EPJ Web of Conferences **90**, 04002 (2015)
DOI: 10.1051/epjconf/20159004002

Possible Implication of a Single Nonextensive p_T Distribution for Hadron Production in High-Energy pp Collisions *

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Abstract. Multiparticle production processes in pp collisions at the central rapidity region are usually considered to be divided into independent "soft" and "hard" components. The first is described by exponential (thermal-like) transverse momentum spectra in the low-p_T region with a scale parameter T associated with the temperature of the hadronizing system. The second is governed by a power-like distributions of transverse momenta with power index n at high-p_T associated with the hard scattering between partons. We show that the hard-scattering integral can be approximated as a nonextensive distribution of a quasi-power-law containing a scale parameter T and a power index $n = 1/(q-1)$, where q is the nonextensivity parameter. We demonstrate that the whole region of transverse momenta presently measurable at LHC experiments at central rapidity (in which the observed cross sections varies by 14 orders of magnitude down to the low p_T region) can be adequately described by a single nonextensive distribution. These results suggest the dominance of the hard-scattering hadron-production process and the approximate validity of a "no-hair" statistical-mechanical description of the p_T spectra for the whole p_T region at central rapidity for pp collisions at high-energies.

1 Introduction

Particle production in pp collisions comprises of many different mechanisms in different parts of the phase space. We shall be interested in particle production in the central rapidity region where it is customary to divide the multiparticle production into independent soft and hard processes populating different parts of the transverse momentum space separated by a momentum scale p_0. As a rule of thumb, the spectra of the soft processes in the low-p_T region are (almost) exponential, $F(p_T)\sim\exp(-p_T/T)$, and are usually associated with the thermodynamical description of the hadronizing system, the fragmentation of a flux tube with a transverse dimension, or the production of particles by the Schwinger mechanism [1–5]. The p_T spectra of the hard process in the high-p_T region are regarded as essentially power-like, $F(p_T)\sim p_T^{-n}$, and are usually associated with the hard scattering process [6–10]. However, it was found already long time ago that both description could be replaced by simple interpolating formula [11],

$$F(p_T) = A\left(1 + \frac{p_T}{p_0}\right)^{-n}, \tag{1}$$

that becomes power-like for high p_T and exponential-like for low p_T. Notice that for high p_T, where we are usually neglecting the constant term, the scale parameter p_0 becomes irrelevant, whereas for low p_T it becomes, together with power index n, an effective temperature $T = p_0/n$. The same formula re-emerged later to become known as the *QCD-based Hagedorn formula* [12]. It was used for the first time in the analysis of UA1 experimental data [13] and it became one of the standard phenomenological formulas for p_T data analysis.

In the mean time it was realized that Eq. (1) is just another realization of the nonextensive distribution [14] with parameters q and T, and a normalization constant A,

$$F(p_T) = A\left[1 - (1-q)\,\frac{p_T}{T}\right]^{1/(1-q)}, \tag{2}$$

that has been widely used in many other branches of physics. For our purposes, both formulas are equivalent with the identification of $n = 1/(q-1)$ and $p_0 = nT$, and we shall use them interchangeably. Because Eq. (2) describes nonextensive systems in statistical mechanics, the parameter q is usually called the *nonextensivity parameter*. As one can see, Eq. (2) becomes the usual Boltzmann-Gibbs exponential distribution for $q \to 1$, with T becoming the temperature. Both Eqs. (1) and (2) have been widely used

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in the phenomenological analysis of multiparticle productions (cf., for example [15–28])[1].

We shall demonstrate here that, similar to the original ideas presented in [11, 12], the whole region of transverse momenta presently measurable at LHC experiments (which spans now enormous range of ∼14 orders of magnitude in the measured cross-sections down to the low-p_T region) [17–19] can be adequately described by a *single quasi-power law distribution*, either Eq. (1) or Eq. (2). We shall offer a possible explanation of this phenomenon by showing that the hard-scattering integral can be cast approximately into a non-extensive distribution form and that the description of a single nonextensive p_T distribution for the p_T spectra over the whole p_T region suggests the dominance of the hard-scattering process at central rapidity for high-energy pp collisions.

2 Questions associated with a Single Nonextensive distribution for p_T spectra in pp collisions

The possibility of two components in the transverse spectra implies that its complete description will need two independent functions with different sets of parameters, each dominating over different regions of the transverse momentum space. The presence of two different components will be indicated by gross deviations when the spectrum over the whole transverse space is analyzed with only a single component. An example for the presence of two (or more) components of production processes can be clearly seen in Fig. 1 of [31], in the p_T spectra in central (0-6%) PbPb collisions at $\sqrt{s_{NN}}$=2.76 TeV from the ALICE Collaboration, where two independent functions are needed to describe the whole spectra as described in [32, 33].

For our purposes in studying produced hadrons in pp collisions, where the high p_T hard-scattering component is expected to have a power-law form with a power index n, either (1) or (2) can be written as

$$E\frac{d\sigma}{d^3p} = \frac{A}{\left(1+\frac{m_T-m}{nT}\right)^n}, \qquad (3)$$

where n is the power index, T is the 'temperature' parameter, and m and m_T= $\sqrt{m^2+p_T^2}$ are the rest mass and transverse mass of the produced hadrons which are taken to be the dominant particles, the pions. It came as a surprise to us that for pp collisions at $\sqrt{s_{NN}}$ = 7 TeV, the p_T spectra within a very broad range, from 0.5 GeV up to 181 GeV, in which cross section varies by 14 orders of magnitude, can still be described well by a single nonextensive formula with power index n = 6.6 [34]. The good fits to the p_T spectra over such a large range of p_T with only three parameters, (A, n, T), raise intriguing questions :

- Why are there only three degrees of freedom in the spectra over such a large p_T domain? Does it imply that there is only a single component, the hard scattering process, contributing dominantly over the whole p_T domain? If so, are there supporting experimental evidences from other correlation measurements?
- Mathematically, the power index n is related to the parameter $q = 1 + 1/n$ in non-extensive statistical mechanics [14]. What is the physical meaning of n? If n is related to the power index of the parton-parton scattering law, then why is the observed value so large, n∼7, rather than $n \sim 4$ as predicted naively by pQCD?
- Are the power indices for jet production different from those for hadron production? If so, why ?
- Do multiple parton collisions play any role in modifying the power index n?
- In addition to the power law $1/p_T^n$, does the differential cross section contain other additional p_T-dependent factors? If they are present, how do they change the power index?

These questions were discussed and, at least partially, answered in [35]. Before proceeding to our main point of phenomenological considerations we shall first recapitulate briefly the main results of this attempt to reconcile, as far as possible, the nonextensive distribution with the QCD where, as shown in [35], the only relevant ingredients from QCD are hard scatterings between constituents resulting in the production of jets which further undergo fragmentation, showering, and hadronization to become the observed hadrons.

