and } \forall e \in \{c, v\}$$

Government spending: subsidies, consumption, and investment in production and infrastructure. The unit subsidies are

$$\hat{S}^{k,e}_{U,t} = \sum_{j\in\{HHS,VAS,FV,IMS\}} s_{U,j} \tilde{Z}^{k,e}_{j,t} \tag{136}$$
$$\forall k \in \{FGS, VAS\} \text{ and } \forall e \in \{c, v\}.$$

Government consumption depends on output and is subject to an auto-regressive policy shock,

$$p^{CG}_{Y,t} \frac{\tilde{Z}^{GV}_{CG,t}}{\tilde{Z}^{GV}_{CG}} = \left(\frac{p_{GDP}\tilde{GDP}_t}{\tilde{GDP}}\right)^{-\phi_{CY}} \varepsilon_{CG,t}. \tag{137}$$

Public investment in production in sector k, ETS-compliance regime e, and use type j normalized by trend growth is

$$p^{IV}_{Y,t} \frac{\tilde{Z}^{GV}_{IV,Pj,k,e,t}}{\tilde{Z}^{GV}_{IV,Pj,k,e}} = \left(\frac{p_{GDP}\tilde{GDP}_t}{\tilde{GDP}}\right)^{-\phi_{IY,c}} \varepsilon_{IG,Pj,k,e,t} \tag{138}$$
$$\forall k \in VAS \text{ and } \forall e \in \{c, v\} \text{ and } \forall j \in \{e, f, g\}.$$

Public investment in infrastructure of type c is

$$p^{IV}_{Y,t} \frac{\tilde{Z}^{GV}_{IV,Ic,t}}{\tilde{Z}^{GV}_{IV,Ic}} = \left(\frac{p_{GDP}\tilde{GDP}_t}{\tilde{GDP}}\right)^{-\phi_{IY,c}} \varepsilon_{IG,Ic,t} \tag{139}$$
$$\forall c \in PIC.$$

The law of motion of the public capital stock for production in sector k, ETS-compliance regime e, and use type j normalized by trend growth is

$$\tilde{K}^{GV}_{Pj,k,e,t} = \left(1 - \frac{\tau_I}{2}\left(\frac{\tilde{Z}^{GV}_{IV,Pj,k,e,t}}{\tilde{Z}^{GV}_{IV,Pj,k,e,t-1}} - 1\right)^2\right) \tilde{Z}^{GV}_{IV,Pj,k,e,t} + (1-\delta)\tilde{K}^{GV}_{Pj,k,e,t-1}/g_{t-1} \tag{140}$$
$$\forall k \in VAS \text{ and } \forall e \in \{c, v\} \text{ and } \forall j \in \{e, f, g\}.$$

The law of motion of the stock of public infrastructure of type c normalized by trend growth is

$$\tilde{K}^{GV}_{Ic,t} = \left(1 - \frac{\tau_I}{2}\left(\frac{\tilde{Z}^{GV}_{IV,Ic,t}}{\tilde{Z}^{GV}_{IV,Ic}} - 1\right)^2\right)\tilde{Z}^{GV}_{IV,Ic,t} + (1-\delta)\tilde{K}^{GV}_{Ic,t-1}/g_{t-1} \tag{141}$$

$\forall c \in PIC.$

5.2 Monetary authority (MT)

The monetary authority sets the policy rate following a Taylor rule. The policy rate responds to consumer price inflation and real GDP, both relative to their respective steady-state values and in changes over time:

$$\frac{R_t}{R} = \left(\frac{R_{t-1}}{R}\right)^{\rho_R}\left(\frac{\Pi_{C,t-1}}{\Pi_Y}\right)^{\phi_{R\Pi}(1-\rho_R)}\left(\frac{\tilde{GDP}_t}{\tilde{GDP}}\right)^{\phi_{RY}(1-\rho_R)}\left(\frac{\Pi_{C,t}}{\Pi_{C,t-1}}\right)^{\phi_{R\Delta\Pi}}\left(\frac{\tilde{GDP}_t}{\tilde{GDP}_{t-1}}\right)^{\phi_{R\Delta Y}}\varepsilon_{R,t} \tag{142}$$

