EPJ Web of Conferences **13**, 05001 (2011)
DOI: 10.1051/epjconf/20111305001


# Nonextensive statistical mechanics: Applications to high energy physics

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**Abstract.** Nonextensive statistical mechanics was proposed in 1988 on the basis of the nonadditive entropy $S_q = k\,[1-\sum_i p_i^q]/(q-1)$ ($q \in \mathcal{R}$) which generalizes that of Boltzmann-Gibbs $S_{BG} = S_1 = -k\sum_i p_i \ln p_i$. This theory extends the applicability of standard statistical mechanics in order to also cover a wide class of anomalous systems which violate usual requirements such as ergodicity. Along the last two decades, a variety of applications have emerged in natural, artificial and social systems, including high energy phenomena. A brief review of the latter will be presented here, emphasizing some open issues.

## 1 Introduction

Standard statistical mechanics is based on the Boltzmann-Gibbs (BG) entropy $S_{BG} = -k\sum_{i=1}^{W} p_i \ln p_i$ ($\sum_{i=1}^{W} p_i = 1$), where $W$ is the number of microscopic configurations of the system. This extremely powerful theory — one of the pillars of contemporary physics — has exhibited very many successes along 140 years, in particular through its celebrated distribution for thermal equilibrium $p_i \propto e^{-\beta E_i}$, $E_i$ being the energy of the corresponding microstate. However, as any other human intellectual construct, it has a restricted domain of validity. For nonlinear dynamical many-body systems the usual requirement is *ergodicity*, which is guaranted by strong chaos (i.e., by a *positive* maximal Lyapunov exponent for classical systems). For nonergodic systems (typically for systems whose maximal Lyapunov exponent *vanishes*), which is quite frequently the case of the so-called complex systems, there is no general reason for legitimately using the BG theory. For (some of) such anomalous systems, a generalization of the BG theory has been proposed in 1988 [1]. It is frequently referred to as *nonextensive statistical mechanics* [2–4] because the total energy of such systems typically is *nonextensive*, i.e., *not proportional* to the total number of elements of the system. This generalized theory is based on the entropy

$$S_q = k\,\frac{1-\sum_i p_i^q}{q-1} \quad (q \in \mathcal{R};\ S_1 = S_{BG}) \tag{1}$$

It can be straightforwardly verified that, if $A$ and $B$ are two probabilistically independent systems (i.e., if $p_{ij}^{A+B} = p_i^A p_j^B$), then

$$\frac{S_q(A+B)}{k} = \frac{S_q(A)}{k} + \frac{S_q(B)}{k} + (1-q)\frac{S_q(A)}{k}\frac{S_q(B)}{k}\,, \tag{2}$$

which exhibits that, in contrast with $S_{BG}$ which is additive, the entropy $S_q$ is nonadditive for $q \neq 1$. This nonadditivity will in fact enable it to be *extensive* (i.e., proportional to the number of elements of the system) for various classes of systems (see for instance [5,6]).

## 2 Connection to Thermodynamics

To generalize BG statistical mechanics for the canonical ensemble (from [7]), we optimize $S_q$ with the constraints

$$\sum_{i=1}^{W} p_i = 1 \tag{3}$$

and

$$\sum_{i=1}^{W} P_i E_i = U_q\,, \tag{4}$$

where

$$P_i \equiv \frac{p_i^q}{\sum_{j=1}^{W} p_j^q} \quad \left(\sum_{i=1}^{W} P_i = 1\right) \tag{5}$$

is the so-called *escort distribution* [8]. It follows that $p_i = \frac{P_i^{1/q}}{\sum_{j=1}^{W} P_j^{1/q}}$. There are various converging reasons for being appropriate to impose the energy constraint with the $\{P_i\}$ instead of with the original $\{p_i\}$. The full discussion of this delicate point is beyond the present scope. However, some of these intertwined reasons are explored in [2]. By imposing Eq. (4), we follow [7], which in turn reformulates the results presented in [1,9]. The passage from one to the other of the various existing formulations of the above optimization problem are discussed in detail in [7,10].

