

# **Aplicações da Teoria de de Broglie-Bohm: Transferência de não-equilíbrio quântico, perturbações cosmológicas fora do equilíbrio e a hipótese geométrica.**

**Francisco Bento Lustosa**

Bolsista PCID-C

**Supervisor: Nelson Pinto-Neto**

Centro Brasileiro de Pesquisas Físicas (CBPF)

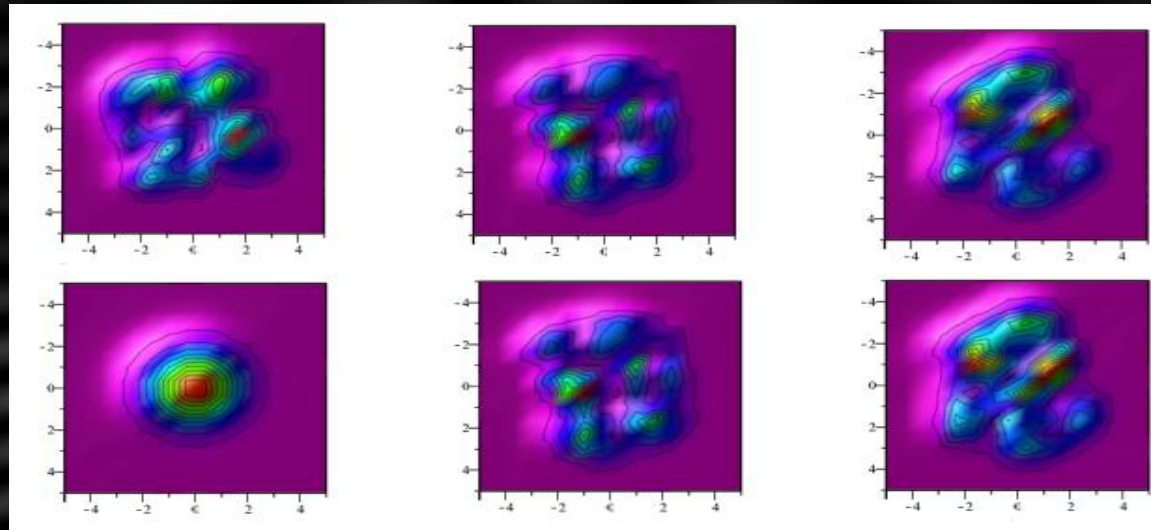
Coordenação de Cosmologia, Astrofísica e Interações Fundamentais (COSMO)

**JORNADA PCI-CBPF 2022**



**CBPF**

# Evolution of quantum nonequilibrium for coupled harmonic oscillators



**Francisco Bento Lustosa**

Centro Brasileiro de Pesquisas Físicas (CBPF)

Coordenação de Cosmologia, Astrofísica e Interações Fundamentais (COSMO)  
em colaboração com Nelson Pinto-Neto (CBPF) e Antony Valentini (Clemson)

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# Summary:

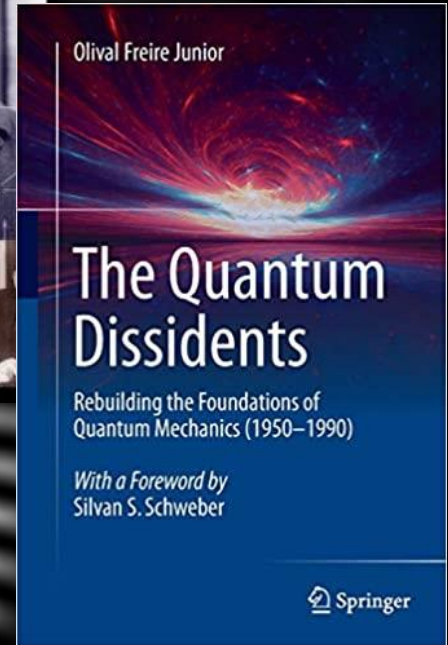
- **Introduction**
- **The de Broglie-Bohm Theory**
- **The Quantum Equilibrium Hypothesis**
- **Quantum nonequilibrium in the early universe**
- **Interactions and quantum relaxation**
- **Conclusions**

# Introduction

- The **Copenhagen Interpretation** of quantum mechanics is extremely successful in making experimental probabilistic predictions but is also plagued with **fundamental inconsistencies**, the most glaring one being the **measurement problem**.
- The **de Broglie-Bohm (dBB) theory** provides a complete, **non-local and deterministic** description of non-relativistic quantum mechanics, including a coherent description of the measurement process.
- The dBB theory has been **successfully applied to cosmology** where it can be shown that singularities are generally avoided by quantum effects.
- In the dBB theory the relation between the wave function and probabilities given by **the Born rule describes a quantum equilibrium state** that is dynamically reached by a process of *quantum relaxation* (**Quantum Equilibrium Hypothesis**).
- Quantum nonequilibrium might have existed at the **very early universe** and, if complete quantum relaxation is avoided, it could leave **detectable signatures of violations of the Born rule** in the CMB or in some relic systems.