As in Smets and Wouters (2007), we assume a premium which captures *flight to safety* and drives a wedge between the policy rate and the risk-free rate:

$$R_t = \left(1 - \Gamma_B \varepsilon^{1/\Gamma_B}_{\Gamma B,t}\right) R_{Ld,t}. \tag{143}$$

where the exponent of the premium shock ensures that the variance of the shock does not depend on the level of the premium Γ_B. Since a positive shock increases the required return on domestic assets and the cost of capital, it reduces current consumption and investment simultaneously and helps explaining the co-movement of consumption and investment.

The MT issues high-powered money to households and transfers profits to the GV. The budget constraint implies

$$\hat{J}^{MT}_t + \frac{1}{\Pi_{C,t}}\left(\hat{M}^{HS}_{t-1} + \hat{M}^{LS}_{t-1}\right)/g_{t-1} = \hat{M}^{HS}_t + \hat{M}^{LS}_t. \tag{144}$$

5.3 Low-skilled wage authority

The wages for low-skilled labor are administered. One may think of a minimum wage of formal employment. We assume that the productivity corrected nominal wage inflation depends on its lagged value, the employment on the extensive margin (Phillips curve) and lagged price inflation (wage indexation).

$$\frac{\Pi^{LS}_{Y,t}}{\Pi^{LS}_Y} = \left(\frac{\Pi_{L,t-1}}{\Pi^{LS}_Y}\right)^{\rho_W}\left(\frac{e_{e,t}}{e_e}\right)^{\phi_{WE}(1-\rho_W)}\left(\frac{\Pi_{C,t-1}}{\Pi_Y}\right)^{\phi_{WP}(1-\rho_W)}\varepsilon^{LS}_{P,t}. \tag{145}$$

where the relationship between wage inflation, price inflation, and the low-skilled real wage is given by

$$\frac{\Pi^{LS}_{Y,t}}{\Pi_{C,t}} = \frac{\tilde{p}^{LS}_{Y,t}}{\tilde{p}^{LS}_{Y,t-1}} g_{t-1}. \tag{146}$$

5.4 ETS authority

The ETS authority controls the maximum share of abatement-backed carbon offsets in total emission permits, ω_{COx}, the maximum share of reforestation-backed carbon offsets in total emission permits, ω_{COs}, the share of freely allocated carbon credits, ω_{CCg}, the share of firms under ETS compliance for every sector $k \in \{FGS, VAS\}$, α^k_G, and the supply of government-issued carbon credits,

$$\frac{\tilde{CC}_t}{\tilde{CC}} = \varepsilon_{CC,t}. \tag{147}$$

6 Remaining market clearing conditions

This section states all those market-clearing conditions which have not yet been imposed above. Note that we have already imposed market clearing conditions, for example, for the supply of differentiated good varieties and the corresponding demand by the aggregators. We have also implicitly imposed market clearings conditions in the deposit, loan, bond, and stock markets.

Recalling that the output of the final good sectors is used only domestically, FGS market clearing implies

$$\tilde{Y}_t^{CH} = \tilde{Z}_{CH,t}^{HS}, \tag{148}$$

$$\tilde{Y}_t^{CL} = \tilde{Z}_{CL,t}^{HL}, \tag{149}$$

$$\tilde{Y}_t^{CG} = \tilde{Z}_{CG,t}^{GV}, \tag{150}$$

$$\tilde{Y}_t^{IV} = \sum_{k \in VAS} \sum_{e \in \{c,u\}} \sum_{j \in \{e,f,g\}} \left(\tilde{Z}_{IV,Pj,t}^{k,e} + \tilde{Z}_{IV,Pj,k,e,t}^{GV} \right) + \tilde{Z}_{IV,Pg,t}^{FV} + \sum_{c \in PIC} \tilde{Z}_{IV,Ic,t}^{GV}. \tag{151}$$