The entropy optimization yields, for the stationary state,

$$p_i = \frac{e_q^{-\beta_q (E_i - U_q)}}{\bar{Z}_q}\,, \tag{6}$$

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with

$$\beta_q \equiv \frac{\beta}{\sum_{j=1}^{W} p_j^q} , \quad (7)$$

and

$$\bar{Z}_q \equiv \sum_i^W e_q^{-\beta_q (E_i - U_q)} , \quad (8)$$

$\beta$ being the Lagrange parameter associated with the constraint (4). Eq. (6) makes explicit that the probability distribution is, for fixed $\beta_q$, invariant with regard to the arbitrary choice of the zero of energies. The stationary state (or (meta)equilibrium) distribution (6) can be rewritten as follows:

$$p_i = \frac{e_q^{-\beta'_q E_i}}{Z'_q} , \quad (9)$$

with

$$Z'_q \equiv \sum_{j=1}^{W} e_q^{-\beta'_q E_j} , \quad (10)$$

and

$$\beta'_q \equiv \frac{\beta_q}{1 + (1-q)\beta_q U_q} . \quad (11)$$

The form (9) is particularly convenient for many applications where comparison with experimental or computational data is involved. Also, it makes clear that $p_i$ asymptotically decays like $1/E_i^{1/(q-1)}$ for $q > 1$, and has a cutoff for $q < 1$, instead of the exponential decay with $E_i$ for $q = 1$.

The connection to thermodynamics is established in what follows. It can be proved that

$$\frac{1}{T} = \frac{\partial S_q}{\partial U_q} , \quad (12)$$

with $T \equiv 1/(k\beta)$. Also we prove, for the free energy,

$$F_q \equiv U_q - T S_q = -\frac{1}{\beta} \ln_q Z_q , \quad (13)$$

where

$$\ln_q Z_q = \ln_q \bar{Z}_q - \beta U_q . \quad (14)$$

This relation takes into account the trivial fact that, in contrast with what is usually done in BG statistics, the energies $\{E_i\}$ are here referred to $U_q$ in (6). It can also be proved

$$U_q = -\frac{\partial}{\partial \beta} \ln_q Z_q , \quad (15)$$

as well as relations such as

$$C_q \equiv T \frac{\partial S_q}{\partial T} = \frac{\partial U_q}{\partial T} = -T \frac{\partial^2 F_q}{\partial T^2} . \quad (16)$$

In fact, the entire Legendre transformation structure of thermodynamics is $q$-invariant, which is both remarkable and welcome.

## 3 Applications

### 3.1 In diverse systems

The nonadditive entropy $S_q$ and its associated nonetensive statistical mechanics have been applied to a wide variety of natural, artificial and social systems. Among others we may mention (i) The velocity distribution of (cells of) *Hydra viridissima* follows a $q = 3/2$ probability distribution function (PDF) [11]; (ii) The velocity distribution of (cells of) *Dictyostelium discoideum* follows a $q = 5/3$ PDF in the vegetative state and a $q = 2$ PDF in the starved state [12]; (iii) The velocity distribution in defect turbulence [13]; (iv) The velocity distribution of cold atoms in a dissipative optical lattice [14]; (v) The velocity distribution during silo drainage [15,16]; (vi) The velocity distribution in a driven-dissipative 2D dusty plasma, with $q = 1.08 \pm 0.01$ and $q = 1.05 \pm 0.01$ at temperatures of $30000\,K$ and $61000\,K$ respectively [17]; (vii) The spatial (Monte Carlo) distributions of a trapped $^{136}Ba^+$ ion cooled by various classical buffer gases at $300\,K$ [18]; (viii) The distributions of price returns and stock volumes at the stock exchange, as well as the volatility smile [19–22]; (ix) Biological evolution [23]; (x) The distributions of returns in the Ehrenfest's dog-flea model [24,25]; (xi) The distributions of returns in the coherent noise model [26]; (xii) The distributions of returns of the avalanche sizes in the self-organized critical Olami-Feder-Christensen model, as well as in real earthquakes [27]; (xiii) The distributions of angles in the $HMF$ model [28]; (xiv) Turbulence in electron plasma [29]; (xv) The relaxation in various paradigmatic spin-glass substances through neutron spin echo experiments [30]; (xvi) Various properties directly related with the time dependence of the width of the ozone layer around the Earth [31]; (xvii) Various properties for conservative and dissipative nonlinear dynamical systems [32–41]; (xviii) The degree distribution of (asymptotically) scale-free networks [42,43]; (xix) Tissue radiation response [44]; (xx) Overdamped motion of interacting particles [45]; (xxi) Rotational population in molecular spectra in plasmas [46]. The systematic study of metastable or long-living states in long-range versions of magnetic models such as the Ising [47] and Heisenberg [48] ones, or in hydrogen-like atoms [49–51] might provide further illustrations.