3 Approximate Hard-Scattering Integral

The answers to the questions posed above will be facilitated with an approximate analytical form of the hard-scattering integral. We start with the relativistic hard-scattering model as proposed in [6][2] and examined in [5, 10, 35]. We consider the collision of projectiles A and B in the center-of-mass frame at an energy $\sqrt{s}$ in the reaction $A + B \to c + X$, with c coming out at midrapidity, $\eta\sim 0$. Upon neglecting the intrinsic transverse momentum and rest masses, the differential cross section in the lowest-order parton-parton elastic collisions is given by

$$\frac{E_c d^3\sigma(AB \to cX)}{dc^3} = \sum_{ab}\int dx_a dx_b G_{a/A}(x_a)G_{b/B}(x_b) \times\frac{E_c d^3\sigma(ab \to cX')}{dc^3}. \qquad (4)$$

The parton-parton invariant cross section is related to $d\sigma(ab{\to}cX')/dt$ by

$$E_c\frac{d^3\sigma(ab \to cX')}{dc^3} = \frac{\hat{s}}{\pi}\frac{d\sigma(ab \to cX')}{dt}\delta(\hat{s}+\hat{t}+\hat{u}), \qquad (5)$$

where

$$\hat{s} = (a+b)^2,\ \ \hat{t} = (a-b)^2,\ \ \hat{u} = (b-c)^2. \qquad (6)$$

[1]For those who would like to use Eq. (2) in the context of nonextensive thermodynamics (as is done, for example, in [25, 26]) references in [29] provide arguments that this is fully legitimate. Outside the physics of multiparticle production, this approach is much better known and commonly used (see for example, [14, 30] for details and references).

[2]For the history of the power law, see [7].

In the infinite momentum frame the momenta can be written as

$$\begin{aligned} a &= \left(x_a\frac{\sqrt{s}}{2},\ \boldsymbol{O}_T,\ x_a\frac{\sqrt{s}}{2}\right), \\ b &= \left(x_b\frac{\sqrt{s}}{2},\ \boldsymbol{O}_T, -x_b\frac{\sqrt{s}}{2}\right), \\ c &= \left(x_c\frac{\sqrt{s}}{2}+\frac{c_T^2}{2x_c\sqrt{s}},\ \boldsymbol{c}_T,\ x_c\frac{\sqrt{s}}{2}-\frac{c_T^2}{2x_c\sqrt{s}}\right). \end{aligned}$$

We denote light-cone variable x_c of the produced parton c as $x_c = (c_0 + c_z)/\sqrt{s}$. The constraint of $\hat{s}+\hat{t}+\hat{u} = 0$ gives

$$x_a\,(x_b) = x_c + \frac{c_T^2}{\left(x_b - \frac{c_T^2}{x_c s}\right)s}. \tag{7}$$

We consider only the special case of c coming out at $\theta_c = 90^o$, in which $x_c = \frac{c_T}{\sqrt{s}}$, $x_a(x_b) = x_c + x_c^2/(x_b - x_c)$ and $x_a = x_b = 2x_c$. We have therefore

$$\left.\frac{E_c d^3\sigma(AB\to cX)}{dc^3}\right|_{y\sim 0} = \sum_{ab}\int dx_b dx_a G_{a/A}(x_a) G_{b/B}(x_b) \times \frac{x_a x_b \delta(x_a - x_a(x_b))}{\pi(x_b - c_T^2/x_c s)}\frac{d\sigma(ab\to cX')}{dt},$$

where $\mathcal{G}_a(x_a) = x_a G_{a/A}(x_a)$ and $\mathcal{G}_b(x_b) = x_a G_{b/B}(x_b)$. After integrating over x_a, we obtain

$$\left.\frac{E_C d^3\sigma(AB\to cX)}{dc^3}\right|_{y\sim 0} = \sum_{ab}\int dx_b \frac{\mathcal{G}_a(x_a(x_b))\mathcal{G}_b(x_b)}{\pi(x_b - c_T^2/x_c s)} \times \frac{d\sigma(ab\to cX')}{dt}. \tag{8}$$

To integrate over x_b, we use the saddle point method, write $\mathcal{G}_a(x_a(x_b))\mathcal{G}_b(x_b) = e^{f(x_b)}$, and expand $f(x_b)$ about its minimum at x_{b0}. We obtain then that

$$\int dx_b e^{f(x_b)} g(x_b) \sim e^{f(x_{b0})} g(x_{b0}) \sqrt{\frac{2\pi}{-\partial^2 f(x_b)/\partial x_b^2|_{x_b=x_{b0}}}}. \tag{9}$$

For simplicity, we assume $G_{a/A}$ and $G_{b/B}$ to have the same form. At $\theta_c \sim 90^0$ in the CM system, the minimum value of $f(x_b)$ is located at

$$x_{b0} = x_{a0} = 2x_c, \tag{10}$$

and we get the hard-scattering integral

$$E_C\left.\frac{d^3\sigma(AB\to cX)}{dc^3}\right|_{y\sim 0} \sim \sum_{ab} B[x_{a0}G_{a/A}(x_{a0})][x_{b0}G_{b/B}(x_{b0})] \times \frac{d\sigma(ab\to cX')}{dt} \tag{11}$$

where

$$B = \frac{1}{\pi(x_b - c_T^2/x_c s)}\sqrt{\frac{2\pi}{-\partial^2 f(x_b)/\partial x_b^2|_{x_b=x_{b0}}}}. \tag{12}$$

For the case of $\mathcal{G}_a(x_a) = x_a G_{a/A}(x_a) = A_a(1-x_a)^{g_a}$, we find

$$E_C\left.\frac{d^3\sigma(AB\to cX)}{dc^3}\right|_{y\sim 0} \sim \sum_{ab} A_a A_b \frac{(1-x_{a0})^{g_a+\frac{1}{2}}(1-x_{b0})^{g_b+\frac{1}{2}}}{\sqrt{\pi g_a}\sqrt{x_c(1-x_c)}} \times \frac{d\sigma(ab\to cX')}{dt}. \tag{13}$$

If the basic process $ab \to cX'$ is $gg \to gg$ or $ab \to cX'$ is $qq' \to qq'$, the cross sections at $\theta_c \sim 90^o$ [36] are

$$\begin{aligned} \frac{d\sigma(gg\to gg)}{dt} &\sim \frac{9\pi\alpha_s^2}{16c_T^4}\left[\frac{3}{2}\right]^3, \\ \frac{d\sigma(qq'\to qq')}{dt} &\sim \frac{4\pi\alpha_s^2}{9c_T^4}\frac{5}{16}. \end{aligned} \tag{14}$$

In both cases, the differential cross section behave as $d\sigma(ab\to cX')/dt \sim \alpha_s^2/(c_T^2)^2$.

4 Parton Multiple Scattering

As the collision energy increases, the value of x_c gets smaller and the number of partons and their density increase rapidly. Thus the total hard-scattering cross section increases as well [8]. The presence of a large number of partons in the colliding system results in multiple hard-scatterings of projectile parton on partons from target nucleon.

We find that for the process of $a \to c$ in the collision of a parton a with a target of A partons in sequence without a centrality selection, the $\boldsymbol{c}_T$-distribution is given by [35]

$$\begin{aligned} \frac{d\sigma_H^{(tot)}(a\to c)}{d\boldsymbol{c}_T} &= A\frac{\alpha_s^2}{c_T^4}\int d\boldsymbol{b}\,T(b) \\ &+\frac{A(A-1)}{2}\frac{16\pi\alpha_s^4}{c_T^6}\cdot\ln\{\frac{c_T}{2p_0}\}\int d\boldsymbol{b}[T(b)]^2\cdot \\ &+\frac{A(A-1)(A-2)}{6}\frac{936\pi^2\alpha_s^6}{c_T^8}[\ln\frac{c_T}{3p_0}]^2\int d\boldsymbol{b}[T(b)]^3, \end{aligned} \tag{15}$$

where the terms on the right-hand side correspond to collisions of the incident parton with one, two and three target partons, respectively. Here, the quantity A is the number of partons in the nucleon as a composite system and is the integral of the parton density over the parton momentum fraction. This result shows that without centrality selection in minimum-biased events, the differential cross section will be dominated by the contribution from a single parton-parton scattering that behaves as α_s^2/c_T^4 (cf. previous analysis on the multiple had-scattering process in [37–39]). Multiple scatterings with $N > 1$ scatterers contribute to terms of order α_s^{2N} $[\ln(C_T/Np_0)]^{N-1}/c_T^{2+2N}$ [35].