Market clearing in the value-added sector k implies

$$\tilde{Y}_t^{k} = \sum_{j \in \{FGS,VAS\}} \sum_{e \in \{c,u\}} \tilde{Z}_{k,t}^{j,e} + Z_{k,t}^{kW} \qquad \forall k \in VAS. \tag{152}$$

Clearing of the market for ETS-compliance-regime-specific output implies that the supply of the output of sector k in compliance regime e equals the demand for this output by the ETS-bundler in k,

$$\tilde{Y}_t^{k,e} = \tilde{Z}_{e,t}^{k} \qquad \forall k \in \{FGS, VAS\} \text{ and } \forall e \in \{c,u\}. \tag{153}$$

The market for import goods k clears when the supply of import goods k by the sector-specific import good aggregator equals the demand from all FGS- and VAS-firms,

$$\tilde{Y}_t^{kM} = \sum_{j \in \{FGS,VAS\}} \sum_{e \in \{c,u\}} \tilde{Z}_{kM,t}^{j,e} \qquad \forall k \in \{VAS, CB\}. \tag{154}$$

Market clearing for the value-added composite produced by the foreign-owned value-added sector implies

$$\tilde{Y}_t^{FV} = \sum_{j \in \{FGS,VAS\}} \sum_{e \in \{c,u\}} \tilde{Z}_{FV,t}^{j,e}. \tag{155}$$

The market for domestic carbon extraction clears when supply equals domestic demand

$$\tilde{Y}_t^{CB} = \sum_{k \in \{FGS,VAS\}} \tilde{Z}_{CB,t}^{k}. \tag{156}$$

The high-skilled labor-market-clearing condition reads

$$\tilde{Y}_t^{HS} = \sum_{j \in \{FGS,VAS\}} \sum_{e \in \{c,v\}} \tilde{Z}_{HS,t}^{j,e} + \tilde{Z}_{HS,t}^{FV}. \tag{157}$$

In scenarios with an ETS, the compliance market clears. The supply of government-issued carbon credits equals the emissions of regulated firms that cannot be offset by abatement-backed carbon offsets. In scenarios without an ETS, the price of carbon credits is zero. That is,

$$\begin{aligned}
&\tilde{CC}_t = (1 - \omega_{COx} - \omega_{COs}) \sum_{k \in \{FGS, VAS\}} G_t^{k,c} && \text{if } \alpha_G^k > 0 \text{ for any } k \in \{FGS, VAS\} \\
&p_{CC,t} = 0 && \text{else.}
\end{aligned} \tag{158}$$

In scenarios with an ETS and a positive share of carbon offsets in total emission permits, the voluntary markets clear. The supply of abatement-backed carbon offsets issued by unregulated firms equals the emissions of regulated firms that can be offset by abatement-backed carbon offsets. The same holds for the reforestation-backed carbon offsets. In scenarios without an ETS, the price is carbon offsets is zero. That is,

$$\begin{aligned}
&\sum_{k \in \{FGS, VAS\}} \tilde{COx}_t^{k,v} = \omega_{COx} \sum_{k \in \{FGS, VAS\}} \tilde{G}_t^{k,c} && \text{if } \alpha_G^k > 0 \text{ for any } k \in \{FGS, VAS\} \text{ and } \omega_{COx} > 0 \\
&p_{COx,t} = 0 && \text{else}
\end{aligned} \tag{159}$$

and

$$\begin{aligned}
&\sum_{k \in \{FGS, VAS\}} \tilde{COs}_t^{k,v} = \omega_{COs} \sum_{k \in \{FGS, VAS\}} \tilde{G}_t^{k,c} && \text{if } \alpha_G^k > 0 \text{ for any } k \in \{FGS, VAS\} \text{ and } \omega_{COs} > 0 \\
&p_{COs,t} = 0 && \text{else.}
\end{aligned} \tag{160}$$

7 Quantity indices and relative price deflators

We construct quantity indices and relative price deflators for baskets of goods which are measured in different units. We define the consumption quantity index and relative price deflator such that

$$p_{CON,t} \tilde{CON}_t = p_{Y,t}^{CH} \tilde{Y}_t^{CH} + p_{Y,t}^{CL} \tilde{Y}_t^{CL} \tag{161}$$

where we take the consumer price deflator as the numéraire,

$$p_{CON,t} = 1. \tag{162}$$

That is, we normalize all price levels by the consumption price deflator. All prices are expressed in levels relative to deflator of the consumption basket.