# The de Broglie–Bohm Theory

- In the Solvay Conference of 1927 Louis de Broglie presented his **Pilot wave theory** that described quantum **systems as composed by a particle and a guiding wave**.
- Contrary to popular belief, there was **no consensus around the correct description of quantum phenomena**. However, shortly after the conference de Broglie abandoned his theory dissatisfied by the definition of the pilot wave.
- In the 1950's, motivated by discussions with Einstein, **David Bohm developed a quantum theory with "hidden" variables** using the same basic ideas of de Broglie.
- Bohm proposed a **stochastic process** through which the **Born rule could be dynamically reached**.



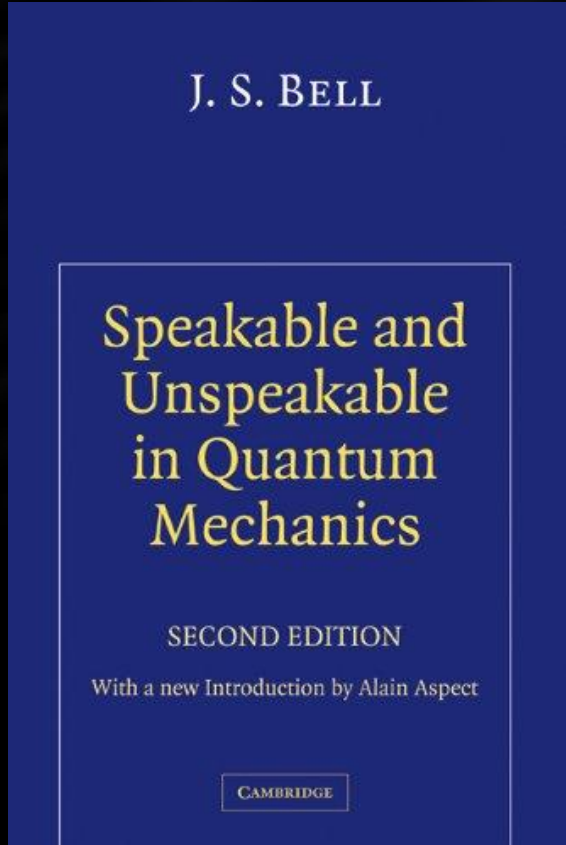


# The de Broglie–Bohm Theory

- The 2022 **Nobel prize** in physics was given to the physicists that **experimentally showed the violations of Bell's inequalities**.
- It is commonly said that these violations **prove the impossibility of hidden variables theories**.
- Quite to the contrary, **John Bell was influenced by Bohm's ideas** and his inequalities served to show that **hidden variable theories had to be non-local** (or violate statistical independence; superdeterminism).



# The de Broglie–Bohm Theory



“But in 1952 I saw the impossible done. It was in papers by David Bohm. **Bohm showed explicitly how** parameters could indeed be introduced, into nonrelativistic wave mechanics, with the help of which the indeterministic description could be transformed into a deterministic one. More importantly, in my opinion, the subjectivity of the orthodox version, **the necessary reference to the ‘observer’, could be eliminated.**

Moreover, **the essential idea was** one that had been advanced already by de Broglie in 1927, **in his ‘pilot wave’ picture.**

But why then had Born not told me of this ‘pilot wave’? If only to point out what was wrong with it? Why did von Neumann not consider it? More extraordinarily, why did people go on producing ‘impossibility’ proofs, after 1952, and as recently as 1978? When even Pauli, Rosenfeld, and Heisenberg, could produce no more devastating criticism of Bohm’s version than to brand it as ‘metaphysical’ and ‘ideologica’? Why is the pilot wave picture ignored in textbooks?

**Should it not be taught, not as the only way, but as an antidote to the prevailing complacency? To show that vagueness, subjectivity, and indeterminism, are not forced on us by experimental facts, but by deliberate theoretical choice?”**

(John Bell in “On the impossible pilot wave”)

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# The de Broglie–Bohm Theory

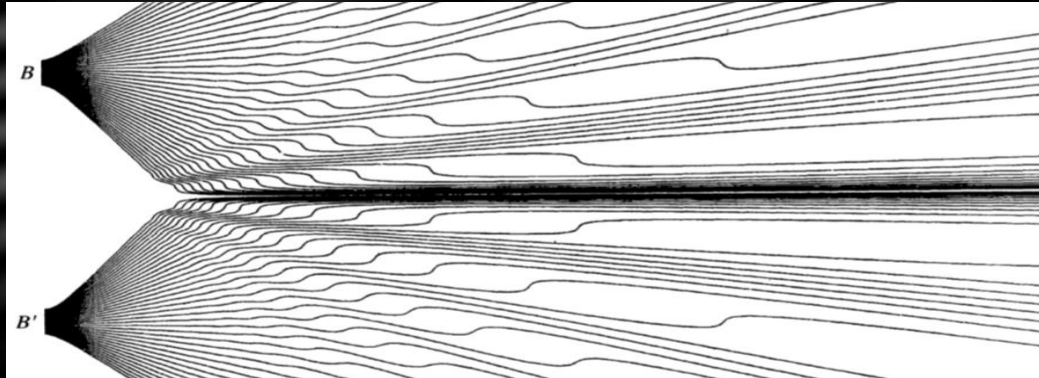
- An isolated **physical system consists** of a **wave** propagating through spacetime **and a point particle** that evolves continuously guided by the pilot wave.
- The wave is a solution to the usual Schrödinger equation:

$$i\hbar\frac{\partial}{\partial t}\psi(x,t) = \left[ -\frac{\hbar^2}{2m}\nabla^2 + V(x,t) \right] \psi(x,t)$$