### 3.2 In high energy physics

Connections of nonextensive statistics with a specific area of solar physics, astrophysics, high energy physics, and related areas, were pioneered by Quarati and collaborators (see [52], among others), who advanced the possibility of this theory being useful in the discussion of the flux of solar neutrinos. A few years later, it was realized that the transverse momenta distribution of the hadronic jets resulting from electron-positron annihilation are well described by distributions associated with $q$-exponentials [53,54]: see Figs. 1 and 2. The energy distribution of cosmic rays has been satisfactorily fitted in [55,56] with distributions related to $q$-exponentials: see Fig. 3. The distributions of returns of magnetic field fluctuations in the solar wind plasma as observed in data from Voyager 1 [57] and from Voyager 2 [58] has provided the values associated with the so called $q$-triplet: see Figs. 4 e 5. Similar results have been obtained in the study of interstellar turbulence [59] (see Figs. 6 and 7), in X-ray-emitting binary systems [60] (see Fig. 8), and in the distribution of stellar rotational velocities in the Pleiades [61].

It is important to address here the fact that the distribution of transverse momenta in high-energy collisions of proton-proton, and heavy nuclei (e.g., Pb-Pb and Au-Au) have received and are receiving great attention [62–68]: see illustrative examples in Figs. 9-15. Several such data have been summarized in [69]: see Fig. 16. We realize that for such collisions the typical values of $q$ are usually close to 1.10, apparently never above say 1.20-1.25. It remains as a challenging problem to precisely understand why (Is it a hadronization of quark matter in a sort of metastable state before attaining ergodicity?). In any case, it was shown in [71] that QCD calculations and $q$-statistical calculations can be consistent for $q \simeq 1.1$: see Fig. 17.

### Acknowledgments

I gratefully acknowledge wonderful hospitality received in Budapest at the occasion of the HCBM meeting, very especially from Prof. T.S. Biro. Naturally, the authors of the various figures that I have reproduced in the present brief review are acknowledged as well.