5 The Power Index in Jet Production

From the above results one gets the approximate analytical formula for hard-scattering invariant cross section $\sigma_{\rm inv}$, for $A + B \to c + X$ at midrapidity, $\eta \sim 0$, equal to

$$E_c\left.\frac{d^3\sigma(AB\to cX)}{dc^3}\right|_{y\sim 0} \propto \frac{\alpha_s^2(1-x_{a0}(c_T))^{g_a+\frac{1}{2}}(1-x_{b0}(c_T))^{g_b+\frac{1}{2}}}{c_T^4\sqrt{c_T/\sqrt{s}}\sqrt{1-x_c}}. \tag{16}$$

The power index n has here the value $4 + 1/2$. Its value can be extracted by plotting $(\ln\sigma_{\rm inv})$ as a function of $(\ln c_T)$ (then the slope in the linear section gives the value of n, and the variation of $(\ln\sigma_{\rm inv})$ at large $(\ln c_T)$ gives the value of g_a and g_b). One can also consider for this purpose

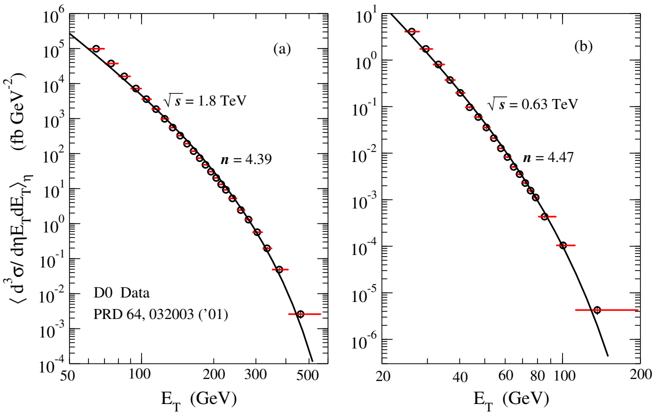

Figure 1. (Color online) Comparison of the relativistic hard-scattering model results for jet production, Eq. (19) (solid curves), with experimental $d\sigma/d\eta E_T dE_T$ data from the D0 Collaboration [45], for hadron jet production within $|\eta|$<0.5, in $\bar{p}p$ collision at (a) $\sqrt{s}$=1.80 TeV, and (b) $\sqrt{s}$=0.63 TeV.

a fixed x_c and look at two different energies (as suggested in [40]),

$$\frac{\ln[\sigma_{\rm inv}(\sqrt{s_1}, x_c)/\sigma_{\rm inv}(\sqrt{s_2}, x_c)]}{\ln[\sqrt{s_2}/\sqrt{s_1}]} \sim n(x_c) - \frac{1}{2}. \quad (17)$$

We follow an alternative method and analyze the p_T spectra using a running coupling constant,

$$\alpha_s(Q^2(c_T)) = \frac{12\pi}{27\ln(C + Q^2/\Lambda^2_{\rm QCD})}, \quad (18)$$

where we have chosen $\Lambda_{\rm QCD}$ to be 0.25 GeV to give $\alpha_s(M_Z^2)$ = 0.1184 [41]. We identify Q as c_T and have chosen C=10 both to give $\alpha_s(Q{\sim}\Lambda_{\rm QCD}) \sim 0.6$ in hadron spectroscopy studies [42] and to regularize the coupling constant for small values of $Q(c_T)$. We search for n by writing the invariant cross section Eq. (16) for jet production as

$$E_c \frac{d^3\sigma(AB{\to}cX)}{dc^3}\bigg|_{y\sim 0} \propto \frac{\alpha_s^2(Q^2(c_T))(1-x_{a0}(c_T))^{g_a+\frac{1}{2}}(1-x_{b0}(c_T))^{g_b+\frac{1}{2}}}{c_T^n\sqrt{1-x_c}}. \quad (19)$$

Table 1. The power index for jet production in $\bar{p}p$ and pp collisions

Collaboration	$\sqrt{s}$	R	η	n
D0 [45]	$\bar{p}p$ at 1.80 TeV	0.7	$\lvert\eta\rvert < 0.7$	4.39
D0 [45]	$\bar{p}p$ at 0.63 TeV	0.7	$\lvert\eta\rvert < 0.7$	4.47
ALICE [46]	pp at 2.76 TeV	0.2	$\lvert\eta\rvert < 0.5$	4.78
ALICE [46]	pp at 2.76 TeV	0.4	$\lvert\eta\rvert < 0.5$	4.98
CMS [47]	pp at 7 TeV	0.5	$\lvert\eta\rvert < 0.5$	5.39

In the literature [43, 44] the index g_a for the structure function of a gluon varies from 6 to 10. Following [43] we shall take g_a = 6. As shown in Fig. 1 and Table I, data from D0 [45] on $d\sigma/d\eta E_T dE_T$ for hadron jet production within $|\eta|$<0.5 can be fitted with n=4.39 for $\bar{p}p$ collisions at $\sqrt{s}$=1.8 TeV, and with n=4.47 for $\bar{p}p$ collisions at $\sqrt{s}$=0.630 TeV. In other comparisons with the ALICE data for jet production in pp collisions at $\sqrt{s}$ = 2.76 TeV at the LHC within $|\eta|$ < 0.5 [46], the power index is n=4.78 for R = 0.2, and is n=4.98 for R = 0.4 (Table I). The power index is n=5.39, for CMS jet differential cross section in pp collisions at $\sqrt{s}$ = 7 TeV at the LHC within $|\eta|$ < 0.5 and R = 0.5 [47]. This latter n value exceeds slightly the expected value of n = 4.5.

Except for the CMS data at 7 TeV that may need further re-examination, the power indices extracted for hadron jet production and listed in Table I are in approximate agreement with the value of n=4.5 in Eq. (16) and with previous analysis of Arleo *et al.* [40], indicating the approximate validity of the hard-scattering model for jet production in hadron-hadron collisions, with the predominant α_s^2/c_T^4 parton-parton differential cross section as predicted by pQCD.

6 Change of the Power Index n from Jet Production to Hadron Production

The results in the last section indicates that the simple hard-scattering model, i.e., Eq. (18), adequately describes the power index of $n \sim 4.5$ for jet production in high-energy pp collisions. However, the power index for hadron production is considerable greater, in the range of $n \sim$ 6 – 10 [34, 40]. What is the origin of the increase in the power index n?

A jet c evolves by fragmentation, showering, and hadronization to turn the jet into a large numbers of hadrons in a cone along the jet axis. The showering of the partons will go through many generations of branching. If we label the (average) momentum of the i-th generation parton by $p_T^{(i)}$, the showering can be represented as $c_T \to p_T^{(1)} \to p_T^{(2)} \to p_T^{(3)} \to ... \to p_T^{(\lambda)}$. Each branching will kinematically degrade the momentum of the showering parton by a momentum fraction, $\zeta = p_T^{(i+1)}/p_T^{(i)}$. At the end of the terminating λ-th generation of the showering, and hadronization, the p_T of a produced hadron is related to the c_T of the parent parton jet by

$$\frac{p_T}{c_T} \equiv \frac{p_T^{(\lambda)}}{c_T} = \zeta^\lambda. \quad (20)$$

It is easy to prove that if the generation number λ and the fragmentation fraction z are independent of the jet c_T, then the power law and the power index for the p_T distribution are unchanged [35].