- The particle movement is determined by the  $x(t)$  solution to the **guidance equation**:

$$\dot{x} = (1/m)\mathbf{Im} \left( \frac{\nabla\psi(x,t)}{\psi(x,t)} \right) \Big|_{x=x(t)}$$

- The **probabilistic properties** come from a **lack of knowledge** of the initial conditions of the system.

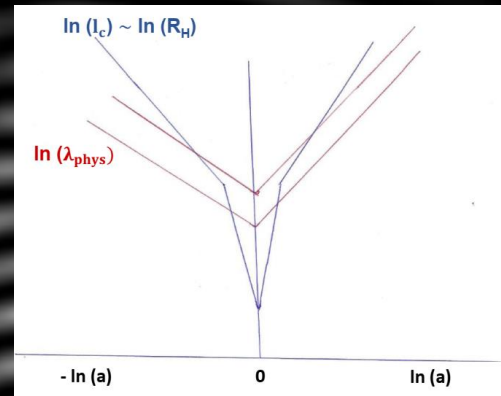
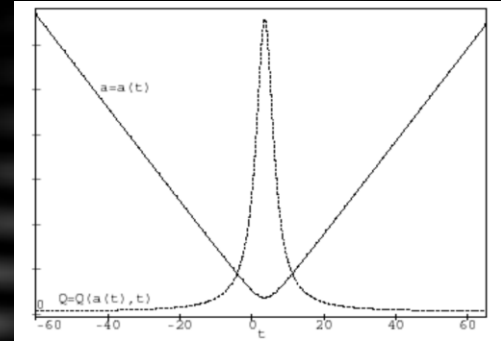


Philippidis and Hiley, 1979



# The de Broglie–Bohm Theory

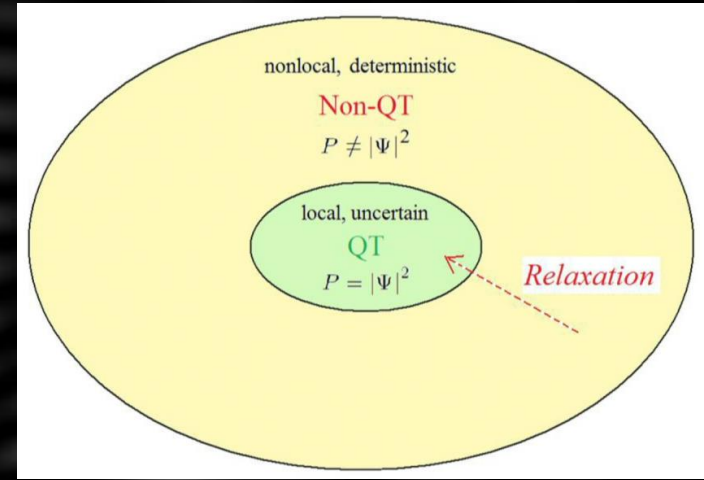
- Application of the **dBB theory to quantum cosmology** has provided natural ways of **solving** some of the problems of the standard cosmology model: **singularity, horizon, flatness...**
- The dBB theory provides a **complete description of quantum-to-classical transition** of cosmological perturbations.
- **Bouncing** models based on the theory have been shown to **reproduce the main features of the CMB**.
- It is possible to construct models **without a scalar field** that reproduce the known cosmological evolution.
- Could the quantum vacuum state that generates the seeds for **cosmological perturbations be initially out of quantum equilibrium**? Could the whole universe be initially out of quantum equilibrium? Could nonequilibrium be generated near the Planck scale during the quantum bounce?



Nelson Pinto-Neto, 2021

# The Quantum Equilibrium Hypothesis

- Motivated by Bohm's ideas to **derive the Born rule** in the de Broglie-Bohm theory Antony Valentini (re)introduced the **concept of quantum equilibrium.**
- Apparent conspiracy: quantum mechanics is non-local but we **cannot use entanglement to send superluminal signals.**
- The de Broglie-Bohm theory is deterministic but we have to **account for statistical uncertainties.**
- We are "trapped" in a quantum equilibrium state.
- How did we get to that state? **Do quantum systems generally relax to equilibrium?**
- Quantum relaxation can be described by the evolution of a **coarse-grained H-function analogous to the one found in classical statistical mechanics** (Paul and Tatyana Ehrenfest, 1959).
- The H-function is zero when the system is in equilibrium, **the way H(t) evolves determines if systems will relax or not.**



$$\frac{\partial |\psi|^2}{\partial t} + \partial_q \cdot (|\psi|^2 \dot{q}) = 0 \quad \frac{\partial \rho}{\partial t} + \partial_q \cdot (\rho \dot{q}) = 0$$