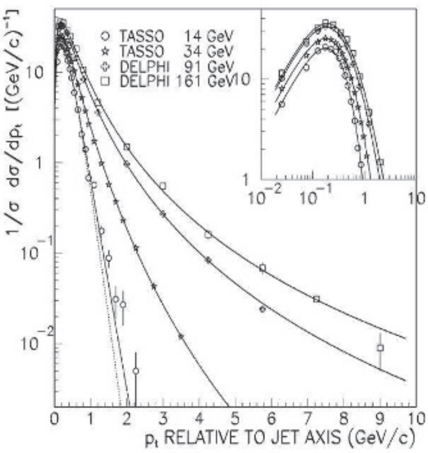


**Fig. 1.** Distributions of transverse momenta for four typical values of the collision energy. See details in [53].

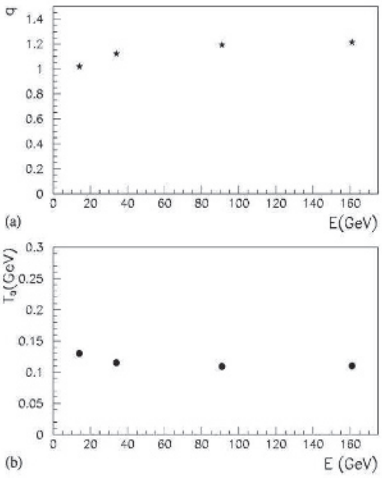


**Fig. 2.** Dependence of the index $q$ (a) and the temperature $T_0$ (b) on the collision energies of Fig. 3. The particular case $q = 1$ corresponds to the Hagedorn 1965 theory. It is advanced in [54] the possibility that $q$ approaches the value 11/9 in the $E \to \infty$ limit. See details in [53].

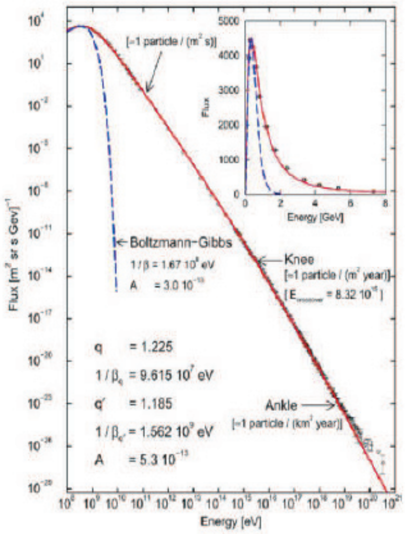


**Fig. 3.** Flux of cosmic rays. Curiously enough, the upper value of the index $q$ is very close to 11/9. See details in [55,56].

## References

1. C. Tsallis, *Possible generalization of Boltzmann-Gibbs statistics*, J. Stat. Phys. **52** (1988) 479-487.
2. C. Tsallis, *Introduction to Nonextensive Statistical Mechanics - Approaching a Complex World* (Springer, New York, 2009).
3. C. Tsallis, *Entropy*, in *Encyclopedia of Complexity and Systems Science*, ed. R.A. Meyers (Springer, Berlin, 2009), 11 volumes [ISBN: 978-0-387-75888-6].
4. A regularly updated bibliography can be seen at http://tsallis.cat.cbpf.br/biblio.htm
5. F. Caruso and C. Tsallis, *Nonadditive entropy reconciles the area law in quantum systems with classical*

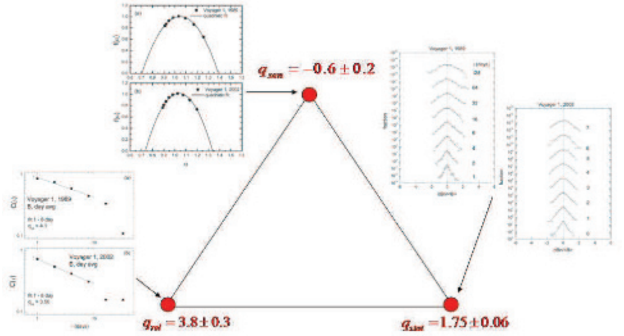


**Fig. 4.** The $q$-triplet as obtained from data of the Voyager 1. See details in [57].

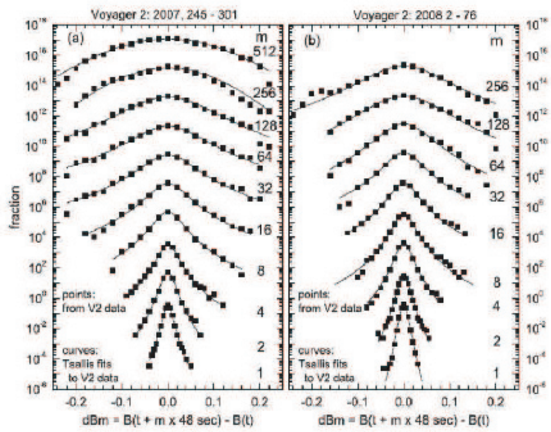


**Fig. 5.** Distributions from which $q_{stat}$ is extracted, from data of the Voyager 2. See details in [58].

*thermodynamics*, Phys. Rev. E **78** (2008) 021101.

