Early warnings of catastrophic changes in ecological systems

Hugo Fort, Raul Donangelo *Montevideo, Rio de Janeiro*
Vasilis Dakos,Egbert van Ness, Marten Scheffer, *Wageningen*

(Theoretical Ecology-2010)

Plan for talk

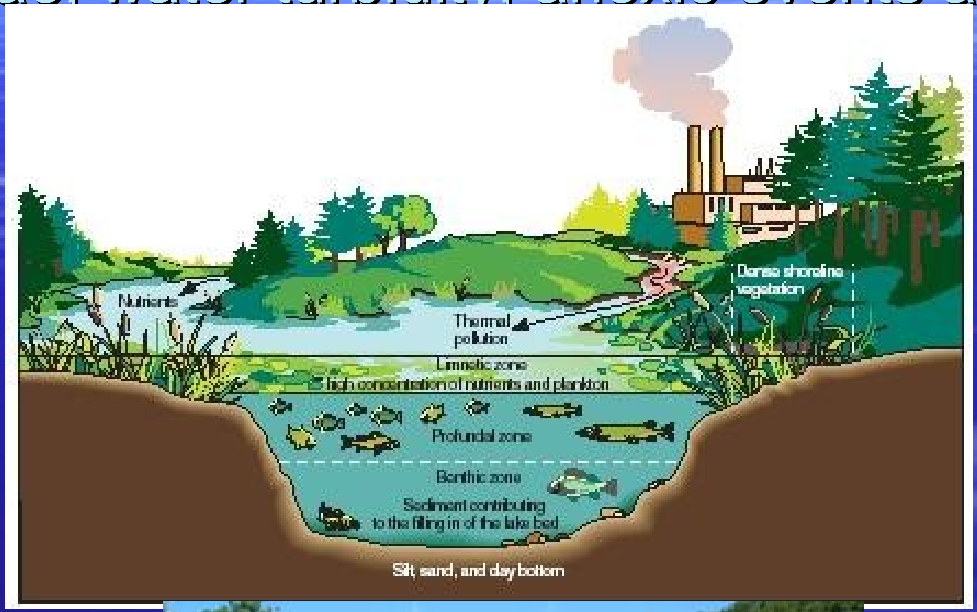
Introduction
Models
Signals
Conclusions

Introduction

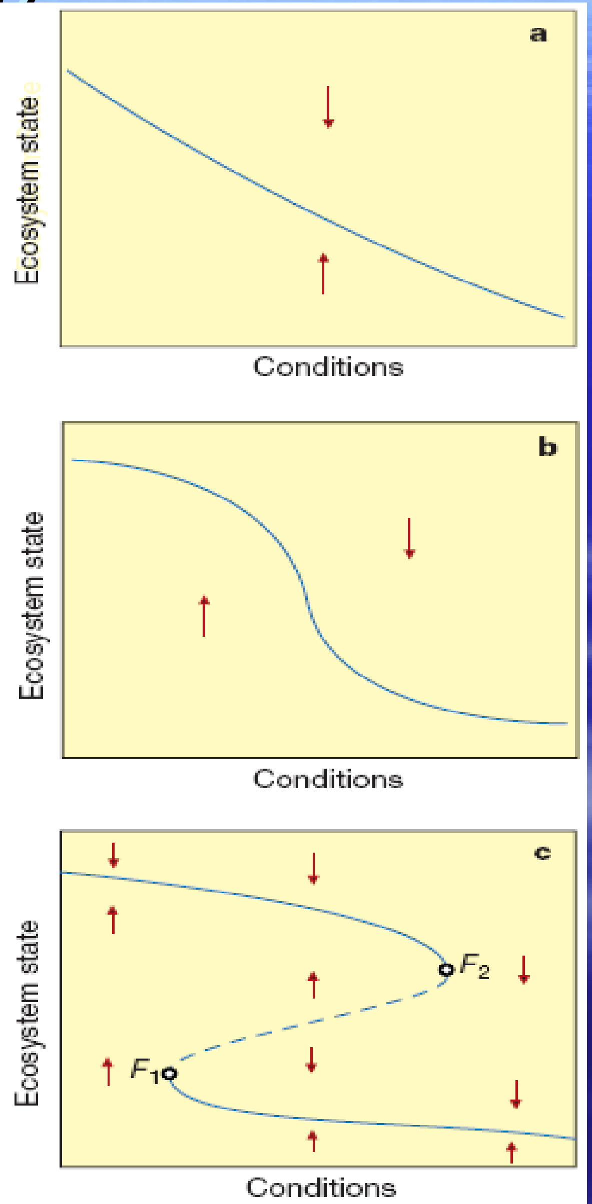
- Sudden extensive ecological changes triggered by small forces have been increasingly identified in a variety of systems.
- Such changes are usually termed as regime shifts between alternative states and their abrupt discontinuous nature makes them challenging to handle from a management point of view.
- Regimes shifts have been described, among others, in lakes, the marine and coastal environment,semi-arid ecosystems.

Example: Lake eutrophication

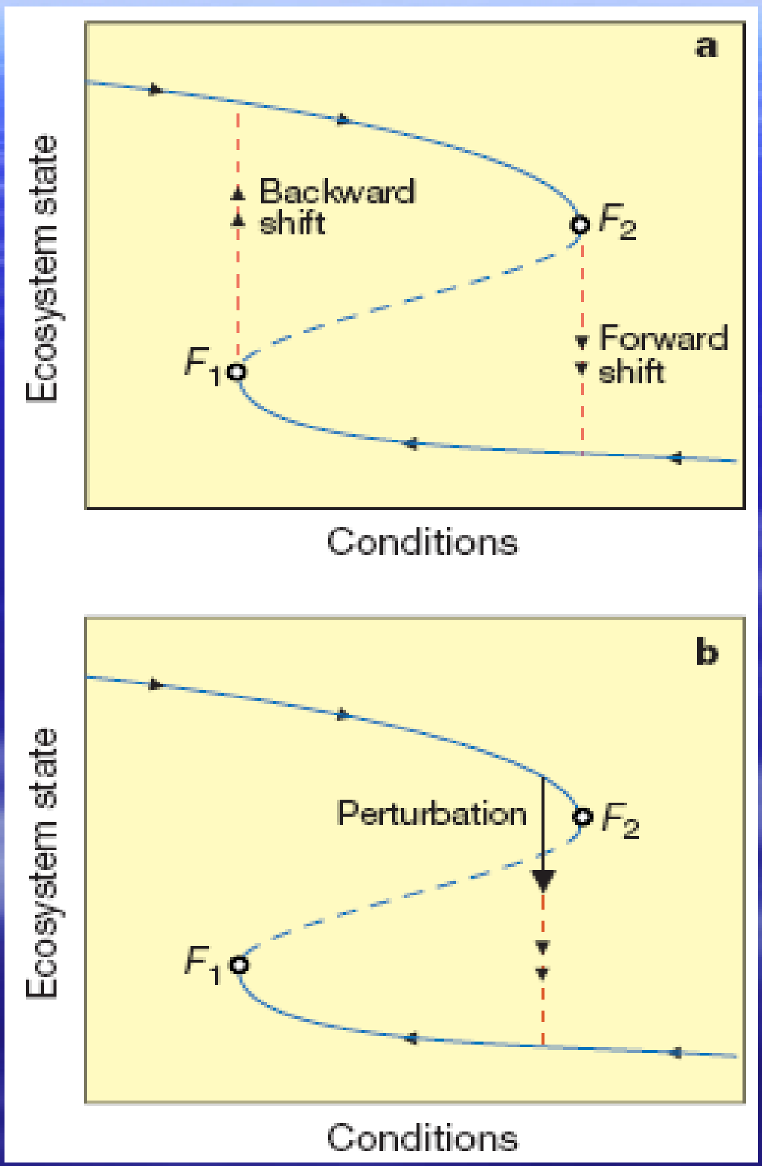
(overenrichment of aquatic ecosystems leading to growth of harmful algae, water turbidity, anoxic events and fish kills)