We note however that in addition to the kinematic decrease of p_T as described by (20), the showering generation number λ is governed by an additional criterion on the virtuality, which measures the degree of the off-the-mass-shell property of the parton. From the different parton showering schemes in the PYTHIA [48], the HERWIG [49], and the ARIADNE [50], we can extract a general picture that the initial parton with a large initial virtuality Q decreases its virtuality by showering until a limit of Q_0 is reached. The downgrading of the virtuality will proceed as $Q=Q^{(0)} \to Q^{(1)} \to Q^{(2)} \to Q^{(3)} \to ... \to Q^{(\lambda)}=Q_0$. There is a one-to-one mapping of the initial virtuality Q with the transverse momentum c_T of the evolving parton as $Q(c_T)$ (or conversely $c_T(Q)$). Because of such a mapping, the decrease in virtuality Q corresponds to a decrease of the corresponding mapped $\tilde{c}_T$ as $c_T=\tilde{c}_T^{(0)} \to \tilde{c}_T^{(1)} \to \tilde{c}_T^{(2)} \to \tilde{c}_T^{(3)} \to ... \to \tilde{c}_T^{(\lambda)}=c_T(Q_0)$, where $\tilde{c}_T^{(i)}=c_T(Q^{(i)})$. The cut-off virtuality Q_0 maps into a transverse momentum $c_{T0}=c_T(Q_0)$. In each successive generation of the showering, the virtuality decreases by a virtuality fraction which corresponds, at least approximately, in terms of the corresponding mapped parton transverse momentum $\tilde{c}_T^{(i)}$, to a decrease by a corresponding transverse momentum fraction, $\tilde{\zeta}=\tilde{c}_T^{(i+1)}/\tilde{c}_T^{(i)}$. The showering will end in λ generations such that

$$\frac{c_{T0}}{c_T} \equiv \frac{\tilde{c}_T(Q^{(\lambda)})}{c_T} = \tilde{\zeta}^\lambda, \tag{21}$$

We can infer a relation between c_T and the number of generations, λ,

$$\lambda = \ln\left(\frac{c_{T0}}{c_T}\right)\Big/\ln\tilde{\zeta}. \tag{22}$$

Thus, the showering generation number λ depends on the magnitude of c_T. On the other hand, kinematically, the showering processes degrades the transverse momentum of the parton c_T to that of the p_T of the produced hadron as given by Eq. (20), depending on the number of generations λ. The magnitude of the transverse momentum p_T of the produced hadron is related to the transverse momentum c_T of the parent parton jet by

$$\frac{p_T}{c_T} = \zeta^\lambda = \zeta^{\ln\frac{c_{T0}}{c_T}/\ln\tilde{\zeta}}. \tag{23}$$

We can solve the above equation for p_T as a function of c_T and obtain

$$\frac{p_T}{c_{T0}} = \left(\frac{c_T}{c_{T0}}\right)^{1-\mu}, \text{ and } \frac{c_T}{c_{T0}} = \left(\frac{p_T}{c_{T0}}\right)^{1/(1-\mu)}, \tag{24}$$

where

$$\mu = \ln\zeta/\ln\tilde{\zeta}, \tag{25}$$

and μ is a parameter that can be searched to fit the data. As a result of the virtuality ordering and virtuality cut-off, the hadron fragment transverse momentum p_T is related to the parton momentum c_T nonlinearly by an exponent $1-\mu$.

After the fragmentation and showering of the parent parton c_T to the produced hadron p_T, the hard-scattering cross section for the scattering in terms of hadron momentum p_T becomes

$$\frac{d^3\sigma(AB\to pX)}{dyd\boldsymbol{p}_T} = \frac{d^3\sigma(AB\to cX)}{dyd\boldsymbol{c}_T}\frac{d\boldsymbol{c}_T}{d\boldsymbol{p}_T} \tag{26}$$

Upon substituting the non-linear relation (24) between the parent parton moment c_T and the produced hadron p_T in Eq. (24), we get

$$\frac{d\boldsymbol{c}_T}{d\boldsymbol{p}_T} = \frac{1}{1-\mu}\left(\frac{p_T}{c_{T0}}\right)^{\frac{2\mu}{1-\mu}}. \tag{27}$$

Therefore under the fragmentation from c to p, the hard-scattering cross section for $AB \to pX$ becomes

$$E_c\frac{d^3\sigma(AB\to pX)}{dp^3}\Big|_{y\sim 0} = \frac{d^3\sigma(AB\to pX)}{dyd\boldsymbol{p}_T}\Big|_{y\sim 0}$$
$$\propto \frac{\alpha_s^2(Q^2(c_T))(1-x_{a0}(c_T))^{g_a+\frac{1}{2}}(1-x_{b0}(c_T))^{g_b+\frac{1}{2}}}{p_T^{n'}\sqrt{1-x_c(c_T)}}, \tag{28}$$

where

$$n' = \frac{n-2\mu}{1-\mu}, \text{ with } n = 4+\frac{1}{2}. \tag{29}$$

Thus, the power index n for jet production can be significantly changed to n' for hadron production because the greater the value of the parent jet c_T, the greater the number of generations λ to reach the produced hadron, and the greater is the kinematic energy degradation. By a proper tuning of μ, the power index can be brought to agree with the observed power index in hadron production. For example, for μ=0.4 one gets n'=6.2 and for $\mu = 0.6$ one gets n'=8.2. Because the parton branching probability, parton kinematic degradation, and parton virtuality degradation depend on the coupling constant and the coupling constant depends on the parton energy, we expect the quantity μ to depend on the pp collision energy. Consequently, n' may change significantly with the collision energy.

7 Regularization of the Hard-Scattering Integral

The power-law (28) has been obtained for high p_T. In order to apply it to the whole range of E_T, we need to regularize it by the replacement,

$$\frac{1}{p_T} \to \frac{1}{1+m_T/m_{T0}}. \tag{30}$$

The quantity m_{T0} measures the average transverse mass of the detected hadron in the hard-scattering process. The differential cross section $d^3\sigma(AB\to pX)/dyd\boldsymbol{p}_T$ in (28) is then regularized as

$$\frac{d^3\sigma(AB\to pX)}{dyd\boldsymbol{p}_T}\Big|_{y\sim 0}$$
$$\propto \frac{\alpha_s^2(Q^2(c_T))(1-x_{a0}(c_T))^{g_a+1/2}(1-x_{b0}(c_T))^{g_b+1/2}}{[1+m_T/m_{T0}]^{n'}\sqrt{1-x_c(c_T)}}. \tag{31}$$

In the above equation for the production of a hadron with a transverse momentum p_T, the variable $c_T(p_T)$ refers to the transverse momentum of the parent jet c_T before fragmentation. We can relate p_T with c_T by using the empirical fragmentation function of Ref. [51] and we get [35]

$$c_T(p_T) \sim p_T\langle\frac{1}{z}\rangle = 2.33\ p_T. \tag{32}$$

This can be regarded as a linearized approximation of Eq. (24), which shows that c_T and p_T are non-linearly related, when we consider the virtuality in the fragmentation process. Comparisons of the theoretical results calculated with Eq. (31) with the experimental hadron transverse momentum distributions in pp collisions at the LHC from the