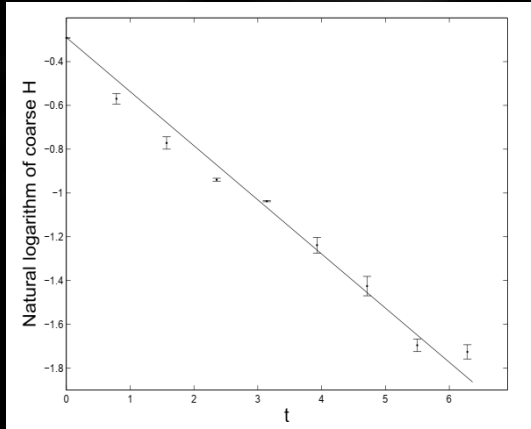
$$\bar{H} = \int \bar{\rho} \ln (\bar{\rho} / |\psi|^2) dx$$

$$\frac{dH}{dt} \leq 0$$

# The Quantum Equilibrium Hypothesis

Quantum relaxation can be studied through numerical simulations. A number of studies have shown that **most systems relax efficiently** and the coarse-grained H-function exponentially decays with time. However, **relaxation could be delayed or prevented in some cases**. Could **nonequilibrium survive the early universe expansion**?

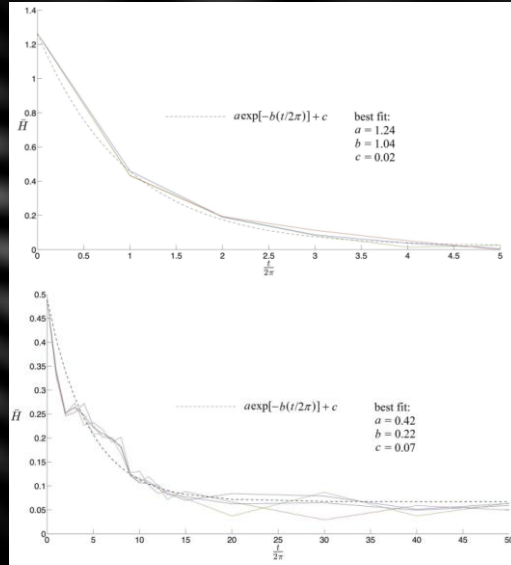
$$\Psi(\mathbf{x}, t) = \frac{2}{\pi\sqrt{M}} \sum_{m,n=1}^{\sqrt{M}} \sin(mx) \sin(ny) \exp i(\theta_{mn} - E_{mn}t).$$



(Valentini e Westman, 2005)

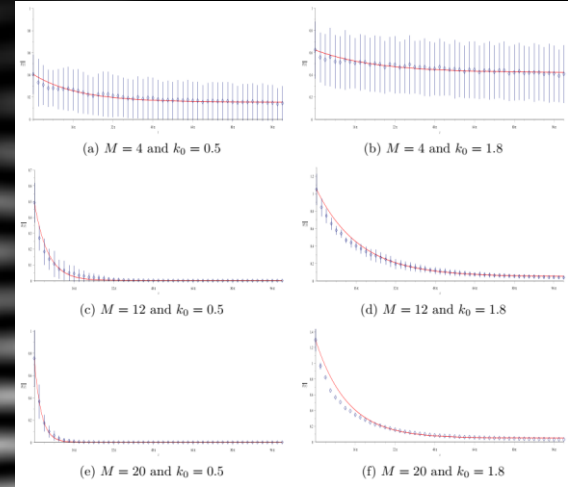
$$\overline{H}(t) \approx \overline{H}_0 e^{-t/\tau}$$

$$\psi(q_1, q_2, t) = \frac{1}{\sqrt{M}} \sum_{m,n=0}^{\sqrt{M}-1} e^{i\theta_{mn}} e^{-i(m+n+1)t} \phi_m(q_1) \phi_n(q_2)$$



(Abraham, Colin e Valentini, 2015)

$$H(x_a, x_b, t) = \frac{p_a^2}{2m} + \frac{m\omega^2 x_a^2}{2} + \frac{p_b^2}{2m} + \frac{m\omega^2 x_b^2}{2} + mkx_a x_b$$

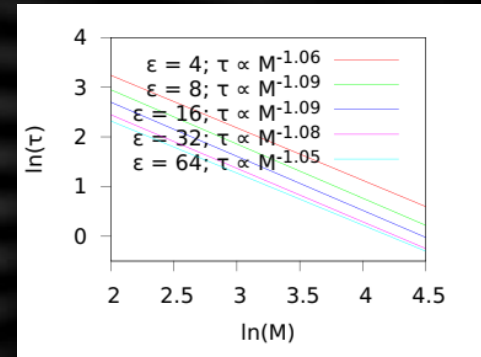


(FBL, Pinto-Neto e Valentini, 2022)