6. A. Saguia and M.S. Sarandy, *Nonadditive entropy for random quantum spin-S chains*, Phys. Lett. A **374**, 3384-3388 (2010).
7. C. Tsallis, R.S. Mendes and A.R. Plastino, *The role of constraints within generalized nonextensive statistics*, Physica A **261** (1998) 534-554.
8. C. Beck and F. Schlogl, *Thermodynamics of Chaotic Systems* (Cambridge University Press, Cambridge, 1993).
9. E.M.F. Curado and C. Tsallis, *Generalized statistical mechanics: connection with thermodynamics*, J. Phys. A **24**, L69 (1991); Corrigenda: **24**, 3187 (1991) and **25**, 1019 (1992).
10. G.L. Ferri, S. Martinez and A. Plastino, *Equivalence of the four versions of Tsallis' statistics*, JSTAT- Journal of Statistical Mechanics: Theory and Experiment 1742-5468/05/P04009 (2005).
11. A. Upadhyaya, J.-P. Rieu, J.A. Glazier and Y. Sawada, *Anomalous diffusion and non-Gaussian velocity dis-*

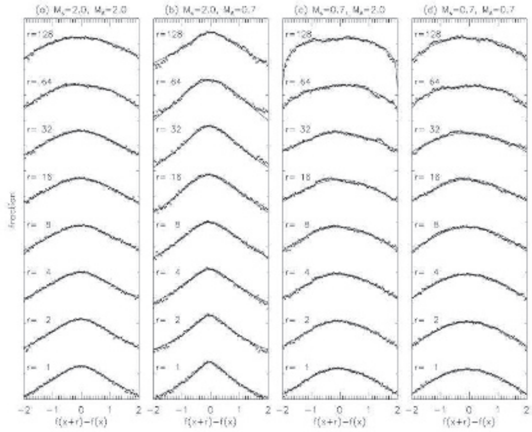


**Fig. 6.** Distributions of column density fluctuations for different spatial separations. See details in [59].

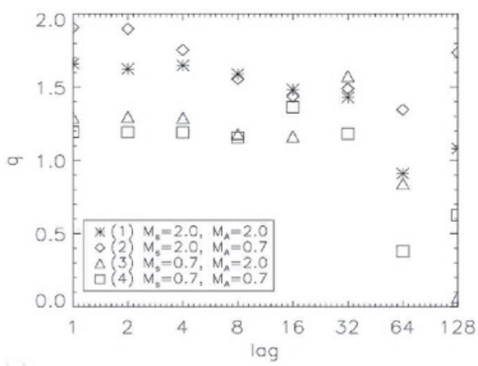


**Fig. 7.** Values of the index $q_{stat}$ corresponding to Fig. 6. See details in [59].

*tribution of Hydra cells in cellular aggregates*, Physica A **293** (2001) 549.

12. A.M. Reynolds, *Can spontaneous cell movements be modelled as Lévy walks?*, Physica A **389**, 273 (2010).
13. K.E. Daniels, C. Beck and E. Bodenschatz, *Defect turbulence and generalized statistical mechanics*, Physica D **193**, 208 (2004).
14. P. Douglas, S. Bergamini and F. Renzoni [2006], *Tunable Tsallis distributions in dissipative optical lattices*, Phys. Rev. Lett. **96**, 110601; G.B. Bagci and U. Tirnakli [2009], *Self-organization in dissipative optical lattices*, Chaos **19**, 033113.
15. R. Arevalo, A. Garcimartin and D. Maza, *Anomalous diffusion in silo drainage*, Eur. Phys. J. E **23**, 191-198 (2007).
16. R. Arevalo, A. Garcimartin and D. Maza, *A non-standard statistical approach to the silo discharge*, in *Complex Systems - New Trends and Expectations*, eds. H.S. Wio, M.A. Rodriguez and L. Pesquera, Eur. Phys.