Taking the steady state as the base and noting that prices are at unity at the steady state, the Laspeyres quantity index for consumption is

$$\tilde{CON}_t = \tilde{Y}_t^{CH} + \tilde{Y}_t^{CL}. \tag{163}$$

We define the export quantity index and relative price deflator such that

$$p_{EXP,t} \tilde{EXP}_t = \sum_{j \in VAS} p_{Y,t}^j \tilde{Z}_{j,t}^{jW} + \mathcal{E}_t p_{Y,t}^{CBW} \tilde{Z}_{CB,t}^{CBW}. \tag{164}$$

The corresponding Laspeyres quantity index for exports is

$$\tilde{EXP}_t = \sum_{j \in VAS} \tilde{Z}_{j,t}^{jW} + \tilde{Z}_{CB,t}^{CBW}. \tag{165}$$

The import quantity index and relative price deflator satisfy

$$p_{IMP,t}\tilde{IMP}_t = \sum_{k\in VAS} p_{Y,t}^{kM}\tilde{Y}_t^{kM}. \tag{166}$$

The corresponding Laspeyres quantity index for imports is

$$\tilde{IMP}_t = \sum_{k\in VAS} \tilde{Y}_t^{kM}. \tag{167}$$

The gross domestic product normalized by the consumer price deflator satisfies

$$p_{GDP,t}\tilde{GDP}_t = p_{CON,t}\tilde{CON}_t + p_{Y,t}^{GV}\tilde{Y}_t^{GV} + p_{Y,t}^{IV}\tilde{Y}_t^{IV} + p_{EXP,t}\tilde{EXP}_t - p_{IMP,t}\tilde{IMP}_t. \tag{168}$$

The corresponding Laspeyres quantity index for the gross domestic product is

$$\tilde{GDP}_t = \tilde{CON}_t + \tilde{Y}_t^{GV} + \tilde{Y}_t^{IV} + \tilde{EXP}_t - \tilde{IMP}_t. \tag{169}$$

8 Zero restrictions

For the sake of a concise presentation of the model, we have treated the household sectors, the final good sectors, and the value-added sectors as symmetric to each other up to the parameterization. For example, the service sector and the fossil fuel sector have been completely identical in the presentation of the model – apart from the parameter values. However, we assume that the service sector does not use carbon inputs while the fossil fuel sector does. This is realized by imposing parameter restrictions. For the sake of completeness, we shall list here all relevant zero-restrictions on parameters. These restrictions collapse the size of the model considerably compared to what is suggested in the model description above.

Regarding tax rates, the only value added taxes we consider are payroll taxes. Unit taxes are levied on fossil fuel inputs (fuel tax) and carbon inputs (carbon tax). Hence,

$$t_{V,j} = t_{V,j^*} = 0 \quad \forall j \in \{HHS, VAS, CB\} \setminus HHS,$$

$$t_{U,j} = t_{U,j^*} = 0 \quad \forall j \in \{HHS, VAS, CB\} \setminus \{FF, CB\}.$$

Regarding production inputs, the domestic country only uses domestic labor and the RW only uses foreign labor $Z_{j^*,t}^{k,e} = Z_{j,t}^{*} = 0\ \forall k \in VAS$ and $\forall e \in \{c, u\}$ and $\forall j \in HHS$. In (67), we restrict

$$\alpha_{W,j}^{k} = \alpha_{W,j}^{*} \to 1 \quad \forall k \in VAS \text{ and } \forall j \in HHS.$$