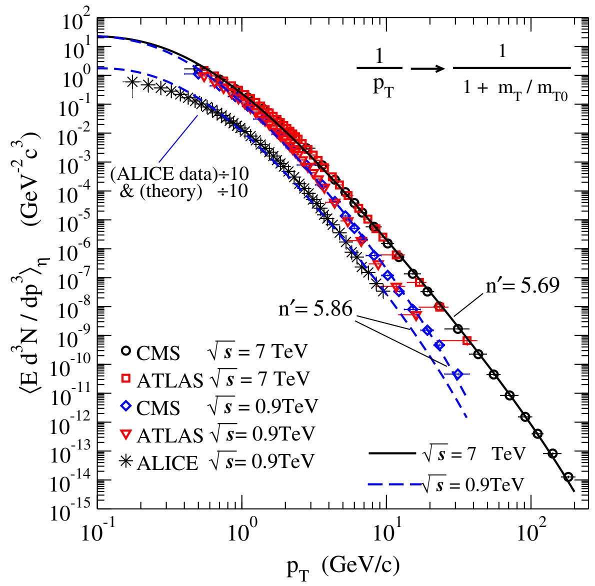

Figure 2. (Color online) Comparison of the experimental $\langle E_p d^3N/dp^3\rangle_\eta$ data for hadron production in pp collisions at the LHC with the relativistic hard-scattering model results of (solid and dashed curves) Eq. (31). The solid line is for $\sqrt{s}$=7 TeV, and the dashed line is for $\sqrt{s}$=0.9 TeV.

CMS [17], ATLAS [18], and ALICE Collaborations [19] are shown in Fig. 2. We find that the experimental data gives n'=5.69 and m_{T0}=0.804 GeV for $\sqrt{s}$=7 TeV and n'=5.86 and m_{T0}=0.634 GeV for $\sqrt{s}$=0.9 TeV. This indicates that there is indeed a systematic change of the power index n from jet production to a larger value n' in hadron production. The fits to the low p_T region for the ALICE data can be improved, with a larger power index n' as we shall see below in Section 9.

8 Further Approximation of the Hard-Scattering Integral

We would like to simplify further the p_T dependencies of the structure function in Eq. (31) and the running coupling constant as additional power indices in such a way that will facilitate subsequent phenomenological comparison. For parton c coming at mid-rapidity, the quantities x_{a0}, x_{b0}, and x_c in Eqs. (10) and (31) are

$$x_{a0} = x_{b0} = 2x_c, \quad \text{and} \quad x_c = \frac{c_T}{\sqrt{s}}. \tag{33}$$

The structure function factor and the denominator factor in Eq. (31) can be approximated for high energies with $\sqrt{s} \gg c_T$ as

$$\frac{(1-x_{a0}(c_T))^{g_a+1/2}(1-x_{b0}(c_T))^{g_b+1/2}}{\sqrt{1-x_c(c_T)}} \sim (1-x_c(c_T))^{2g_a+3/4}.$$

We can relate c_T with p_T by Eq. (32) and further approximate the right-hand side of the above equation in a form that is advantageous for subsequent purposes. For high energy with large $\sqrt{s}$, we make the approximation

$$(1-x_c(c_T))^{2g_a+\frac{3}{4}} = \left(1-\frac{2p_T}{\sqrt{s}}\langle\frac{1}{z}\rangle\right)^{2g_a+3/4} \sim \frac{1}{[1+m_T/m_{T0}]^{n_g}}, \tag{34}$$

where

$$n_g = \frac{2(2g_a+3/4)m_{T0}}{\sqrt{s}}\langle\frac{1}{z}\rangle. \tag{35}$$

We therefore estimate that $n_g \sim 0.04$ and 0.007 for $\sqrt{s}$ = 0.9 and 7 TeV respectively.

The running coupling constant α_s is a monotonically decreasing function of $Q(c_T)$. It can be written approximately as

$$\alpha_s(Q^2(c_T)) \propto \frac{1}{[1+m_T/m_{T0}]^{n_\alpha}}, \tag{36}$$

where n_α can be chosen to minimize errors by matching α_s at two points of p_T. If we match $\alpha_s(p_T)$ at p_T=$\Lambda_{\rm QCD}$=0.25 GeV and at p_T=100 GeV, then n_α = 0.36. If we match α_s at p_T=$\Lambda_{\rm QCD}$ and at p_T=20 GeV, then n_α = 0.46.

As a consequence of the above simplifying approximations, we can write the hard-scattering integral Eq. (31) in the approximate form

$$\left.\frac{d^3\sigma(AB\to pX)}{dyd\boldsymbol{p}_T}\right|_{y\sim 0} = F(p_T) \sim \frac{A}{[1+m_T/m_{T0}]^n}, \tag{37}$$

where

$$n = n' + n_g + n_\alpha, \tag{38}$$

and n' is the power index after taking into account the fragmentation process, n_g the power index from the structure function, and n_α from the coupling constant. We note that the predominant change of the power index from jet production to hadron production arises from the fragmentation process because n_g and n_α are relatively small.

In reaching the above equation, we have approximated the hard-scattering integral $F(p_T)$ that may not be exactly in the form of $1/[1+m_T/m_{T0}]^n$ into such a form. It is easy then to see that upon matching $F(p_T)$ with $A/[1+m_T/m_{T0}]^n$ according to some matching criteria, the hard-scattering integral $F(p_T)$ will be in excess of $1/[1+m_T/m_{T0}]^n$ in some region, and will be in deficit in some other region. As a consequence, the ratio of the hard-scattering integral $F(p_T)$ to the fitting $1/[1+m_T/m_{T0}]^n$ will oscillate as a function of p_T. This matching between the physical hard-scattering outcome that contains all physical effects with the approximation of Eq. (37) may be one of the origin of the oscillations of the experimental fit with the non-extensive distribution (as can be seen below in Fig. 3).

9 Nonextensive Distribution as a Lowest-Order Approximation of the Hard-scattering Integral

In the hard-scattering integral Eq. (37), if we identify

$$n \to \frac{1}{q-1} \quad \text{and} \quad m_{T0} \to \frac{T}{q-1} = nT, \tag{39}$$

and consider produced particles to be relativistic so that m_T~E_T~p_T and E_T~E at mid-rapidity, then we will get the nonextensive distribution of Eq. (2) as the lowest-order approximation for the QCD-based hard-scattering integral.