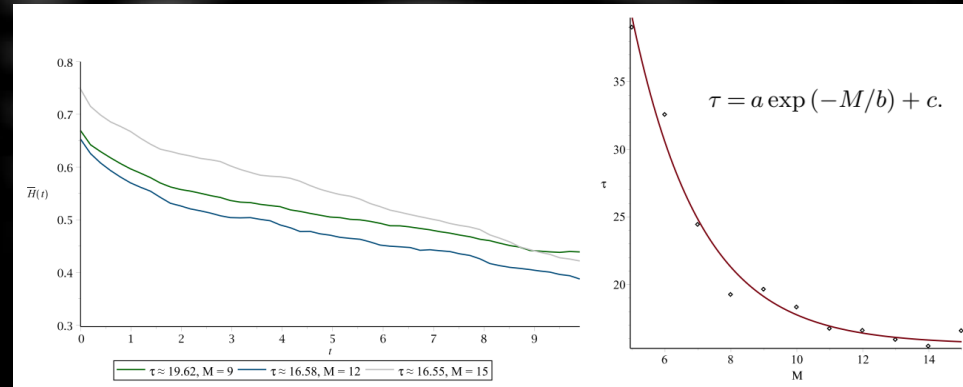
$$\overline{H}(t) \approx (\overline{H}(0) - R) e^{-t/\tau} + R$$

# The Quantum Equilibrium Hypothesis

- The evolution of **the coarse-grained H-function** can be influenced by all the parameters of the system.
- The **coarse-graining method** and the details of the numerical simulations also have to be taken into account (for a review see **Section 2 of FBL, Colin e Bergliaffa 2021**).
- The **relaxation timescale  $\tau$**  and the **nonequilibrium  $R$**  show how efficiently the H-function approaches zero.
- Simulations for nonrelativistic systems have shown **direct correlations between the relaxation timescale and the number of superposed modes of the wavefunction**.
- Most systems relax very efficiently, but complete relaxation may be delayed or even prevented in some situations.



(Towler e Valentini, 2011)



(FBL, Colin e Bergliaffa, 2021)

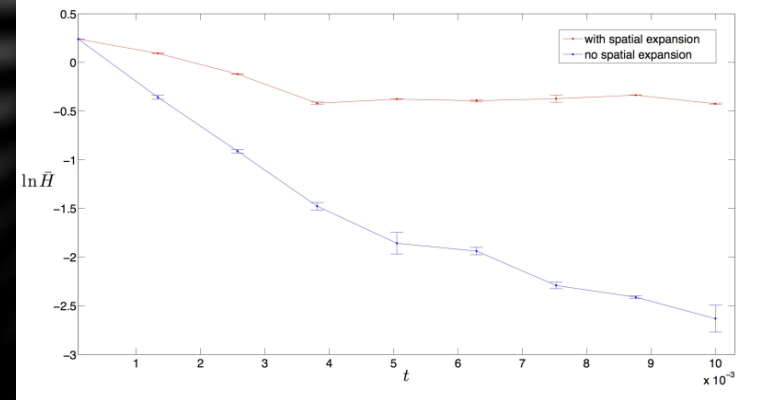
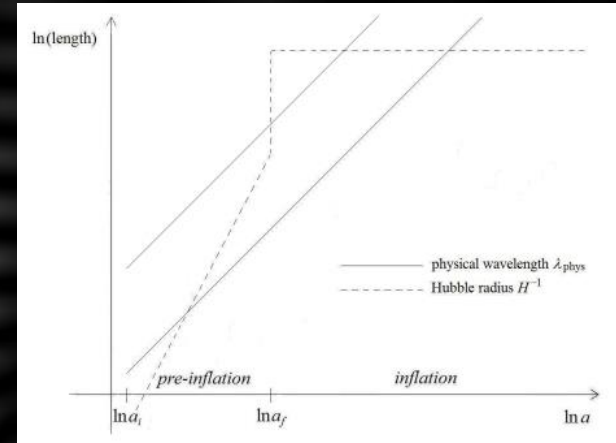
# Quantum nonequilibrium in the early universe

Q: Considering that all observed systems appear to obey the Born rule and simulations indicate that most systems relax very fast ( $\tau \sim 10^{-20}$ s), **how and where/when can we observe the effects of quantum nonequilibrium?**

A: When/where: early universe! How? If the initial quantum state of the inflation (or the universe) was out of equilibrium this could lead to detectable signatures in...

- **CMB** (Colin e Valentini, 2013, 2015, 2016)
- **Relic particles** (Underwood e Valentini, 2014, 2016)
- **Reheating? Structure formation?** (FBL, ????)