Only fossil electricity generation and fossil fuel production use carbon inputs directly: $Z_{j,t}^{k,e} = Z_{j^*,t}^{k,e} = 0$ $\forall k \in VAS \setminus \{FY, FF\}$ and $\forall e \in \{c, u\}$ and $\forall j \in \{CB\}$. In (54), we restrict

$$\alpha_{Y}^{k} \to 1 \quad \forall k \in VAS \setminus \{FY, FF\}.$$

Only VAS-firms are subject to the ETS-regimes of compliance and voluntary. FGS-firms are not subject to the ETS. We restrict

$$\alpha_{G}^{k} \to 0 \quad \forall k \in FGS.$$

Value-added sectors generate process emissions and emissions from fossil-fuel or carbon combustion. No other input to production generates emissions. In (**??**), we therefore set

$$\psi_{Gj}^{k} = 0 \quad \forall k \in VAS \text{ and } \forall j \in \{HHS, VAS\} \setminus \{FF, CB\}.$$

Of the two sectors that use carbon inputs (fossil electricity and fossil fuels) only fossil electricity additionally generates emissions from carbon combustion. The fossil fuel sector does not as carbon is transformed rather than combusted:

$$\psi_{Gj}^{k} = 0 \quad \forall k \in VAS \setminus \{FF\} \text{ and } \forall j \in \{CB\}.$$

χ is such that no steady state profits in FGS and $\chi_{Y}^{k} = 0$ for VAS.

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Table 1A: List of variables with descriptions (Part A)

Variable	Description
Assets and liabilities	
A_t^{HS}	Financial wealth of HS-households
M_t^k	Interest-free liquidity balances of sector $k \in \{HHS, VAS\}$
S_t^k	Equity shares of sector $k \in \{VAS\}$
$K_{P,t}^k$	Private capital stock in sector $k \in \{VAS\}$
$K_{G,t}^k$	Public capital stock in sector $k \in \{VAS\}$
$D_{P,t}^{HS}$	Domestic bank deposits of HS-households
$D_{G,t}^{HS}$	Domestic public debt held by HS-households
$D_{W,t}^{HS}$	Foreign bank deposits of HS-households
$D_{W,t}^*$	Asset position of the RW vis-a-vis the domestic country
$B_{P,t}^k$	Corporate domestic debt of sector $k \in \{VAS\}$
$B_{G,t}^{GV}$	Public domestic debt
$B_{W,t}^k$	Corporate foreign debt of sector $k \in \{VAS\}$
$B_{W,t}^{GV}$	Public foreign debt
$B_{W,t}^*$	Debt position of the RW vis-a-vis the domestic country
Returns, risk premia, and transaction costs	
R_t	Gross policy rate
$R_{B,t}$	Return on deposits and public domestic debt
$R_{S,t}^k$	Return on equity shares in sector $k \in \{VAS\}$
$R_{B,t}^*$	Return on foreign debt in foreign currency
$\Gamma_{S,t}$	Equity risk premium or share of expropriated firm value in sector $k \in \{VAS\}$
$\Gamma_{W,t}$	Foreign debt risk premium vis-a-vis domestic debt
$\Gamma_{v,t}^k$	Consumption and investment finance transaction costs, respectively, in sector $k \in \{HHS, VAS\}$
Adjustment costs	
$\Psi_{P,t}^{HS}$	High-skilled wage adjustment costs
$\Psi_{Z,j,t}^k$	Input adjustment costs in sector $k \in \{FGS, VAS\}$ for input $j \in \{HHS, VAS, VAS^*, CB, CB^*\}$
$\Psi_{P,t}^k$	Price adjustment costs in sector $k \in \{VAS\}$
$\Psi_{u,t}^k$	Capital utilization adjustment costs in sector $k \in \{VAS\}$
Ψ_t^k	Adjustment cost lump-sum rebate in sector $k \in \{HHS, FGS, VAS\}$
Demand for and supply of goods and services	
$Z_{j,t}^k$	Demand of sector $k \in \{HHS, FGS, VAS, GV, ^*\}$ for good/service $j \in \{HHS, VAS, VAS^*, CB, CB^*\}$
Y_t^k	Supply of good/service of sector $k \in \{HHS, FGS, VAS, CB\}$
U_t^k	Material-value added composite in sector $k \in \{VAS\}$
V_t^k	Value added in sector $k \in \{VAS\}$
S_t^k	Capital-energy composite in sector $k \in \{VAS\}$
K_t^k	Capital service in sector $k \in \{VAS\}$
N_t^k	Energy service in sector $k \in \{VAS\}$
F_t^k	Fuel service in sector $k \in \{VAS\}$
E_t^k	Electricity service in sector $k \in \{VAS\}$
L_t^k	Labor service in sector $k \in \{VAS\}$
X_t^k	Material in sector $k \in \{VAS\}$
$W_{j,t}^k$	Domestic-foreign input composite in sector $k \in \{VAS\}$ for input $j \in \{VAS, CB\}$