The convergence of Eq. (37) and Eq. (2) can be considered from the viewpoint of the reduction of a microscopic description to a statistical-mechanical description. From the microscopic perspective, the hadron production in a pp collision is a very complicated process, as evidenced by the complexity of the evolution dynamics in the evaluation of the p_T spectra in explicit Monte Carlo programs, for example, in [48–50]. If one starts from the initial condition of two colliding nucleons, there are many intermediate and complicated processes entering into the dynamics, each of which contain a large set of microscopic and stochastic degrees of freedom. Along the way, there are stochastic elements in the picking of the degree of inelasticity, in picking the colliding parton momenta from the parent nucleons, the scattering of the partons, the showering evolution of scattered partons, the hadronization of the fragmented partons. Some of these stochastic elements cannot be definitive and many different models, sometimes with untestable assumptions, have been put forth. In spite of all these complicated stochastic dynamics, the final result of Eq. (37) of the single-particle distribution can be approximated to depend only on three degrees of freedom, after all is done, put together, and integrated. The simplification can be considered as a "no hair" reduction from the microscopic description to nonextensive statistical mechanics in which all the complexities in the microscopic description "disappear" and are subsumed behind the stochastic processes and integrations. In line with statistical mechanics and in analogy with the Boltzmann-Gibbs distribution, we can cast the hard-scattering integral in the non-extensive form in the lowest-order approximation as [52][3]

$$\left.\frac{dN}{dyd\boldsymbol{p}_T}\right|_{y\sim 0} = \frac{1}{2\pi p_T}\left.\frac{dN}{dydp_T}\right|_{y\sim 0} = Ae_q^{-E/T}, \qquad (40)$$
$$e_q^{-E/T} \equiv \left[1-(1-q)\,E/T\right]^{1/(1-q)}, \;\; e_1^{-E/T} = e^{-E/T},$$

where $E=\sqrt{m^2+\boldsymbol{p}^2}$ and $E=E_T=m_T$ at y=0. Here, the parameter q is related physically to the power index n, the parameter T related to m_{T0} and the average transverse momentum, and the parameter A related to the multiplicity (per unity rapidity) after integration over p_T. Given a physically determined invariant cross section in the log-log plot of the cross section as a function of the transverse hadron energy as in Fig. 3, the slope at large p_T gives the power index n (and q), the average of E_T gives T (and m_{T0}), and the integral over p_T gives A.

Fig. 3 gives the comparisons of the results from Eq. (40) with the experimental p_T spectra at central rapidity obtained by different Collaborations [17–19]. In these calculations, the effective temperature parameter is set equal to T=0.13 GeV, and the parameters of A, q and the corresponding n are given in Table 2. The dashed line (an ordinary exponential of E_T for $q \to 1$) illustrates the large discrepancy if the distribution is described by Boltzmann-Gibbs distribution. The results in Fig. 3 shows that Eq. (40) adequately describes the hadron p_T spectra at central rapidity in high-energy pp collisions. We verify that q increases slightly with the beam energy, but, for the present energies, remains always $q \simeq 1.1$, corresponding to a power index n in the range of 6-8 that decreases as a function of $\sqrt{s}$.

[3]We are adopting the convention of setting both Boltzmann constant k_B and the speed of light c to be unity.

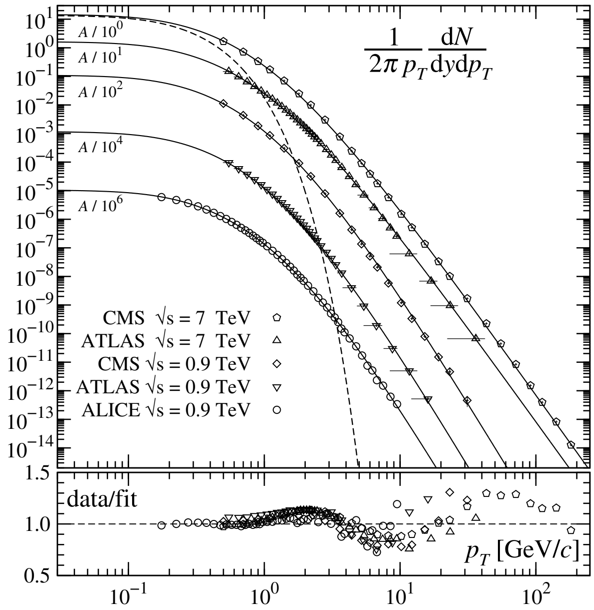

Figure 3. Comparison of Eq. (19) with the experimental transverse momentum distribution of hadrons in pp collisions at central rapidity y. Herein the temperature is set to be the same for all curves and equal T = 0.13 GeV, and the normalization constant in units of GeV$^{-2}/c^3$. The corresponding Boltzmann-Gibbs (purely exponential) fit is illustrated as the dashed curve. For a better visualization both the data and the analytical curves have been divided by a constant factor as indicated. The ratios data/fit are shown at the bottom, where a roughly log-periodic behavior is observed on top of the q-exponential one. Data are taken from [17–19].

Table 2. Parameters used to obtain fits presented in Fig. 3 where we have used T=0.13 GeV. The values of A is in units of GeV$^{-2}/c^3$.

Collaboration	$\sqrt{s}$ [TeV]	A	q	n=1/(q − 1)
CMS [17]	7	38	1.150	6.67
ATLAS [18]	7	43	1.151	6.62
CMS [17]	0.9	30	1.127	7.87
ATLAS [18]	0.9	32	1.124	8.06
ALICE [19]	0.9	27	1.124	8.06

What interestingly emerges from the analysis of the data in high-energy pp collisions is that the good agreement of the present phenomenological fit extends to the whole p_T region (or at least for p_T greater than 0.2 GeV/c, where reliable experimental data are available) [34]. This is being achieved with a single nonextensive distribution. On the other hand, theoretical analysis demonstrates that the hard-scattering integral can be written as a nonextensive distribution with only three degrees of freedom, in the lowest-order approximation. It is reasonable to infer that the dominant mechanism of hadron production over the whole range of p_T at central rapidity and high energies is the hard-scattering process.

The dominance of hard-scattering also for the production of low-p_T hadron in the central rapidity region is supported by two-particle correlation data where the two-particle correlations in minimum p_T-biased data reveals that a produced hadron is correlated with a "ridge" of particles along a wide range of $\Delta\eta$ on the azimuthally away side centering around $\Delta\phi \sim \pi$ [16, 53, 54]. The $\Delta\phi \sim \pi$ (back-to-back) correlation indicates that the correlated pair is related by a collision, and the $\Delta\eta$ correlation in the shape of a ridge indicates that the two particles are partons from the two nucleons and they carry fractions of the longitudinal momenta of their parents, leading to the ridge of $\Delta\eta$ at $\Delta\phi \sim \pi$.

10 Conclusions and Discussions

Particle production in high-energy pp collisions at central rapidity is a complex process that can be viewed from two different and complementary perspectives. On the one hand, there is the successful microscopic description involving perturbative QCD and nonperturbative hadronization at the parton level where one describes the detailed mechanisms of parton-parton hard scattering, parton structure function, parton fragmentation, parton showering, the running coupling constant and other QCD processes. On the other hand from the viewpoint of statistical mechanics, the single-particle distribution can be cast into a form that exhibit all the essential features of the process with only three degrees of freedom. The final result of the process can be summarized, in the lowest-order approximation, by a power index n which can be represented by a nonextensivity parameter q=$(n+1)/n$, the average transverse momentum m_{T0} which can be represented by an effective temperature T=m_{T0}/n, and an multiplicity constant A that is related to the multiplicity per unit rapidity when integrated over p_T. Such a reduction from microscopic description to a statistical mechanical description can be shown both from theoretical considerations by obtaining a simplified and approximate hard-scattering integral, and also by comparing with experimental data. In the process, we uncover the dominance of the hard-scattering hadron-production and the approximate validity of a "no-hair" statistical-mechanical description for the whole transverse momentum region in pp collision at high-energies. We emphasize also that, *in all cases*, the temperature turns out to be one and the same, namely $T = 0.13$ GeV.

What we may extract from the behavior of the experimental data is that scenario proposed in [11, 12] appears to be essentially correct excepting for the fact that we are not facing thermal equilibrium but a different type of stationary state, typical of violation of ergodicity (for a discussion of the kinetic and effective temperatures see [55, 56])).