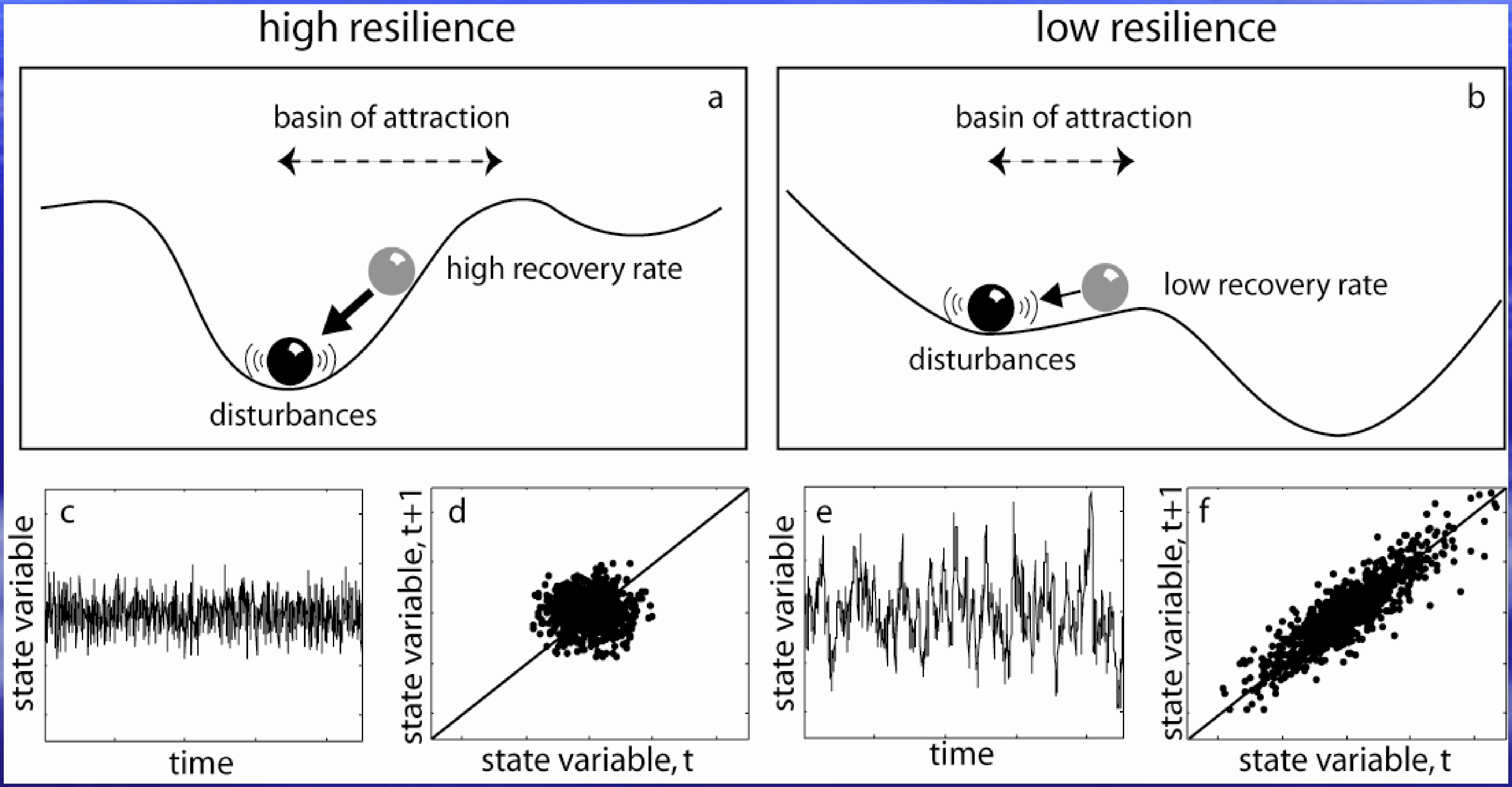
Dependence on external conditions (P loading) for different lake depths



Bifurcations and hysteresis



Basin of Attraction and Resilience Higher σ, time correlations.



General spatial model

- General form of the equation:

$$\frac{dX_{i,j}}{dt} = f(X_{i,j}, p_{i,j}, c) + D(X_{i+1,j} + X_{i-1,j} + X_{i,j+1} + X_{i,j-1} - 4X_{i,j}) + \sigma \frac{dW_{i,j}}{dt}$$

- force diffusion fluctuations
- *D* = diffusion coefficient, σW = white noise
- The force term changes according to the system considered.

Particular cases:
1. Overharvesting model

$$f(X_{i,j},t) = r_{i,j}X_{i,j}\left(1 - \frac{X_{i,j}}{K}\right) - c\frac{X_{i,j}^2}{X_{i,j}^2 + 1}$$

$X_{i,j}$: state variable = resource biomass

K : carrying capacity $[10]$

$r_{i,j}$: maximum growth rate at gridcell (i, j) $[0.6 - 1.0]$

c : control parameter = maximum grazing rate $[1\text{-}3]$

D : dispersion rate $[0\text{-}1]$

σ : standard deviation of white noise $[0.1]$

Particular cases:
2. Eutrophication model

$$f(X_{i,j},t) = a - b_{i,j}X_{i,j} + r\frac{X_{i,j}^{p}}{X_{i,j}^{p}+1}$$

$X_{i,j}$: state variable = nutrient concentration

a : control parameter = nutrient loading rate $[0.1\text{-}0.9]$

$b_{i,j}$: control parameter = maximum grazing rate $[1\text{-}3]$

r : maximum recycling rate $[1.0]$

p : Hill coefficient $[8]$

D : dispersion rate $[0\text{-}1]$

σ : standard deviation of white noise $[0.01]$

Other case:

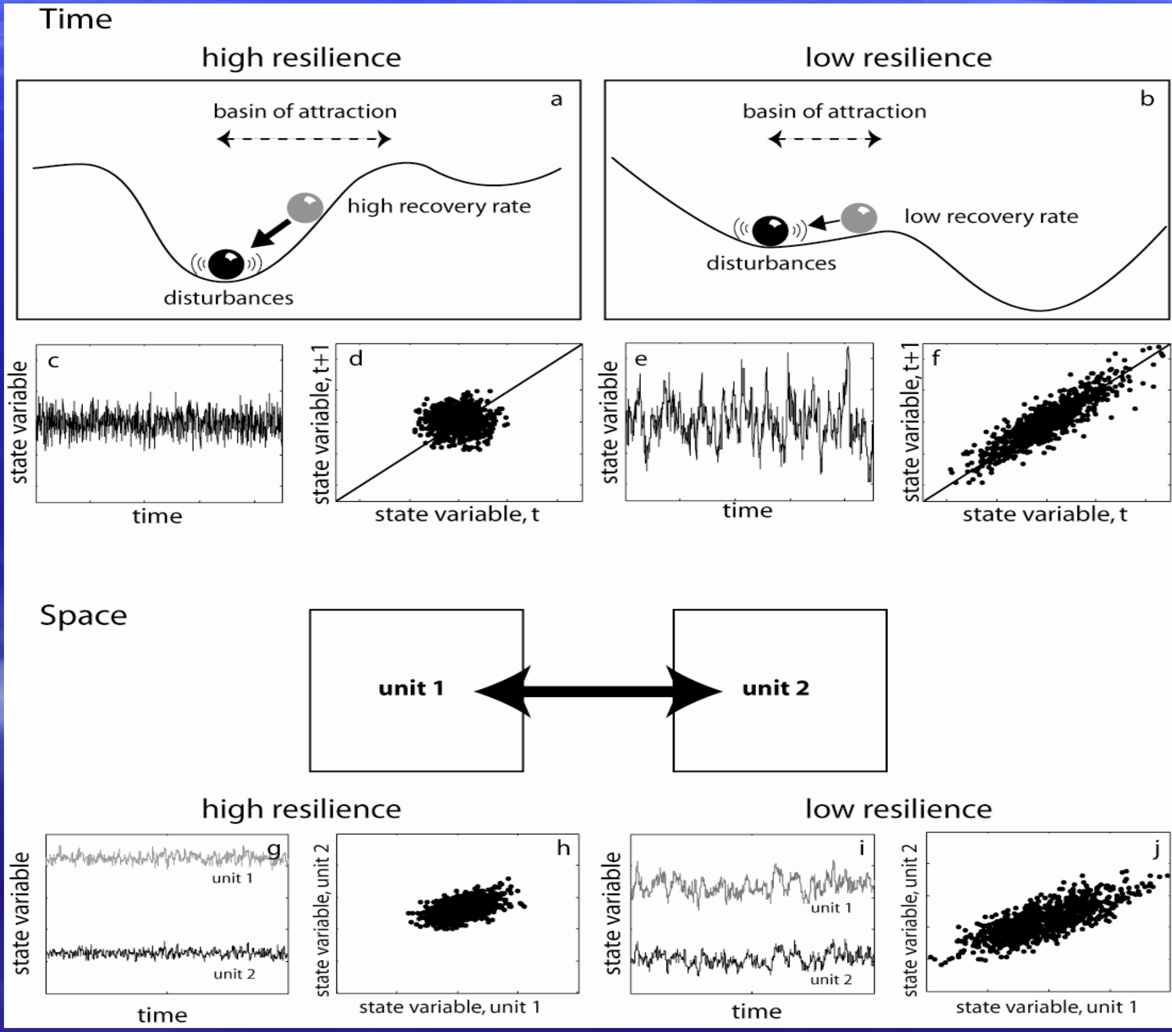
- Vegetation-turbidity model
  Presently being applied to a particular lake (Laguna del Diario, Punta del Este)

First example: overgrazing in a 2-cell spatial model

- Two cells

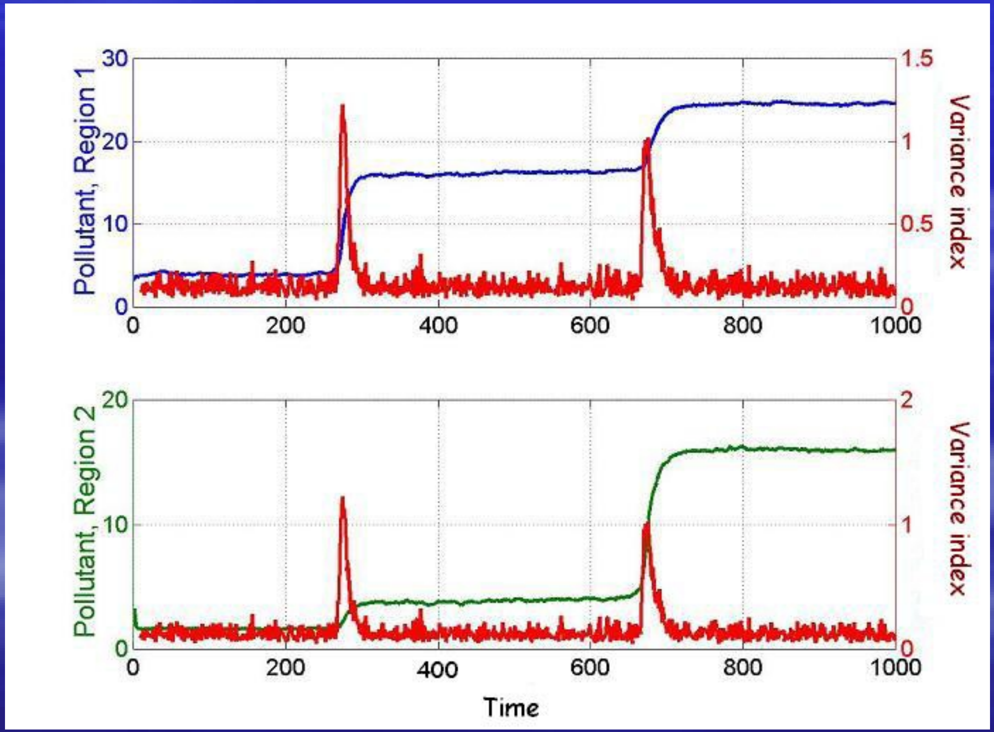
$$\frac{dx_1}{dt} = f(x_1, p_1, c) + D(x_2 - x_1)$$

$$\frac{dx_2}{dt} = f(x_2, p_2, c) + D(x_1 - x_2)$$



TEMPORAL Early Warnings
→Mean Field models
→ temporal variance

$$\sigma_t^2 = \frac{\sum_{t'=t-\tau}^{t} s(0,0;t')^2 - \left(\sum_{t'=t-\tau}^{t} s(0,0;t')\right)^2}{\tau}$$



Brock, W. A., and S. R. Carpenter. 2006. Variance as a leading indicator of regime shift in ecosystem services. *Ecology and Society* 11(2): 9

Spatial model applied to lake eutrophication

$$\frac{dX(x,y;t)}{dt} = a(t) - bX(x,y;t) + r\frac{X(x,y;t)^8}{X(x,y;t)^8 + 1} + 4D\nabla^2 X$$

We work on a spatial square lattice:

Thus, we have a cellular automaton with update rule given by

$$X_{ij}(t+1) = X_{ij}(t) + a_{ij}(t) - bX_{ij}(t) + r\frac{X_{ij}(t)^8}{X_{ij}(t)^8 + 1} +$$

$$D\left[X_{i+1j}(t) + X_{i-1j}(t) + X_{ij+1}(t) + X_{ij-1}(t) - 4X_{ij}(t)\right]$$