A\*: Nonequilibrium could be generated near **black hole horizons or at the Planck scale** (Valentini, 2007, 2014 + Kandhadai e Valentini, 2019 + Valentini, 2021)



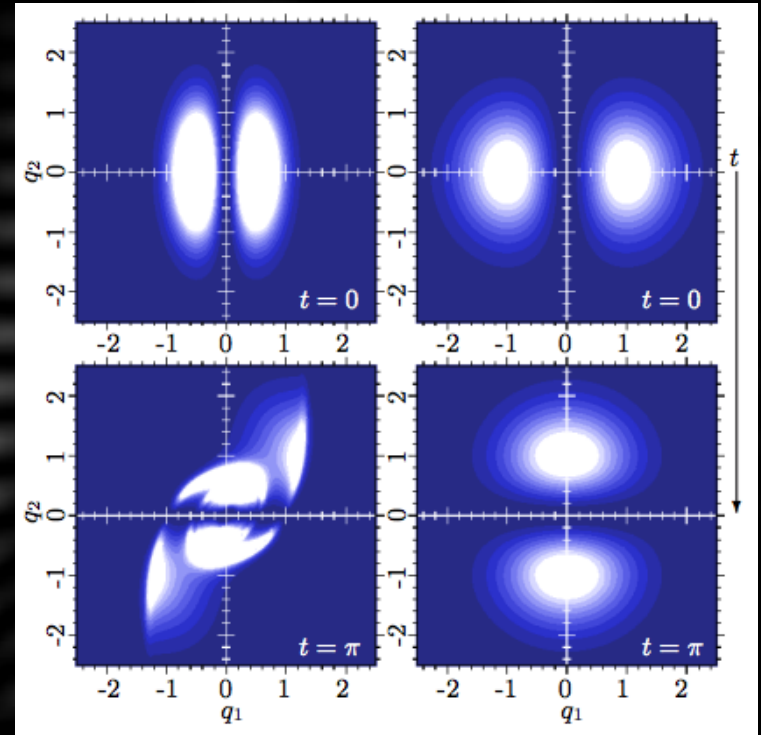
(Colin e Valentini, 2013)

Evolution of quantum nonequilibrium for coupled harmonic oscillators



# Quantum nonequilibrium in the early universe

- If **nonequilibrium** in super-Hubble modes of the inflaton survives the inflationary phase it **could be transferred to other particle species through decaying processes.**
- For two bosonic coupled scalar fields it was shown that **if one of the fields is** in a excited state **out of equilibrium** the decay process leads to **the whole system being out of equilibrium.**
- **But how interactions affect the relaxation process?** If the nonequilibrium excited states are created during reheating interactions between particles might accelerate the relaxation process.

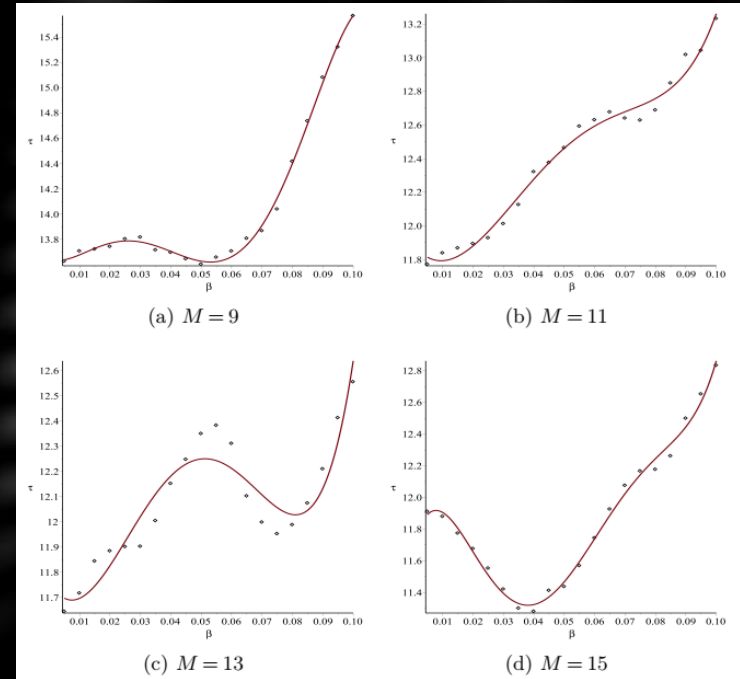


(Underwood e Valentini, 2015)

# Interactions and quantum relaxation

- The **relaxation** process seems to be **accelerated for more complex** systems, which lead to the expectation that **interactions might wash away** any initial **nonequilibrium**.
- However, simulations for a system of two 1D harmonic oscillators with a time-dependent coupling have indicated the opposite: **increase in the coupling constant lead to increase in the relaxation timescale**.
- Previous studies for the scalar field already indicated that some type of **time-dependence might prevent relaxation**.
- Time independent **interactions** would accelerate or delay quantum relaxation?

$$H(x_a, x_b, t) = \frac{p_a^2}{2m} + \frac{p_b^2}{2m} + \frac{m\omega^2 x_a^2}{2} + \frac{m\omega^2 x_b^2}{2} + 2m\beta t x_a x_b$$



(FBL, Colin e Bergliaffa, 2021)

# Interactions and quantum relaxation

$$H(x_a, x_b, t) = \frac{p_a^2}{2m} + \frac{m\omega^2 x_a^2}{2} + \frac{p_b^2}{2m} + \frac{m\omega^2 x_b^2}{2} + mkx_a x_b,$$

$$H(x_1, x_2, t) = \sum_{r=1}^2 H_r = \sum_{r=1}^2 \left( \frac{p_r^2}{2m} + m\Omega_r^2 x_r^2 \right),$$

$$\Omega_r = \sqrt{(m\omega^2 \pm k/2)}.$$

$$\Psi(x_a, x_b, t) = \sum_{n_1, n_2} c_{n_1, n_2} \psi_{n_1}(x_1(x_a, x_b), t) \psi_{n_2}(x_2(x_a, x_b), t),$$