As a concluding remark, we note that the data/fit plot in the bottom part of Fig. 3 exhibit an intriguing rough log-periodicity oscillations, which suggest corrections to the lowest-order approximation of Eq. (37) and some hierarchical fine-structure in the quark-gluon system where hadrons are generated. This behavior is possibly an indication of some kind of fractality in the system. Indeed, the concept of *self-similarity*, one of the landmarks of fractal structures, has been used by Hagedorn in his definition of fireball, as was previously pointed out in [21] and found in analysis of jets produced in pp collisions at LHC [57]. This small oscillations have already been preliminary discussed in Section 8 and in [58, 59], where the authors were able to mathematically accommodate these observed oscillations essentially allowing the index q in the very same Eq. (40) to be a complex number[4] (see also Refs. [60, 61]; more details on this phenomenon, including also discussion of its presence in recent AA data, can be found in [33]).

Acknowledgments: The research of CYW was supported in part by the Division of Nuclear Physics, U.S. Department of Energy, and the research of GW was supported in part by the National Science Center (NCN) under contract Nr 2013/08/M/ST2/00598 (Polish agency). Two of us (L.J.L.C. and C.T.) have benefited from partial financial support from CNPq, Faperj and Capes (Brazilian agencies). One of us (CT) acknowledges partial financial support from the John Templeton Foundation.

References

[1] B. Andersson, G. Gustafson, T. Sjöstrand, Z. Phys. C **20**, 317 (1983); B. Andersson, G. Gustafson, G. Ingelman, T. Sjöstrand, Phys. Rep. **97**, 31 (1983); T. Sjöstrand, M. Bengtsson, Comput. Phys. Commun. **43**, 367 (1987); B. Andersson, G. Gustavson, B. Nilsson-Alqvist, Nucl. Phys. B **281**, 289 (1).

[2] J. Schwinger, Phys. Rev. 82, 664 (1951).

[3] R. C. Wang and C. Y. Wong, Phys. Rev. **D 38**, 348 (1988).

[4] G. Gatoff and C. Y. Wong, Phys. Rev. **D 46**, 997 (1992); and C. Y. Wong and G. Gatoff, Phys. Rep. **242**, 1994, 489 (1994).

[5] C. Y. Wong, *Introduction to High-Energy Heavy-Ion Collisions*, World Scientific Publisher, 1994.

[6] R. Blankenbecler, S. J. Brodsky, Phys. Rev. D **10**, 2973 (1974); R. Blankenbecler, S. J. Brodsky, J. Gunion, Phys. Rev. D **12**, 3469 (1975); E. A. Schmidt, R. Blankenbecler, Phys. Rev. D **15**, 332 (1977); R. Blankenbecler, Lectures presented at Tübingen University, Germany, June 1977, SLAC-PUB-2077 (1977).

[7] J. Rak, M. J. Tannenbaum, *High-p_T Physics in the Heavy Ion* , Cambridge University Press, Cambridge, 2013.

[8] T. Sjöstrand, M. van Zijl, Phys. Rev. D **36**, 2019 (1987); R. Corke, T. Sjöstrand, JHEP **1001**, 035 (2010) and **1103**, 032 (2011) [arxiv : 1011.1759] ; T. Sjöstrand and P. Z. Skands, Eur. Phys. J. **C39**, 129 (2005), [arXiv :hepph /0408302] ; T. Sjöstrand

[4] It should be noted here that other alternative to complex q would be log-periodic fluctuating scale parameter T, such possibility was discussed in [59].

and P. Z. Skands, JHEP 03, 053 (2004), [arXiv :hep-ph/0402078] ; R. Corke and T. Sjöstrand, JHEP 1001, 035 (2010).

[9] X.N. Wang and M. Gyulassy, Phys. Rev. **D44**, 3501 (1991) ; X.N. Wang and M. Gyulassy, Phys. Rev. **D45**, 734 (1992).

[10] C. Y. Wong, H. Wang, Phys. Rev. C **58**, 376 (1998).

[11] C. Michael, L. Vanryckeghem, J. Phys. G **3**, L151 (1977); C. Michael, Prog. Part. Nucl. Phys. **2**, 1 (1979).

[12] R. Hagedorn, Riv. Nuovo Cimento **6**, 1 (1983.

[13] G. Arnison et al. (UA1 Collaboration), Phys. Lett. B **118**, 167 (1982).

[14] C. Tsallis, J. Stat. Phys. **52**, 479 (1988) and Eur. Phys. J. **A40**, 257 (2009); M. Gell-Mann and C. Tsallis eds., *Nonextensive Entropy – Interdisciplinary Applications* (Oxford University Press, New York, 2004). Cf. also C. Tsallis, *Introduction to Nonextensive Statistical Mechanics - Approaching A Complex World*, Springer, New York, 2009. A regularly updated bibliography on nonadditive entropies and nonextensive statistical mechanics is available at http://tsallis.cat.cbpf.br/biblio.htm.

[15] A. Adare *et al*, (PHENIX Collaboration), Phys. Rev. D **83**, 052004 (2011) and Phys. Rev. C **83**, 064903 (2011).

[16] J. Adams *et al*. (STAR Collaboratotion), Phys. Rev. D **74**, 032006 (2006).

[17] V. Khachatryan *et al*. (CMS Collaboration), JHEP **02**, 041 (2010) and Phys. Rev. Lett. **105**, 022002 (2010); V. Khachatryan *et al*. (CMS Collaboration), JHEP **08**, 086 (2011).

[18] G. Aad *et al*. (ATLAS Collaboration), New J. Phys. **13**, 053033 (2011).

[19] K. Aamodt *et al*. (ALICE Collaboration), Phys. Lett. **B693**, 53; Eur. Phys. J. C **71**, 1594 (2011), 1655 (2010).

[20] I. Bediaga, E. M. F. Curado, J. M. de Miranda, Physica A **286**, 156 (2000);

[21] C. Beck, Physica A **286**, 164 (2000).

[22] M. Rybczyński, Z. Włodarczyk, G. Wilk, Nucl. Phys. B (Proc. Suppl.) **97**, 81 (2001); F. S. Navarra, O. V. Utyuzh, G. Wilk, Z. Włodarczyk, Phys. Rev. D **67**, 114002 (2003); G. Wilk, Z. Włodarczyk, J. Phys. G **38** 065101 (2011), Eur. Phys. J. **A40**, 299 (2009) and Eur. Phys. J. A **48**, 161 (2012), Cent. Eur. J. Phys. **10**, 568 (2012); M. Rybczyński, Z. Włodarczyk, G. Wilk, J. Phys. G **39**, 095004 (2012); M. Rybczyński, Z. Włodarczyk, Eur. Phys. J. C **74**, 2785 (2014.

[23] T. Wibig, J. Phys. G **37**, 115009 (2010) and Eur. Phys. J. C **74**, 2966 (2014).

[24] K. Ürmösy, G. G. Barnaföldi, T. S. Biró, Phys. Lett. **B701**, 111 (2012), and **B718** 125 (2012); T.S. Biró, G.G. Barnaföldi, P. Ván, Eur. Phys. J. A **49**, 110 (2013) and Physica A **417**, 215 (2015).

[25] J. Cleymans, D. Worku, J. Phys. G **39**, 025006 (2012) and Eur. Phys. J. A **48**, 160 (2012); M. D. Azmi, J. Cleymans, J. Phys. G **41** 065001 (2014).