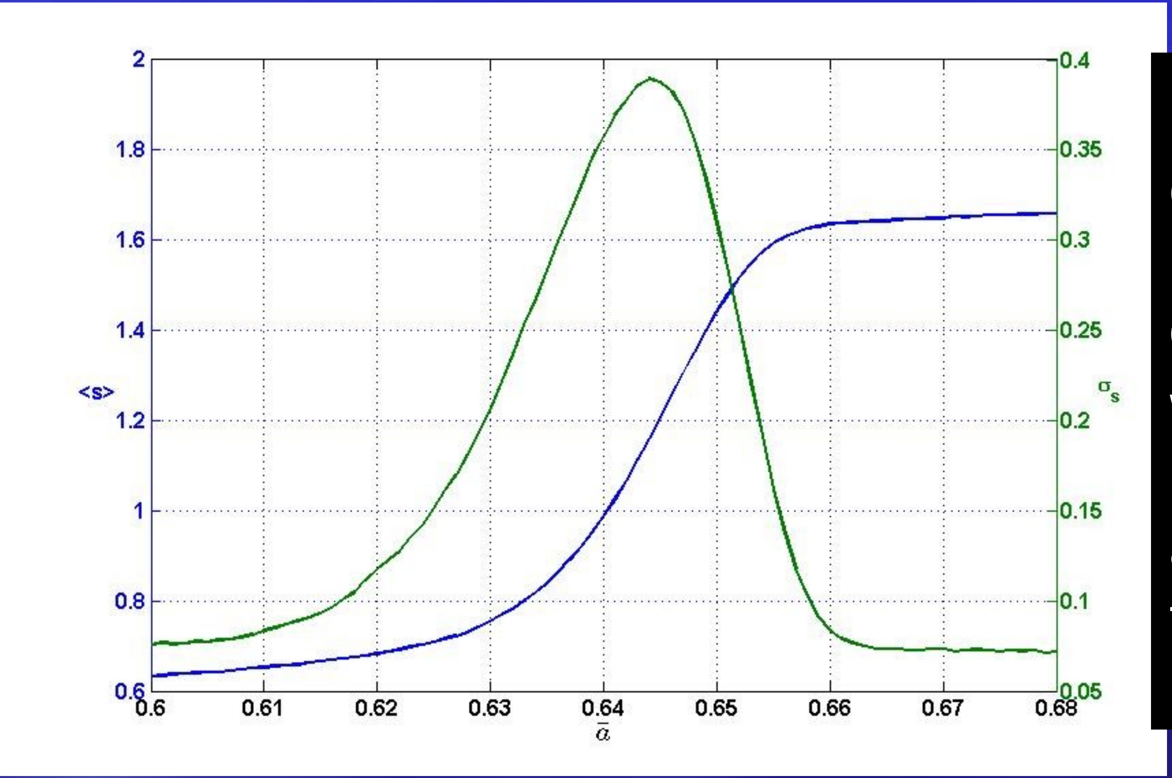
$a_{ij}(t) = \bar{a}(t) \pm \delta a = 0.375 \pm 0.125$, $r = 1$, $D = 0.1$ … $D = 1$

$\bar{a}(t)$ varies in steps of $d\bar{a} = 0.001$. This value of $d\bar{a}$ was estimated from Carpenter 2005 to represent one year in the evolution of lake Mendota.

Other indicators:

I. The ***spatial variance*** of $X_{ij}(t)$, σ_s^2, defined as

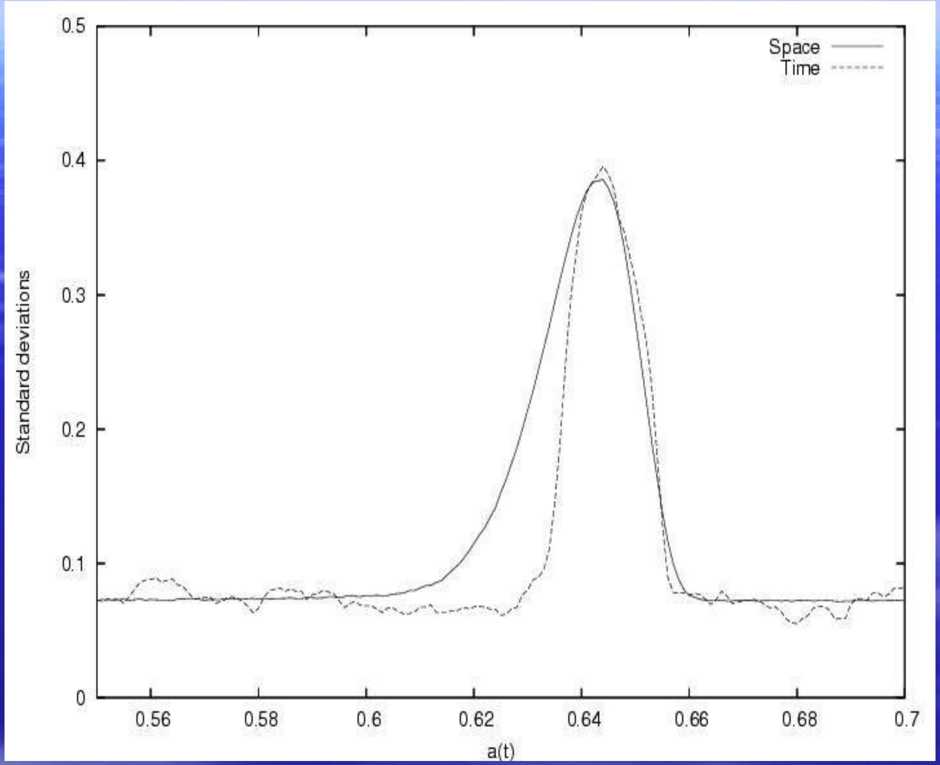
$$\sigma_s^2 \equiv \left\langle X^2 \right\rangle - \left\langle X \right\rangle^2 = \frac{\sum_{i,j=1}^{L} X_{ij}(t)^2 - \left(\sum_{i,j=1}^{L} X_{ij}(t) \right)^2}{L^2}$$



σ_s appears to be an early signal:
it increases by 20% over the initial value when $\bar{a} \approx 0.615$ (year 240)
and has its maximum for $\bar{a} \approx 0.644$ (year 269).

$\langle X \rangle$ (blue) and σ_s (green) vs. $\bar{a}(t)$ for $D = 0.1$.

