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Relativistic de Broglie-Bohm interpretation of quantum mechanics for the scalar field in the Rindler spacetime

Abstract

In this work we investigate the behavior of the wave functional associated with a massless scalar field in Rindler spacetime using the de Broglie-Bohm (dBB) interpretation of quantum mechanics. First, we use the Schrödinger picture to obtain the wave functional associated with the Minkowski vacuum and express it in Rindler coordinates. Then we calculate the associated Bohmian trajectories and analyze their behavior in terms of the Hamilton-Jacobi equation with a supplementary quantum potential. Finally, we calculate the power spectrum and compare it to the result of a classical scalar field at finite temperature.

The Rindler wave functional

An observer in Minkowski spacetime with a constant acceleration a with respect to some inertial reference frame is described by Rindler coordinates (τ, ξ) with line element $ds^2 = e^{2a\xi}(-d\tau^2 + d\xi^2)$, so that the Rindler metric is conformal to Minkowski metric.

Consider the following action for a massless field

$$S = \frac{1}{2}\int dt dx \left\{ \left(\frac{\partial \phi}{\partial t}\right)^2 - \left(\frac{\partial \phi}{\partial x}\right)^2 \right\}. \tag{1}$$

In terms of Minkowski modes, ϕ admits the half-Fourier expansion [1]

$$\phi(t, x) = \sqrt{\frac{2}{\pi}} \int_0^\infty dk \sin(kx) \phi_k^M(t), \tag{2}$$

while for Rindler modes ϕ can be expressed as

$$\phi(\tau, \xi) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} dk' e^{ik'\xi} \phi_{k'}^R(\tau). \tag{3}$$

Applying a product decomposition $\Psi[\phi, t] = \prod_{k>0} \Psi_k[\phi_k^M, t]$, we obtain a Schrödinger equation for each Ψ_k. When we express the Minkowski modes in terms of the Rindler modes, the vacuum solution reads

$$\Psi_k^M[\phi_k^R, \phi_k^{R*}, \tau] = \exp\left[-k f_k(\tau)\phi_k^R \phi_k^{R*} + i\Omega_k(\tau)\right], \tag{4}$$

with $f_k(\tau) = \coth\left(\frac{\pi k}{2a} + ik\tau\right)$ and $\Omega_k(\tau) = -\ln\left[\sinh\left(\frac{\pi k}{2a} + ik\tau\right)\right]$. Note that it is possible to recover the usual Bose-Einstein distribution with Unruh temperature using the expectation value of the Rindler number operator in Minkowski vacuum [2].

De Broglie-Bohm interpretation of Quantum Mechanics

In order to pass to the dBB interpretation [3] we rewrite the wave functional (4) in the polar form $\Psi_k = R_k e^{iS_k}$. Then the Schrödinger equation yields the two real equations

$$\frac{\partial S_k}{\partial \tau} + \frac{\partial S_k}{\partial \phi_k^{R*}}\frac{\partial S_k}{\partial \phi_k^R} + k^2 \left|\phi_k^R\right|^2 + Q_k = 0, \tag{5}$$

$$\frac{\partial R_k^2}{\partial \tau} + \frac{\partial}{\partial \phi_k^R}\left(R_k^2 \frac{\partial S_k}{\partial \phi_k^{R*}}\right) + \frac{\partial}{\partial \phi_k^{R*}}\left(R_k^2 \frac{\partial S_k}{\partial \phi_k^R}\right) = 0. \tag{6}$$

he first one can be interpreted as a Hamilton-Jacobi equation for S_k with a supplementary quantum potential

$$Q_k = -\frac{1}{R_k}\frac{\partial^2 R_k}{\partial \phi_k^{R*} \partial \phi_k^R}. \tag{7}$$

The second equation can be viewed as a continuity equation and can give a probabilistic interpretation to R_k^2. Classical limit is given when $Q \approx 0$.

The Bohmian field ϕ_k is obtained by integration of the guidance equations

$$\frac{\partial \phi_k^R}{\partial \tau} = \frac{\partial S_k}{\partial \phi_k^{R*}}, \qquad \frac{\partial \phi_k^{R*}}{\partial \tau} = \frac{\partial S_k}{\partial \phi_k^R}, \tag{8}$$

and it is equal to

$$\phi_k(\tau) = \frac{1}{\sqrt{2k}} \frac{\left(e^{2\pi k/a} - 2e^{\pi k/a}\cos(2k\tau) + 1\right)^{1/2}}{\left(e^{\pi k/a} - 1\right)}, \tag{9}$$

where S_k the phase in the wave functional (4).