[26] A. Deppman, Physica A **391**, 6380 (2012) and J. Phys. G **41**, 055108 (2014); I. Sena, A. Deppman, Eur. Phys. J. A **49**, 17 (2013). A. Deppman, L. Marques, E. Andrade-II, A. Deppman, Phys. Rev. D **87**, 114022 (2013).

[27] P. K. Khandai, P. Sett, P. Shukla, V. Singh, Int. J. Mod. Phys. A **28**, 1350066 (2013) and J. Phys. G **41**, 025105; Bao-Chun Li.,Ya-Zhou Wang, Fu-Hu Liu, Phys. Lett. B **725**, 352 (2013).

[28] D. B. Walton, J. Rafelski, Phys. Rev. Lett. **84**, 31 (2000).

[29] O. J. E. Maroney, Phys. Rev E **80** (2009) 061141; T. S. Biró, K. Ürmössy and Z. Schram, J. Phys. G **37** (2010) 094027; T. S. Biró and P. Ván, PhysṘev. E **83** (2011) 061147; T. S. Biró and Z. Schram, Eur. Phys. J. Web Conf. **13**, (2011) 05004; T. S. Biró, *Is there a Temperature? Conceptual Challenges at High Energy, Acceleration and Complexity* (Springer, New York Dordrecht Heidelberg London, 2011); P. Ván, G. G. Barnaföldi, T. S. Biró and K. Ürmössy, J. Phys.: Conf. Ser. **394** (2012) 012002.

[30] C. Tsallis, Contemporary Physics **55** (3), 179 (2014).

[31] B. Abelev *et al*, (ALICE Collaboration), Phys. Lett. **B 720**, 52 (2013).

[32] K. Urmossy a, T. S. Biró a, G. G. Barnaföldi a and Z. Xu , Arxiv:1405.3963.

[33] Talk by M. Rybczyński in these proceedings, [Arxiv:1411.5148].

[34] C. Y. Wong, G. Wilk, Acta Phys. Pol. B **43**, 2047 (2012).

[35] C. Y. Wong, G. Wilk, Phys. Rev. D **87**, 114007 (2013) and *Relativistic Hard-Scattering and Tsallis Fits to p_T Spectra in pp Collisions at the LHC*, arXiv:1309.7330[hep-ph], to be published in The Open Nuclear & Particle Physics Journal.

[36] R. Gastman, T. T. Wu, *The Ubiquitous Photon*, Clarendon Press, Oxford, 1990.

[37] K. Kastella, Phy. Rev. D **36**, 2734 (1987).

[38] G. Calucci, D. Treleani, Phys. Rev. D **41**, 3367 (1990), **44**, 2746 (1990), D **49**, 138 (1994), D **50**, 4703 (1994), D **63**, 116002 (2001); Int. Jour. Mod. Phys. A **6**, 4375 (1991); A Accardi, D. Treleani, Phys. Rev. D **64**, 116004 (2001).

[39] M. Gyulassy, P. Levai, I. Vitev, Nucl. Phys. B **594**, 371 (2001).

[40] F. Arleo, S. Brodsky, D. S. Hwang, A. M. Sickles, Phys. Rev. Lett. **105**, 062002 (2010).

[41] J. Beringer *et al*., (Particle Data Group), Phys. Rev. D **86**, 010001 (2012).

[42] C. Y. Wong, E. S. Swanson, T. Barnes, Phy. Rev. C, **65**, 014903 (2001).

[43] D. W. Duke, J. F. Owens, Phy. Rev D **30**, 49 (1984).

[44] S. Chekanov *et al*., (ZEUS Collaboration), Phy. Rev. D **67**, 012007 (2003) and Eur. Phys. J. C **42**, 1 (2005).

[45] B. Abbott *et al*. (D0 Collaboration), Phys. Rev. D **64**, 032003 (2001).

[46] B. Abelev *et al*. (ALICE Collaboration), Phys. Lett. **B722**, 262 (2013).

[47] S. Chatrchyan *et al*. (CMS Collaboration), Phys. Rev. Lett. **107**, 132001 (2011).

[48] M. Bengtsson, T. Sjöstrand, Nucl. Phys. B **289**, 810 (1987); E. Norrbin, T. Sjöstrand, Nucl. Phys. B **603**, 297 (2001).

[49] G. Marchesini, B. R. Webber, Nucl. Phys. B **238**, 1 (1984); G. Corcella, I. G. Knowles, G. Marchesini, S. Moretti, K. Odagiri, P. Richardson, M. H. Seymour, B. R. Webber, .

[50] G. Gustafson, Phys. Lett. B **175**, 453 (1986); G. Gustafson, U. Pettersson, Nucl. Phys. B **306**, 746 (1988); L. Lönnblad, Computer Physics Commun. **71**, 15 (1992).

[51] J. Binnewies, B. A. Kniehl and G. Kramer, Z. Phys. **C65**, 471 (1995) .

[52] L. J. L. Cirto, C. Tsallis, C.-Y. Wong, G. Wilk, *The transverse-momenta distributions in high-energy pp collisions - A statistical-mechanical approach*, arXiv:1409.3278 [hep-ph].

[53] B. Abelev *et al*. (ALICE Collaboration), Phys. Rev. D **86**, 112007 (2012).

[54] R. L. Ray, Phys. Rev. D **84**, 034020 (2011); T. A. Trainor, D. J. Prindle, *Improved isolation of the p-p underlying event based on minimum-bias trigger-associated hadron correlations*, arXiv:1310.0408 [hep-ph].

[55] W. Niedenzu, T. Grießer, H. Ritsch, Europhys. Lett. **,96**, 43001 (2011); L. A. Gougam, M. Tribeche, Phys. Plasmas **18**, 062102 (2011); L. A. Rios, R. M. O. Galvão, L. Cirto, ibid. **19**, 034701 (2012); L. J. L. Cirto, V. R. V. Assis, C. Tsallis, Physica A **393**, 286 (2014); H. Christodoulidi, C. Tsallis, T. Bountis, Europhys. Lett. **108**, 40006 (2014).

[56] J. S. Andrade Jr., G. F. T. da Silva, A. A. Moreira, F. D. Nobre, E. M. F. Curado, Phys. Rev. Lett. **105**, 260601 (2010); M. S. Ribeiro, F. D. Nobre, E. M. F. Curado, Eur. Phys. J. B **85**, 399 (2012) and Phys. Rev. E **85**, 021146 (2012); E. M. F. Curado, A. M. C. Souza, F. D. Nobre, R. F. S. Andrade, Phys. Rev. E **89**, 022117 (2014).

[57] G. Wilk, Z. Włodarczyk, Phys. Lett. B **727**, 163 (2013).

[58] G. Wilk, Z. Włodarczyk, Physica A **413**, 53 (2014).

[59] G. Wilk, Z. Włodarczyk, *Log-periodic oscillations of transverse momentum distributions*, ArXiv:1403.3508 [hep-ph].

[60] C. Tsallis, L. R. da Silva, R. S. Mendes, R. O. Vallejos, A. M. Mariz, Phys. Rev. E **56**, R4922 (1997); L. R. da Silva, R. O. Vallejos, C. Tsallis, R. S. Mendes, S. Roux, Phys. Rev. E **64**, 011104 (2001).

[61] D. Sornette, Phys. Rep. **297**, 239 (1998).