- Spatial vs. temporal standard deviations

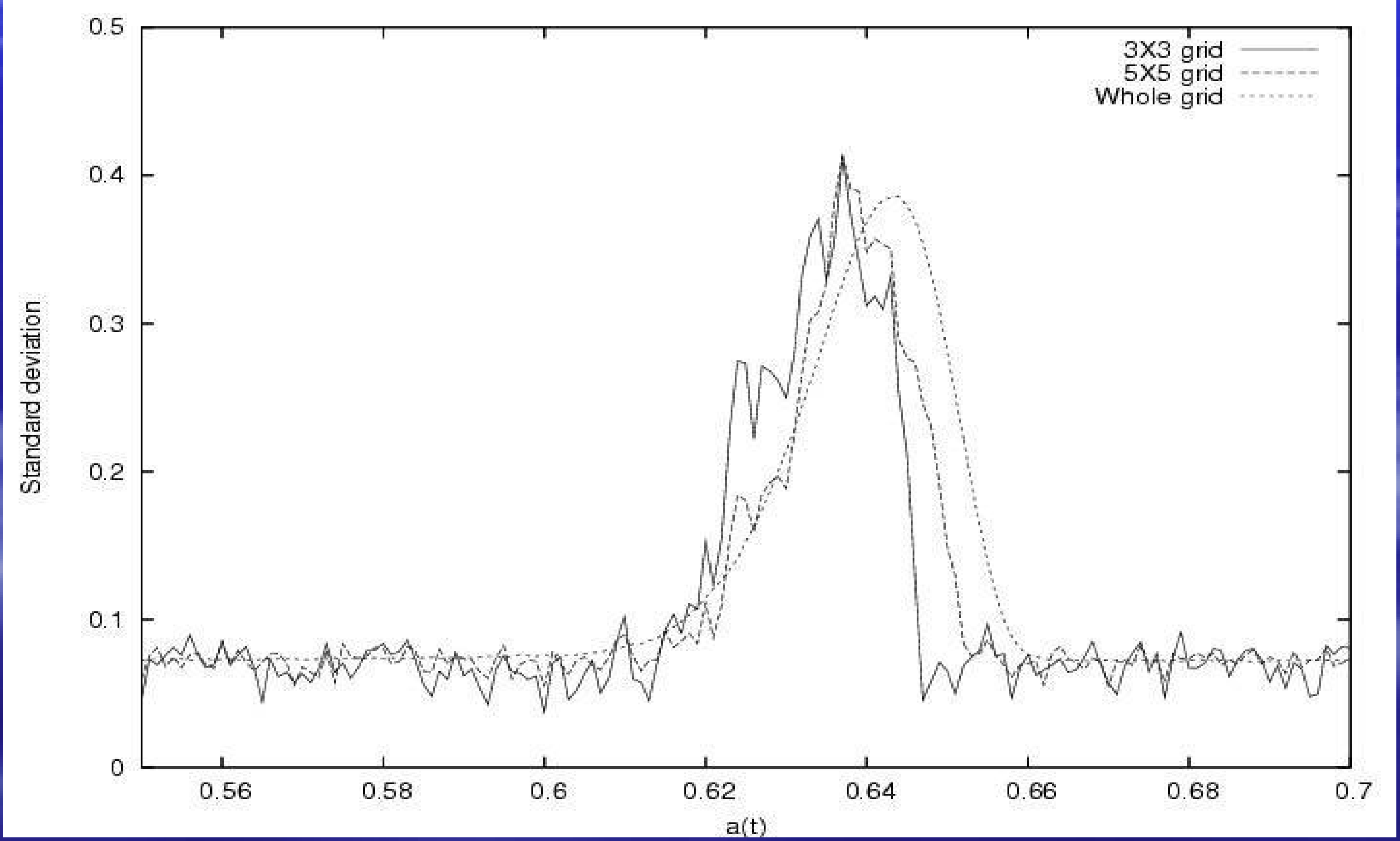


Although the time for the maximum of both σ is the same, σ_s rises before, so it is a better early warning signal for the transition in $X(t)$ than σ_t.

$$\sigma_t^2 = \frac{\sum_{t'=t-\tau}^{t} s(0,0;t')^2 - \left(\sum_{t'=t-\tau}^{t} s(0,0;t')\right)^2}{\tau}$$

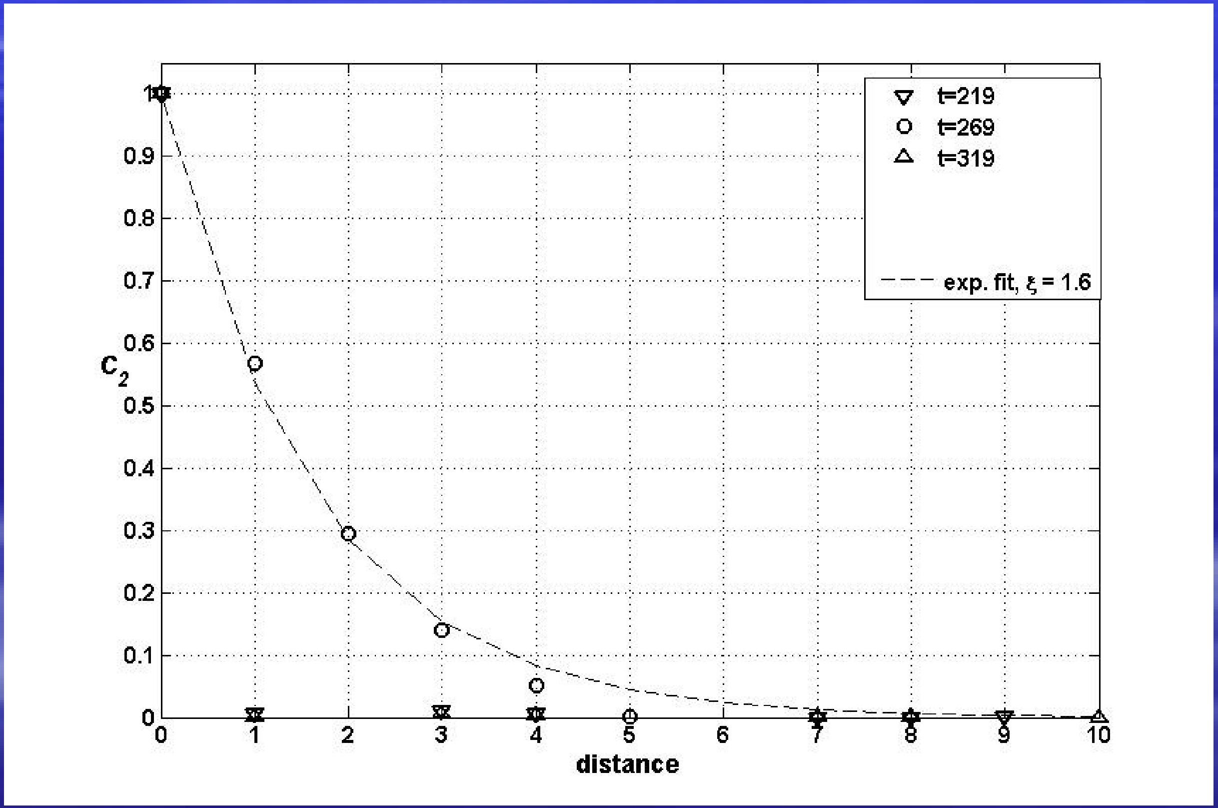
$$\sigma_s^2 = \frac{\sum_{i,j=1}^{L} X_{ij}(t)^2 - \left(\sum_{i,j=1}^{L} X_{ij}(t)\right)^2}{L^2}$$

Signal may be seen with a small grid



- Correlation: ***Two point correlation function***

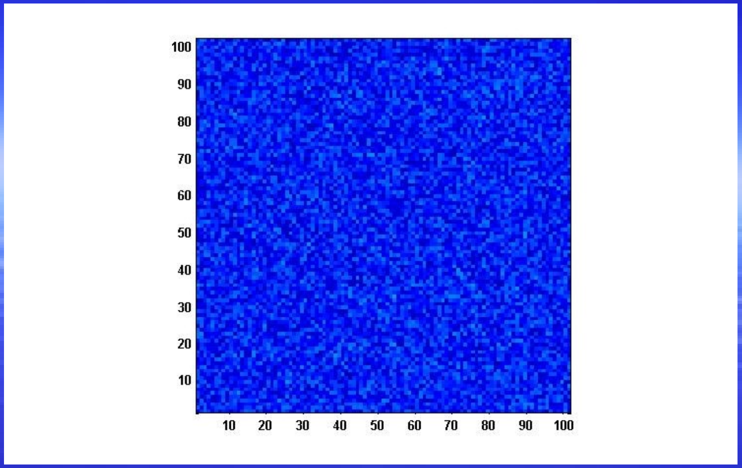
$$C_2(d) = \langle s(x_1, y_1)s(x_2, y_2)\rangle - \langle s(x_1, y_1)\rangle \langle s(x_2, y_2)\rangle$$



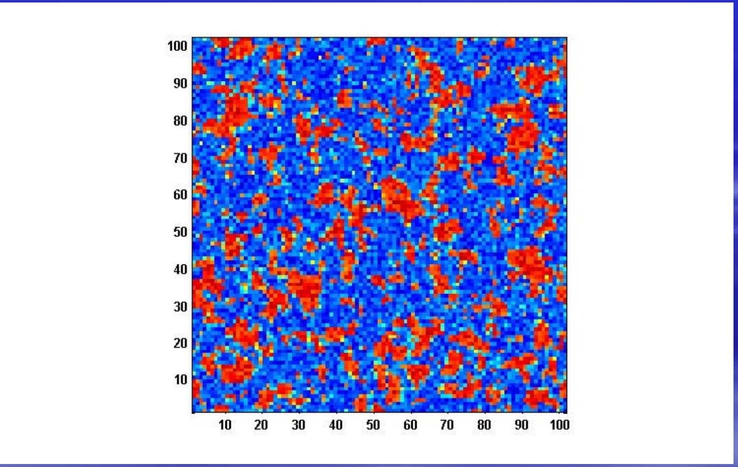
Appearance of clusters

Color scale ranging from blue (s=0.35) to red (s=1.8). $s(x,y;t)$ at times:

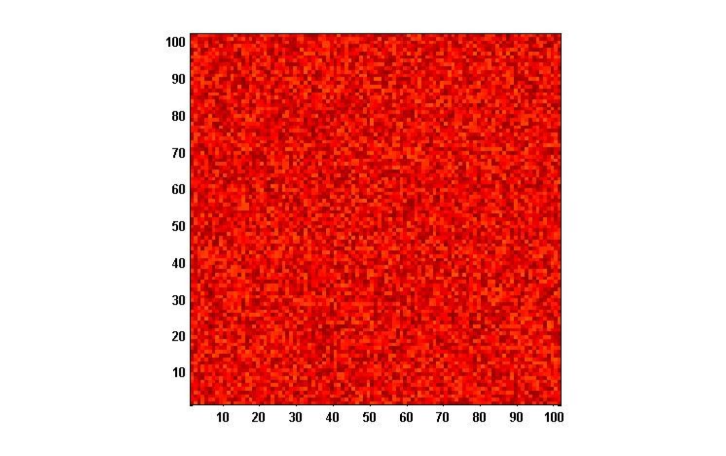
t=219 *i.e.* $\bar{a}(219)=0.594$



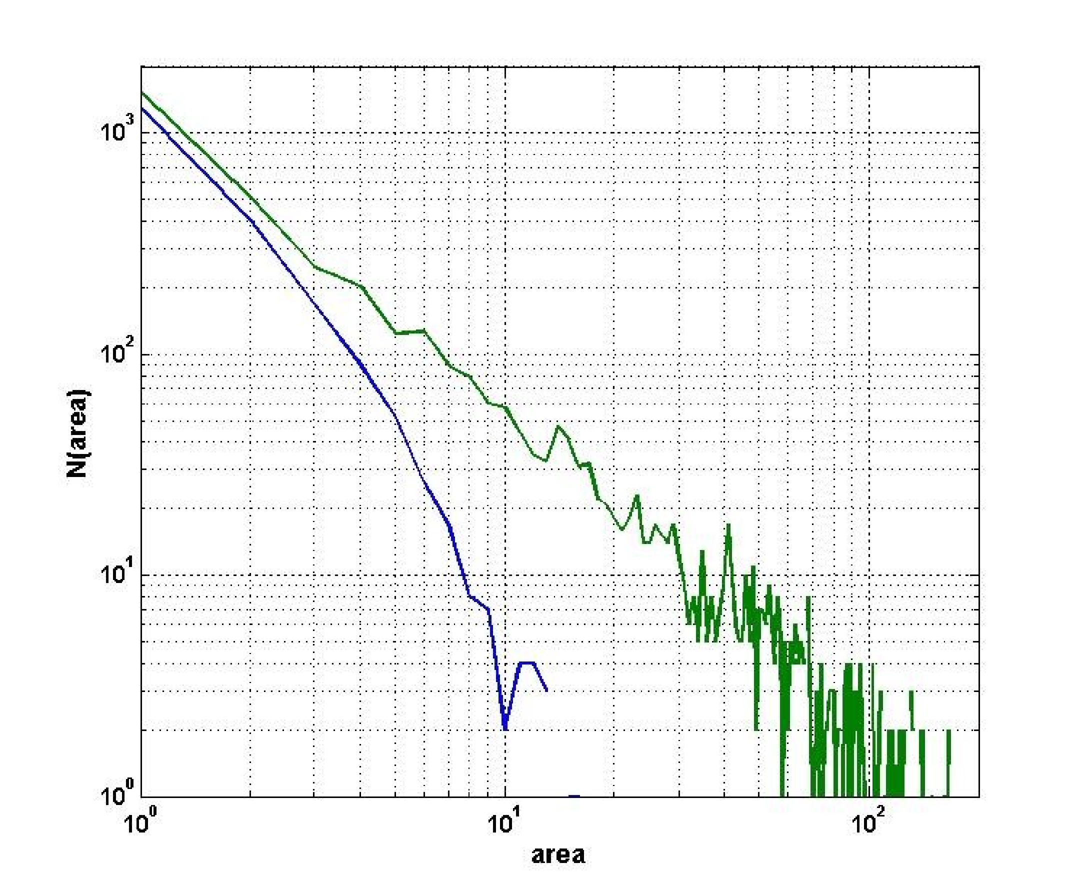
t=269 *i.e.* $\bar{a}(269)=0.644$



t=319 *i.e.* $\bar{a}(319)=0.694$



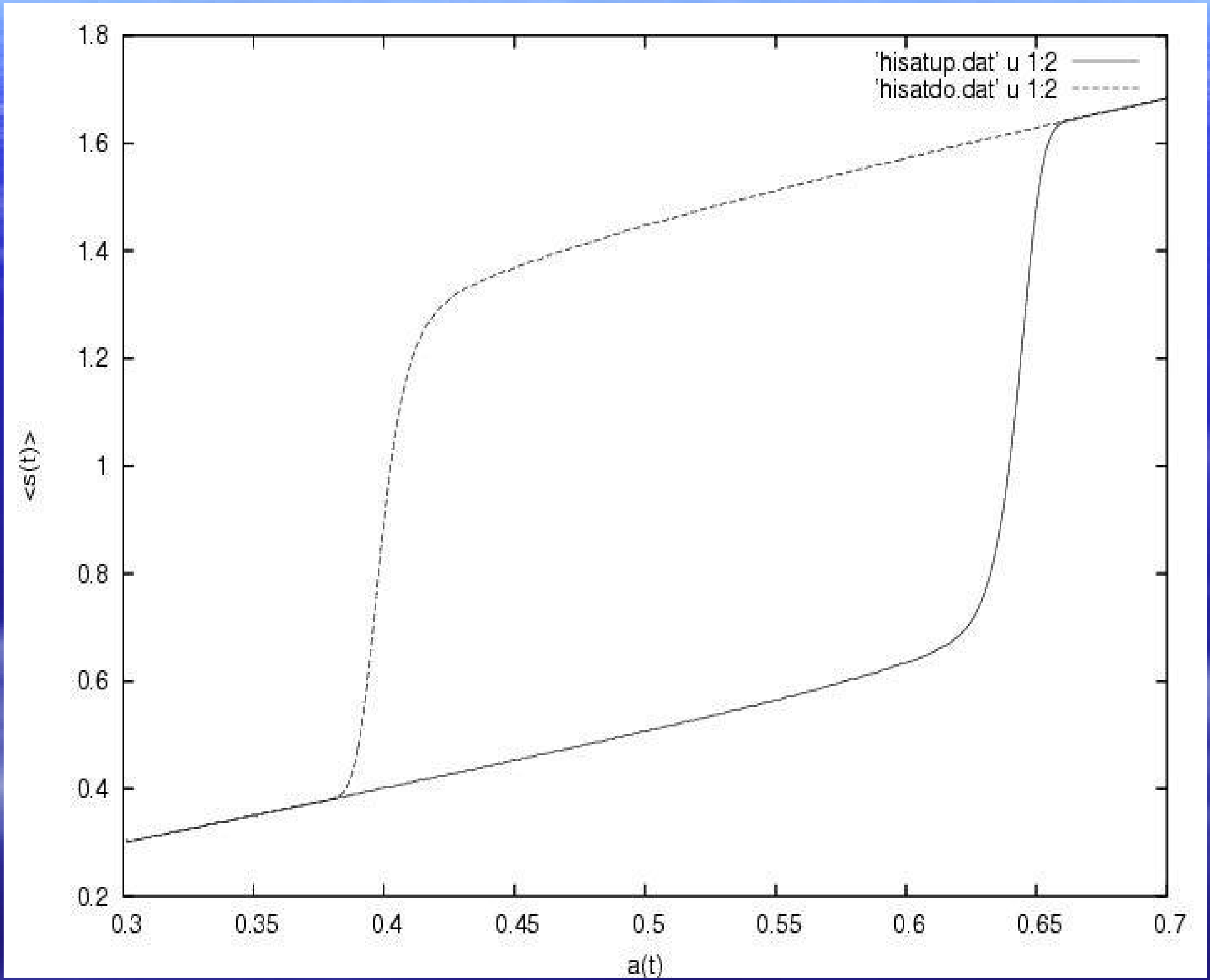
▪ Cluster area distribution