A Variables and parameters

Table 1B: List of variables with descriptions (Part B)

Variable	Description
Prices, shadow prices, inflation rates, and exchange rates	
$p_{G,t}$	Price of GHG emissions
$p^k_{S,t}$	Price of equity shares in sector $k \in \{VAS\}$
$p^k_{Y,t}$	Price of good/service of sector $k \in \{HHS, FGS, VAS, VAS^*, CB, CB^*\}$
Q^k_t	Shadow price of private capital stock in sector $k \in \{VAS\}$
φ^k_t	Marginal cost of production in sector $k \in \{VAS\}$
$\mu^k_{B,t}$	Shadow price of borrowing domestically for sector $k \in \{VAS\}$
$\mu^k_{W,t}$	Shadow price of borrowing abroad for sector $k \in \{VAS\}$
λ^k_t	Shadow price of an additional unit of income for household $k \in \{HHS\}$
$\Pi^{HS}_{Y,t}$	Gross inflation rate of high-skilled wage
$\Pi^{LS}_{Y,t}$	Gross inflation rate of low-skilled wage
$\Pi_{C,t}$	Gross inflation rate of high-skilled consumption goods
$\Pi^*_{Y,t}$	Gross inflation rate in the RW
$\Delta\mathcal{S}_t$	Nominal exchange rate depreciation
$\mathcal{E}_t$	Real exchange rate
Taxes	
$T^k_{C,t}$	Consumption tax revenues in sector $k \in \{HHS\}$
$T^k_{A,t}$	Wealth tax revenues in sector $k \in \{HHS\}$
$T^k_{L,t}$	Wage income tax revenues in sector $k \in \{HHS\}$
$T^k_{R,t}$	Capital income tax revenues in sector $k \in \{HHS\}$
$T^k_{P,t}$	Corporate income tax revenues in sector $k \in \{VAS\}$
$T^k_{V,t}$	Ad-valorem input tax revenues in sector $k \in \{VAS\}$ (payroll tax)
$T^k_{U,t}$	Unit input tax revenues in sector $k \in \{VAS\}$ (fuel tax and carbon tax)
Shock processes	
$\varepsilon^k_{C,t}$	Consumption preference shock for household sector $k \in \{HHS\}$
$\varepsilon^k_{N,t}$	Labor supply preference shock for household sector $k \in \{HHS\}$
$\varepsilon_{\Gamma B,t}$	Flight-to-safety shock to the domestic deposit/debt return
$\varepsilon_{\Gamma W,t}$	Risk premium shock to the foreign deposit/debt return
$\varepsilon_{\Gamma S,t}$	Equity risk premium shock
$\varepsilon^k_{P,t}$	Wage mark-up shock for $k = HS$, wage policy shock for $k = LS$, and price mark-up shock for sector $k \in \{VAS\}$
$\varepsilon^k_{K,t}$	Private and public investment efficiency shock in sector $k \in \{VAS\}$
$\varepsilon_{g,t}$	Permanent labor-embodied productivity shock
$\varepsilon_{CG,t}$	Government consumption shock
$\varepsilon_{IG,k,t}$	Shock to public investment in sector $k \in \{VAS\}$
$\varepsilon_{R,t}$	Monetary policy shock
$\varepsilon^*_{W,k,t}$	Shock to RW's demand for the exports of sector $k \in \{VAS\}$
$\varepsilon^{k^*}_{p,t}$	Shock to the import price of input $k \in \{VAS\}$
$\varepsilon_{R^*,t}$	Monetary policy shock in the RW
$\varepsilon_{\Pi^*_Y,t}$	Shock to inflation in the RW
Miscellaneous	
GDP_t	Gross domestic product
$e_{e,t}$	Extensive margin of employment rate
$e_{i,t}$	Intensive margin of employment rate
z_t	Labor-embodied productivity
g_t	Gross growth rate of labor-embodied productivity
Lam^k_t	Stochastic discount factor in sector $k \in \{VAS\}$
G^k_t	Greenhouse gas emissions in sector $k \in \{VAS\}$
J^k_t	Dividend payments to households by sector $k \in \{VAS\}$
u^k_t	Rate of capital utilization in sector $k \in \{VAS\}$
v^k_t	Expenditure velocity in sector $k \in \{HHS, VAS\}$
x^k_t	Emission abatement effort in sector $k \in \{VAS\}$