Quantum Potential

The quantum potential can be expressed in the form

$$Q_k(\tau) = k\Re[f_k(\tau)] - k^2\Re^2[f_k(\tau)] \left|\phi_k\right|^2. \tag{10}$$

For $\tau = 0$ we have

$$Q_k(\tau = 0) = \frac{k}{2}\frac{e^{3\pi k/a} + 1}{(e^{4\pi k/a} + 1)(e^{\pi k/a} - 1)}. \tag{11}$$

The mode energy for every k is given by

$$E_k(\tau) = k\frac{e^{4\pi k/a} + 1}{(e^{4\pi k/a} - 1)}. \tag{12}$$

In the limit $a \to 0$ the result of Minkowski space is recovered for both Q_k and E_k as expected, since the equations (10) and (12) give

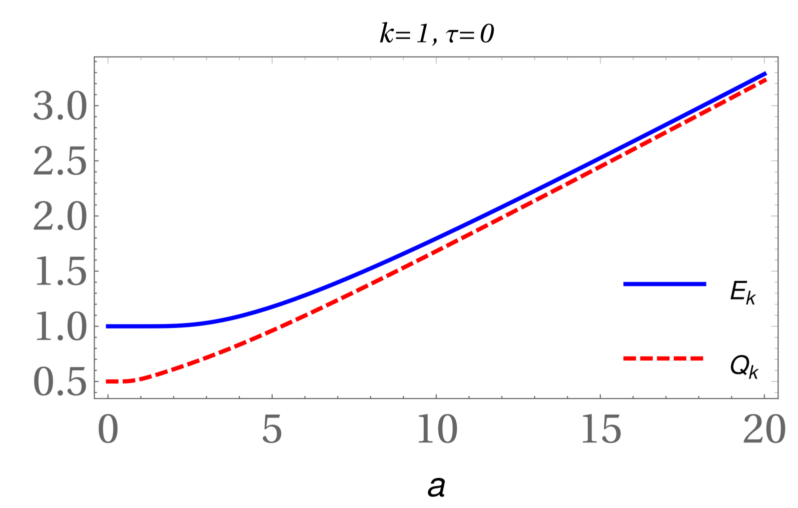
$$\lim_{a \to 0} Q_k(\tau = 0) = \frac{k}{2}, \tag{13}$$

$$\lim_{a \to 0} E_k(\tau = 0) = k. \tag{14}$$

For high temperature regime $T \to \infty$, which is equivalent to $a >> 1$, we have

$$\lim_{a \to \infty} Q_k(\tau = 0) = \lim_{a \to \infty} E_k(\tau = 0) = T, \tag{15}$$

where $T = \frac{a}{2\pi}$ is the Unruh temperature.
These results can be explained by analyzing the behavior of the Bohmian field in these limits. For low values of the acceleration a the field is $\phi_k(\tau = 0) = \frac{1}{\sqrt{2k}}$, so the contribution of the classical potential in the eq.(5) is $\frac{k}{2}$.
For higher values of a, the field can be approximated as $\phi_k(\tau = 0) = \frac{\sqrt{3}}{4\sqrt{T}}$, which means it vanishes as $a \to \infty$. Therefore, in this limit, the quantum potential is the unique contribution to the energy.
We plot the energy E_k and the quantum potential Q_k as functions of a. For low values of a we have $Q_k \approx \frac{E_k}{2}$, while for large accelerations $Q_k \to E_k$.



Statistical Analysis and Power Spectrum

The power spectrum is commonly defined as the reversed Fourier transform of the two point function

$$P(k) = \int d\xi e^{-ik\xi} \left\langle \phi(\xi)\phi(0) \right\rangle_{dBB}, \tag{16}$$