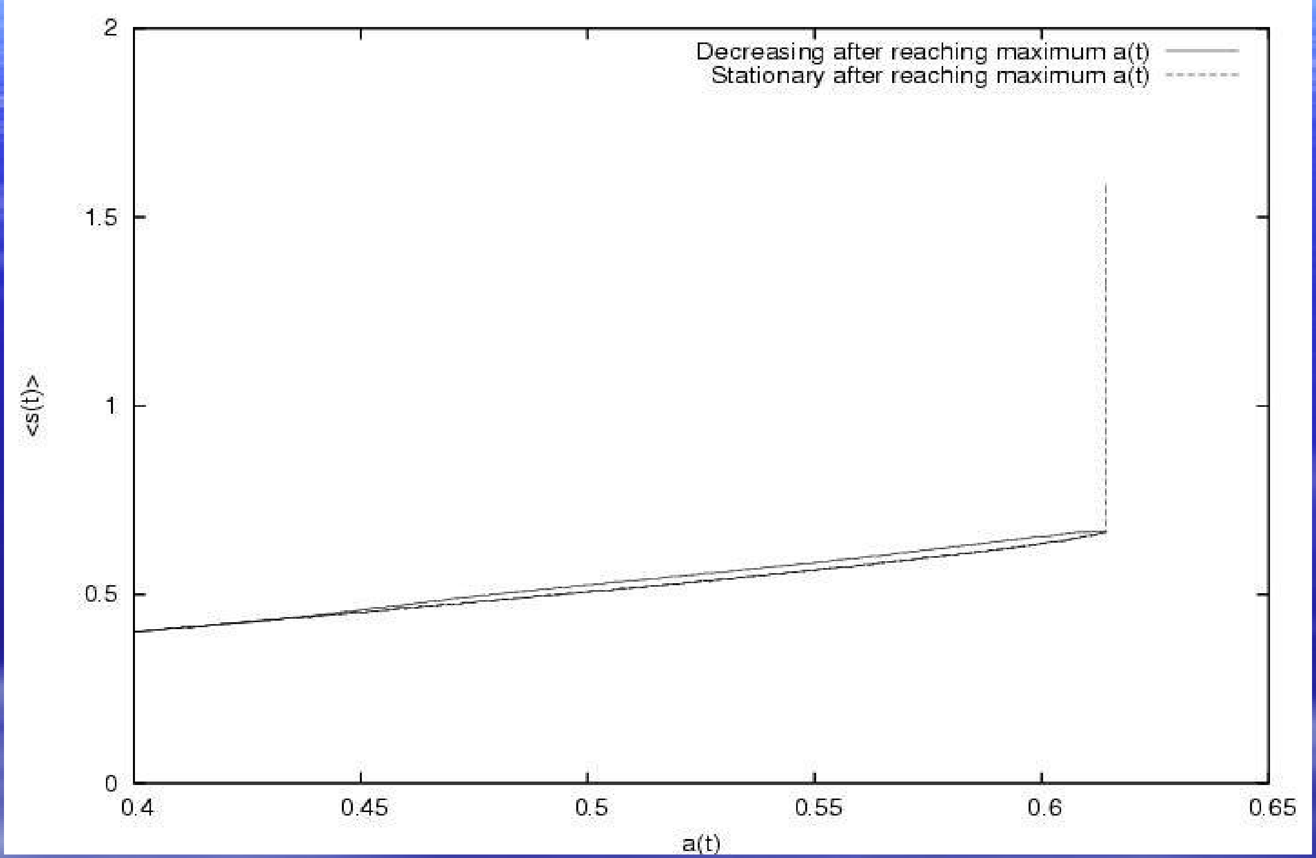
Histograms for t=249 i.e. $\bar{a}(t)$ = 0.624 (blue) and t=269 i.e. $\bar{a}(t)$ = 0.644.

- This cluster distribution follows a power law for $\bar{a}(t)$ = 0.644 (at the peak of both σ_s and σ_t).

Catastrophic transition to eutrophication.
Huge hysteresis.