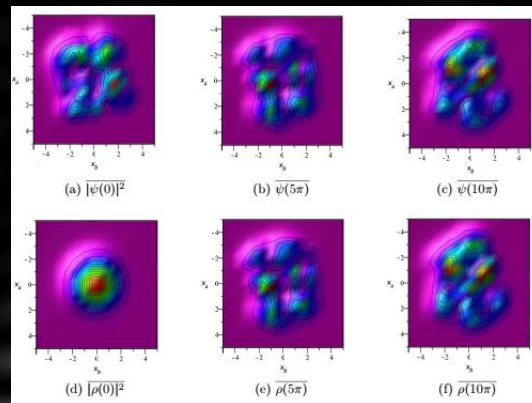
$$i\partial_t \psi_r = \hat{H}_r \psi_r.$$

$$\dot{x}_r = \frac{1}{m} \text{Im} \left( \frac{\partial_r \Psi}{\Psi} \right),$$

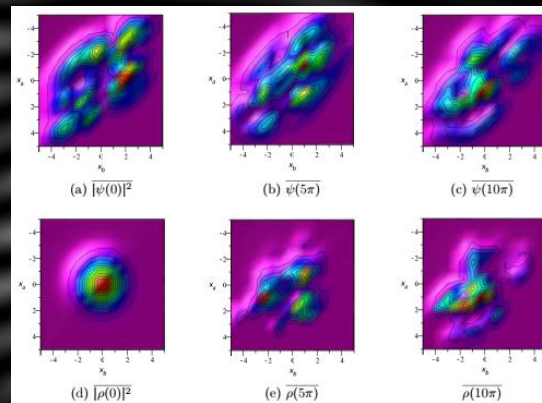
$$(x_a(0), x_b(0)) \rightarrow (x_1(0), x_2(0)),$$

$$(x_1(0), x_2(0)) \rightarrow (x_1(t), x_2(t)),$$

$$(x_1(t), x_2(t)) \rightarrow (x_a(t), x_b(t)).$$



$k=0.1$

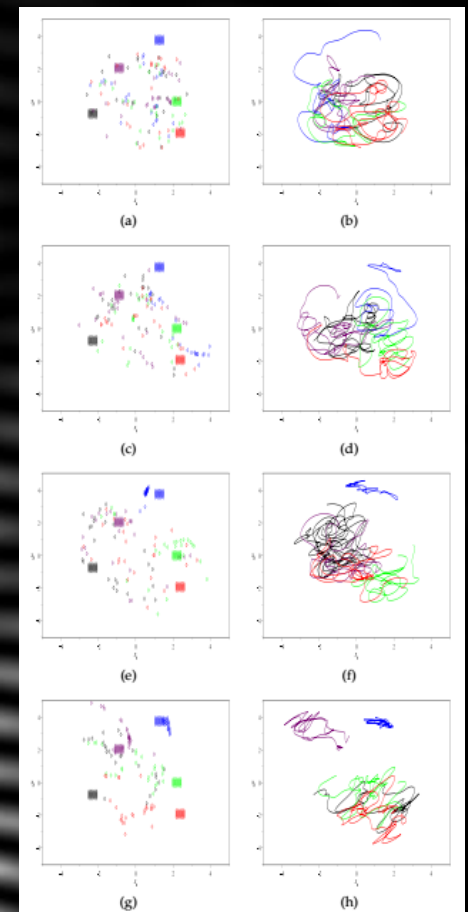


$k=1.8$

(FBL, Pinto-Neto e Valentini, 2022)