Table 2A: List of parameters with descriptions (Part A)

Parameter	Description
Preferences	
β	Inter-temporal discount factor
ψ_A	Wealth preference shifter
ψ_C	Consumption preference shifter
ψ_N	Labor supply preference shifter of household $k \in \{HHS\}$
κ	Consumption habit persistence
ϱ	Inverse elasticity of intertemporal substitution
η	Inverse Frisch elasticity of labor supply of household $k \in \{HHS\}$
ξ	Extensive margin employment adjustment parameter
CES production technology	
ψ_Y	Output scaling parameter
α_Y	Share parameter of material-value added-energy-land composite (vs. extracted carbon) in final output
α_T	Share parameter of material-value added-energy composite (vs. land) in material-value added-energy-land composite
α_U	Share parameter of material composite (vs. value added-energy) in material-value added-energy composite
$\alpha_{X,j}$	Share parameter of inputs from $j \in \{AG, IN, TR, SV\}$ to the material composite
α_S	Share parameter of value added composite (vs. energy) in value added-energy composite
α_O	Share parameter of domestic value added (vs. foreign-owned value added) in value added composite
α_V	Share parameter of general capital service (vs. labor service) in domestic value added composite
α_L	Share parameter of high-skilled labor (vs. low-skilled labor) in labor services
α_N	Share parameter of electricity-capital composite (vs. fuel-capital) in energy composite
α_{EK}	Share parameter of electricity composite (vs. electricity-specific capital) in electricity-capital composite
α_{FK}	Share parameter of fuel composite (vs. fuel-specific capital) in fuel-capital composite
α_E	Share parameter of renewable electricity (vs. fossil electricity) in electricity composite
α_F	Share parameter of renerwable fuels (vs. fossil fuels) in fuel composite
$\alpha_{W,j}$	Share parameter of domestic inputs (vs. imported inputs) from $j \in \{VAS, CB\}$
σ_Y	Substitution elasticity between material-value-added and extracted carbon in sector $k \in \{VAS\}$
	Substitution elasticity between materials and energy in sector $k \in \{FGS\}$
σ_U	Substitution elasticity between materials and value-added in sector $k \in \{VAS\}$
σ_V	Substitution elasticity between capital-energy and labor in sector $k \in \{VAS\}$
σ_S	Substitution elasticity between capital and energy in sector $k \in \{VAS\}$
σ_K	Substitution elasticity between private and public capital in sector $k \in \{VAS\}$
σ_N	Substitution elasticity between fuels and electricity in sector $k \in \{FGS, VAS\}$
σ_F	Substitution elasticity between bio-fuels and fossil fuels in sector $k \in \{FGS, VAS\}$
σ_E	Substitution elasticity between renewable and fossil electricity in sector $k \in \{FGS, VAS\}$
σ_L	Substitution elasticity between high- and low-skilled labor in sector $k \in \{VAS\}$
σ_X	Substitution elasticity between material inputs from sector $j \in \{SV, MA, AG, MI, TR, CN\}$ in sector $k \in \{FGS, VAS\}$
$\sigma_{W,j}$	Substitution elasticity between dom. and for. inputs from $j \in \{SV, MA, AG, MI, TR, CN\}$ in $k \in \{FGS, VAS,^*\}$
Other supply side parameters	
δ	Capital depreciation rate
μ_P^{HS}	High-skilled wage markup
μ_P^k	Price markup in sector $k \in \{VAS\}$
ζ_P	Domestic borrowing constraint parameter in sector $k \in \{VAS\}$
ζ_W	Foreign borrowing constraint parameter in sector $k \in \{VAS\}$
$\psi_{x,j}$	Emission intensity of input $j \in \{ALL\}$ in sector $k \in \{VAS\}$
$\psi_{x,Y}$	Process emission intensity in sector $k \in \{VAS\}$
ω_G^{CB}	Share of government-owned (vs. privately owned) domestic carbon mines
ω_M^{CB}	Share of imported (vs. domestically extracted) carbon input