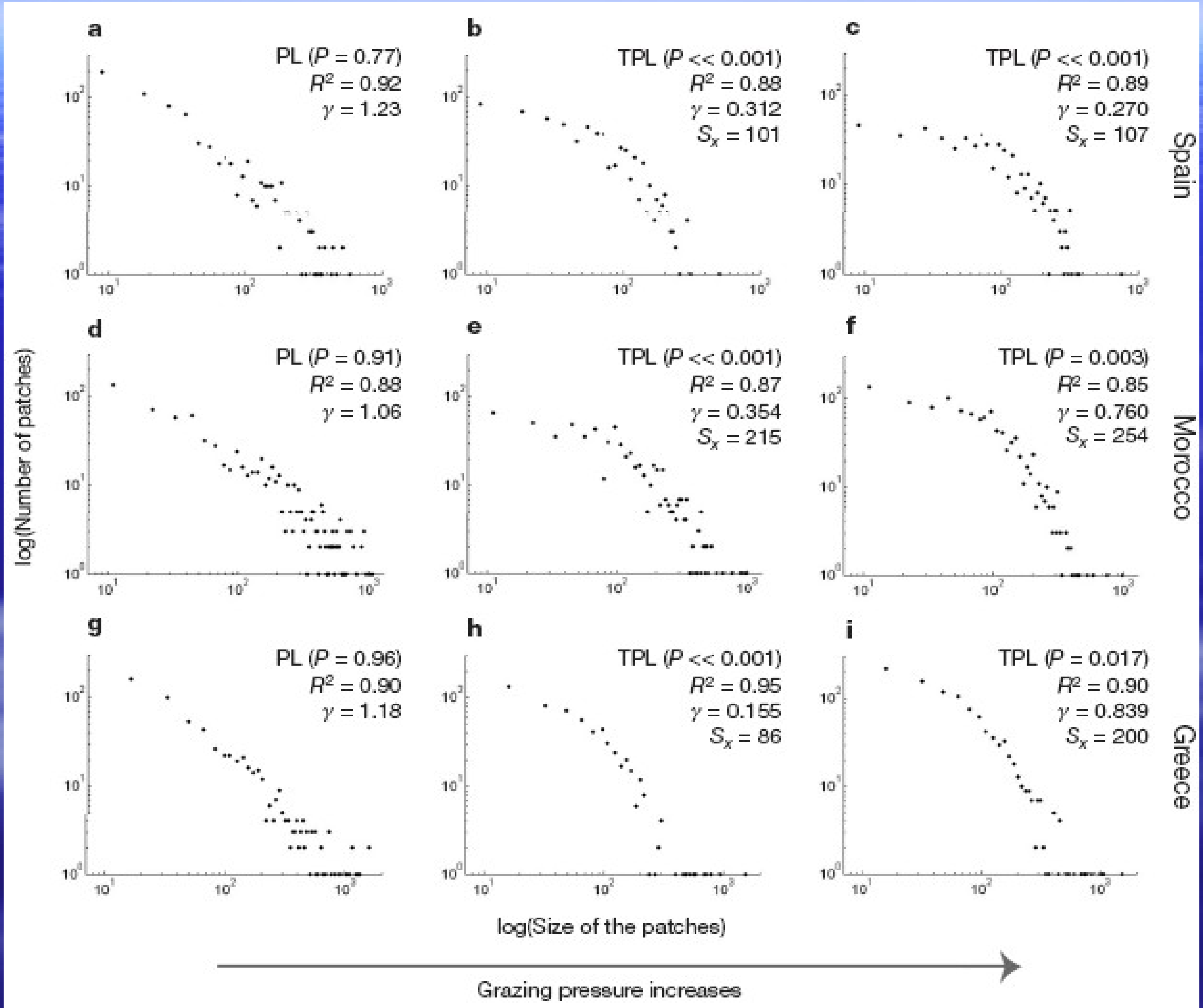


Remedial actions:
Stop P input (*a*(*t*)=const.)
Remove P (d*a*/d*t* < 0)

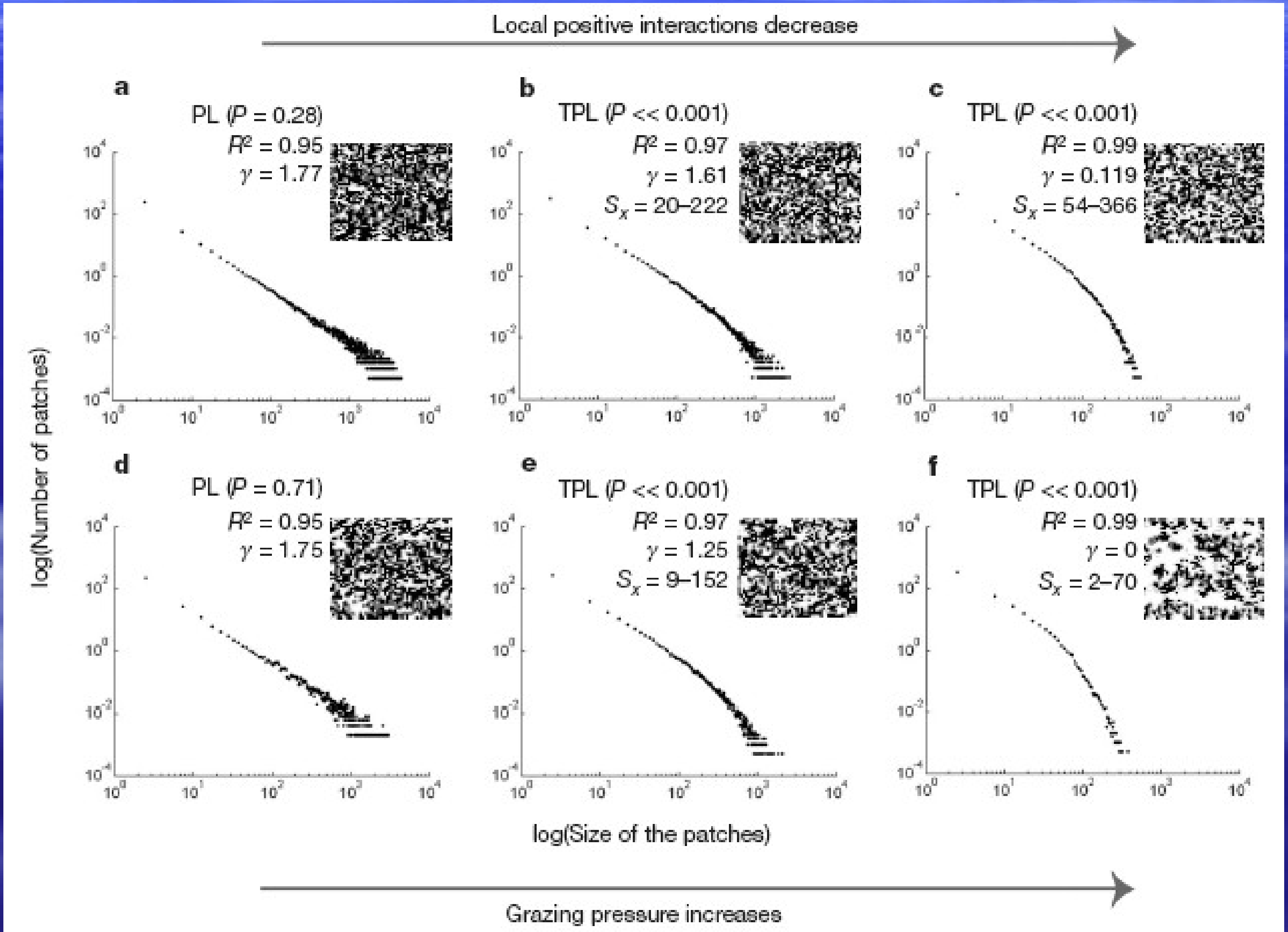


It is not trivial to avoid the catastrophic transition to eutrophication. Remedial actions must be taken as soon as possible.

Desertification



Model results



- Apperance of patches as early warnings (also found in desertification processes):

More detailed model, with P in soil (U), sediment (M), and water (P) dynamically coupled.
S. Carpenter, PNAS 2005

$$\frac{dU}{dt} = W + F - H - cU$$

$$\frac{dP}{dt} = cU - (s + h)P + rMf(P)$$

$$\frac{dM}{dt} = sP - bM - rMf(P)$$

Table 1. Model parameters, nominal values, and source

Symbol	Definition	Units	Nominal value
b	Permanent burial rate of sediment P	y^{-1}	0.001
c	P runoff coefficient	y^{-1}	0.00115
F	Annual agricultural import of P to the watershed per unit lake area	$gm^{-2}y^{-1}$	31.6
h	Outflow rate of P	y^{-1}	0.15
H	Annual export of P from the watershed in farm products, per unit lake area	$gm^{-2}y^{-1}$	18.6
m	P density in the lake when recycling is 0.5 r	gm^{-2}	2.4
r	Maximum recycling rate of P	$gm^{-2}y^{-1}$	0.019
q	Parameter for steepness of $f(P)$ near m	Unitless	8
s	Sedimentation rate of P	$gm^{-2}y^{-1}$	0.7
W	Nonagricultural inputs of P to the watershed prior to disturbance, per unit lake area	$gm^{-2}y^{-1}$	0.147
W_0	Nonagricultural inputs of P to the watershed after disturbance, per unit lake area	$gm^{-2}y^{-1}$	1.55

Areal units are based on the area of the lake, not the area of the watershed.

More detailed models: interactions between the different algal species and turbidity.

Conclusions

- Lake eutrophication amenable to a description in terms of nutrient concentration models.
- Early warnings needed to avoid catastrophic changes. Models can help devise those warnings
- Spatial signals seen before temporal signals.
- Formation of patches of not clear water might be the fastest signal.
- Desertification process has similar characteristics.
- Much room for model improvements and study of other ecosystems.