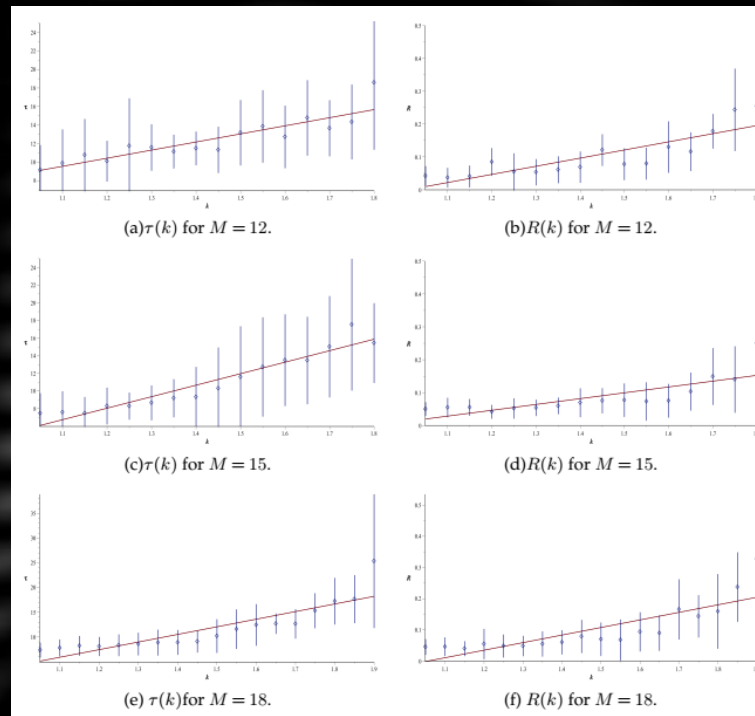
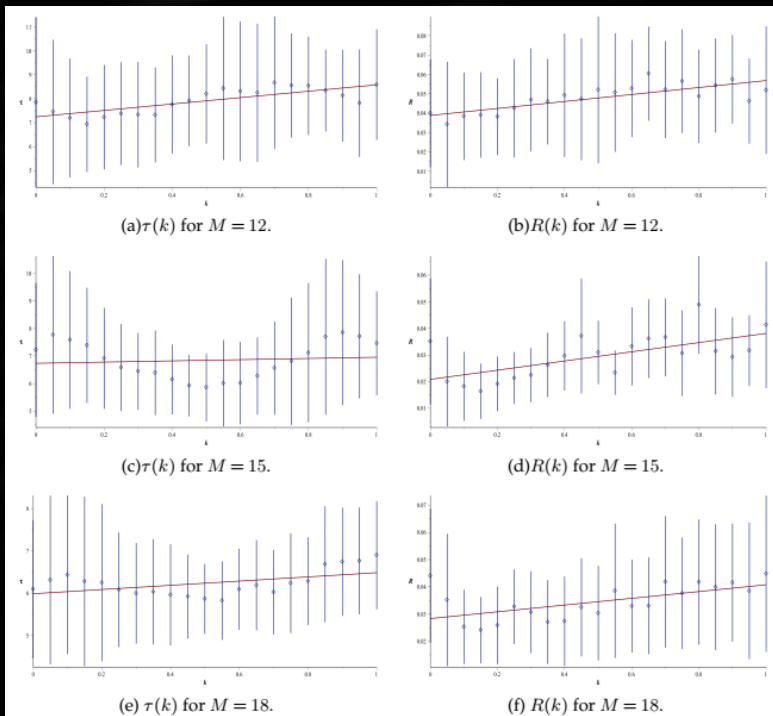
# Interactions and quantum relaxation

- Analysis of the evolution of nonequilibrium distributions, trajectories and of a large number of H-functions all indicated that, contrary to what was expected, **stronger interactions lead to larger relaxation timescales** and leaves open the possibility of **long-lasting nonequilibrium residues**.
- The **character of trajectories** indicates how chaotic the system is. Trajectories that explore **larger regions** of the support region of the wavefunction are a **indication that a system relax efficiently**.
- Our system seems to have two regimes: when  $k < \omega$  the coupling seems to have little to no effect on the trajectories and when  $k > \omega$  the trajectories seem to remain confined to regions close to their initial positions.
- This can be interpreted as: **when interactions dominate** over the fundamental frequencies of the system **quantum relaxation might be suppressed**.



(FBL, Pinto-Neto e Valentini, 2022)

# Interactions and quantum relaxation



Our results show a direct **correlation** between relaxation **timescales** and the **coupling** constant for this system **when  $k > \omega$** . For these values of  $M$  residues are too small but they also seem to be correlated to the values of  $k$ .

$$\overline{H(t)} \approx (\overline{H(0)} - R)e^{-t/\tau} + R$$



# CONCLUSIONS

- The de Broglie-Bohm theory is a **complete description of quantum phenomena that can be applied to cosmology** in a natural and paradox-free way.
- **This theory allows for** violations of the Born rule which can lead to **new physics**. Those violations might have **existed in the very early universe** and could leave detectable signatures.
- Numerical simulations have indicated that **most complex systems should relax** very fast but some factors may delay or prevent complete relaxation.
- **Super-Hubble modes** of the inflaton have been shown to stay out of equilibrium when evolving on an expanding background. If those modes decay into other particles they **could transfer signatures of nonequilibrium**.
- **Interactions** play a fundamental role in these scenarios so their **effects on quantum relaxation should be further investigated**.
- Early results for nonrelativistic systems indicate that **when interaction dominates** over other properties of the system **quantum relaxation is delayed and may be prevented** for simple systems on expanding backgrounds.

# OPEN QUESTIONS

- How would quantum **nonequilibrium** modes of cosmological perturbations evolve **in other cosmological models?**
- What could be the observable effects if quantum nonequilibrium existed during and after reheating/particle creation?
- Is it possible to define quantum **nonequilibrium for the whole universe?**
- How quantum nonequilibrium evolves in other interacting systems? Multi-field inflation, interacting multifluid bounces, **entanglement between perturbation modes...**
- **If everything fails...** can the de Broglie-Bohm theory provide **other predictions** that deviate from the orthodox interpretation? **(CLUE: Times of arrival of spin  $\frac{1}{2}$  particles, Das, Noth e Dürr, 2019).**

**Obrigado!**