Table 2B: List of parameters with descriptions (Part B)

Parameter	Description
Adjustment, transaction, abatement, expropriation cost technologies	
τ_{v1}^{k}	Positive slope parameter in convex transaction cost technology in sector $k \in \{HHS, VAS\}$
τ_{v2}^{k}	Negative slope parameter in convex transaction cost technology in sector $k \in \{HHS, VAS\}$
τ_{P}^{k}	Price adjustment cost scaling parameter in sector $k \in \{HS, VAS\}$ (including high-skilled wage adjustment)
$\tau_{Z,j}$	Input adjustment cost scaling parameter for input $j \in \{ALL\}$
τ_{u1}^{k}	Linear capital utilization adjustment cost scaling parameter in sector $k \in \{VAS\}$
τ_{u2}^{k}	Quadratic capital utilization adjustment cost scaling parameter in sector $k \in \{VAS\}$
τ_{I}^{k}	Investment adjustment cost scaling parameter in sector $k \in \{VAS\}$
τ_{x}^{k}	Abatement cost scaling parameter in sector $k \in \{VAS\}$
ϕ_{x}^{k}	Abatement effort elasticity of abatement costs in sector $k \in \{VAS\}$
τ_{S}^{k}	Expropriation cost scaling parameter in sector $k \in \{VAS\}$
ϕ_{S}^{k}	Expropriation share elasticity of expropriation costs in sector $k \in \{VAS\}$
τ_{W}	External risk premium scaling parameter
ϕ_{W}	Net-investment-position elasticity of the external risk premium
Γ_{B}	Flight-to-safety premium on the domestic deposit/debt return
Policy and policy rules	
t_{C}	Consumption tax rate
t_{A}	Wealth tax rate
t_{L}^{k}	Wage income tax rate for labor $k \in \{HHS\}$
t_{R}	Capital income tax rate
t_{P}	Corporate income tax rate
$t_{V,j}$	Ad-valorem tax rate on input $j \in ALL$ (payroll tax)
$t_{U,j}$	Unit tax rate on input $j \in ALL$ (fuel tax and carbon tax)
Z_{CG}^{GV}	Target government consumption
ϕ_{CY}	Output-gap elasticity of government consumption
$Z_{IV,k}^{GV}$	Target public investment in sector $k \in \{VAS\}$
$\phi_{IY,k}$	Output-gap elasticity of public investment in sector $k \in \{VAS\}$
ω_{B}^{GV}	Share of external public debt in total public debt
R	Target policy rate
Π_{Y}	Price inflation target
GDP	Target gross domestic product
ρ_{R}	Monetary policy inertia
$\phi_{R\Pi}$	Inflation elasticity of the policy rate
ϕ_{RY}	Output-gap elasticity of the policy rate
Π_{Y}^{LS}	Structural low-skilled wage-inflation rate
e_{e}	Non-accelerating inflation rate of employment on extensive margin (1-NAIRU)
e_{i}	Structural rate of employment on intensive margin
ρ_{W}	Low-skilled wage policy inertia
ϕ_{WE}	Extensive employment elasticity of low-skilled wage inflation
ϕ_{WP}	Price inflation elasticity of low-skilled wage inflation (low-skilled wage indexing)