where

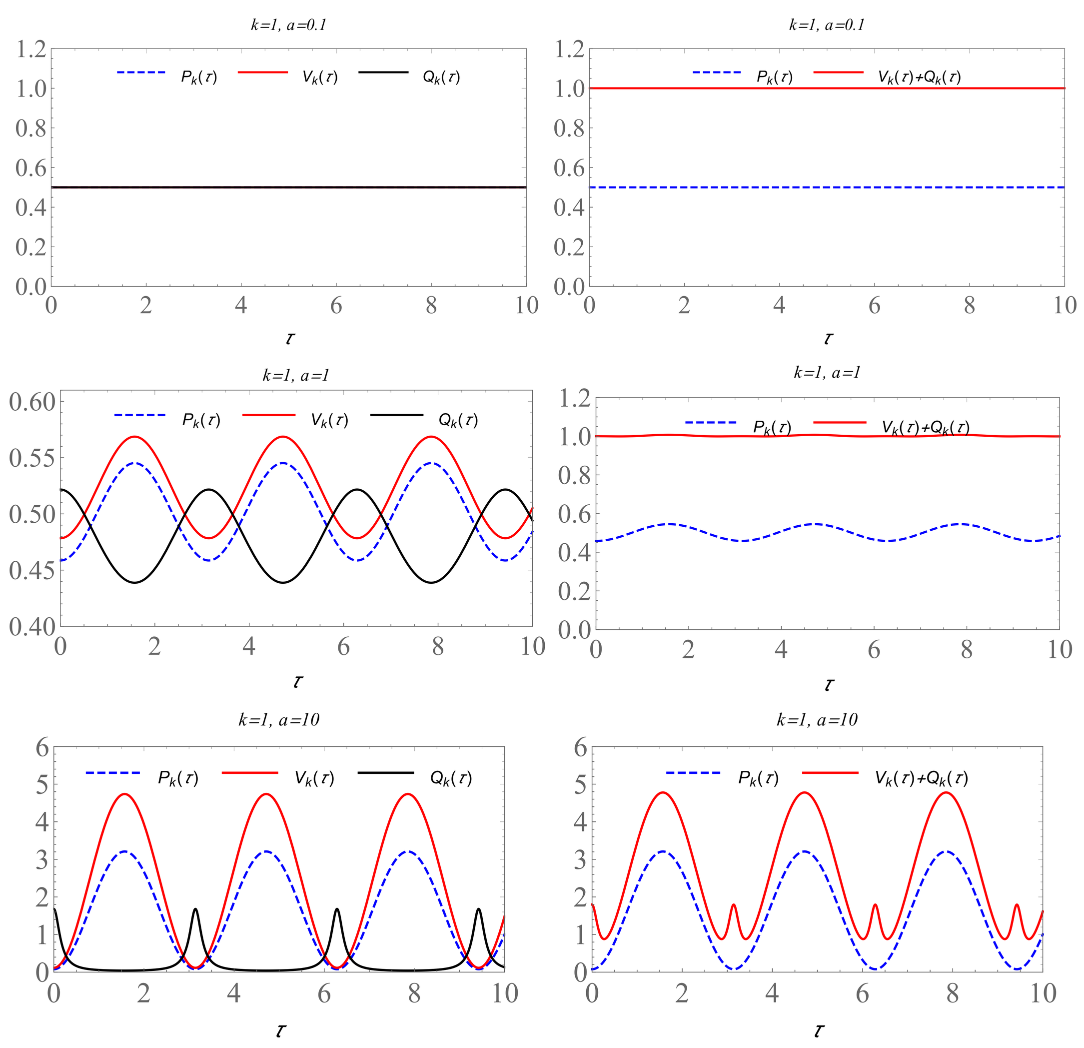
$$\left\langle \phi(\xi)\phi(\xi + \chi) \right\rangle_{dBB} = \frac{1}{2\pi}\int_{-\infty}^{\infty} dk \frac{1}{2k\Re[f_k(\tau)]} e^{-ik\xi}. \tag{17}$$

In the high temperature limit the power spectrum can be approximated as

$$P(k) = \frac{2T}{k^2}\sin^2(k\tau) + \frac{\sin^2(k\tau)}{k}. \tag{18}$$

In the classical thermal field theory the power spectrum for the free massless scalar field is given by $P(k) = \frac{T}{\omega_k^2}\cos(\omega_k t)$ [4]. Note that the result (18) is obtained using the wave functional that describes only positive values of k. Thus, it is necessary to rewrite (18) as $P(k) = \frac{T}{k^2} - \frac{T}{k^2}\cos(k\tau)$.

We plot the behavior of the power spectrum, quantum and classical potentials for different values of a. One can say that the behavior of the power spectrum depends mostly on the classical potential $V_k = k^2 \left|\phi_k^R\right|^2$.



Discussion

In this work we study the Bohmian interpretation of quantum mechanics for a massless scalar field. We calculated the quantum potential and the energy, and verified that in the low acceleration limit, the results of Bohmian mechanics for the Minkowski space are recovered. Then we obtained the power spectrum as the reversed Fourier transform of the two point function. We found out that the behavior of the power spectrum depends mostly on the classical potential that contains a Bohmian field. In the future work we pretend to extend our analysis to both Rindler wedges, and, possibly, to black hole theories.

References

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