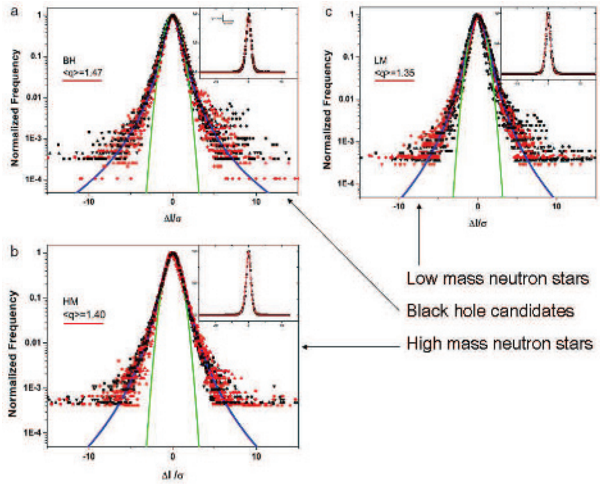


**Fig. 8.** Distributions of intensities of X-ray emission. See details in [60].

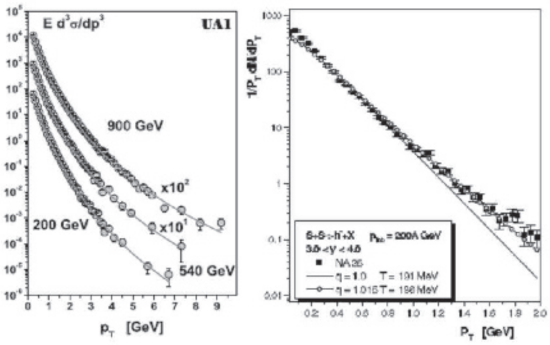


**Fig. 9.** Transverse momenta distributions for different energies. See details in [62].

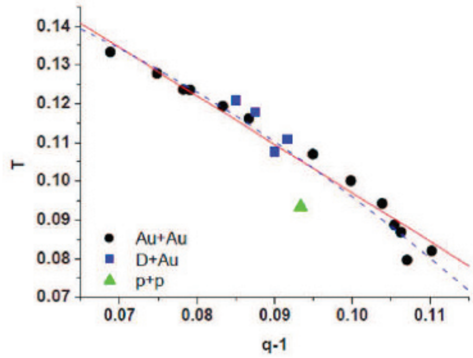


**Fig. 10.** Dependence of the temperature $T$ on the index $q$ for production of negative pions in different reactions. See details in [62].

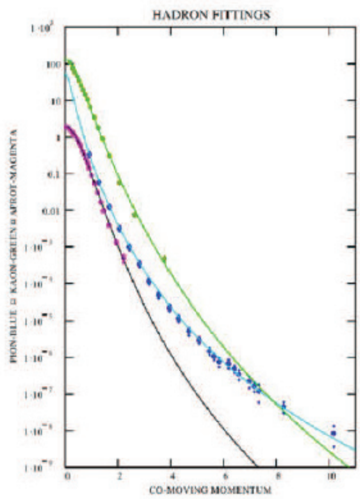


**Fig. 11.** Transverse momenta spectra for pions, kaons and antiprotons in relativistic heavy ions experiments. See details in [63].


J.-Special Topics **143** (2007).

17. B. Liu and J. Goree [2008], *Superdiffusion and non-Gaussian statistics in a driven-dissipative 2D dusty plasma*, Phys. Rev. Lett. **100**, 055003.
18. R.G. DeVoe, *Power-law distributions for a trapped ion interacting with a classical buffer gas*, Phys. Rev. Lett. **102**, 063001 (2009).
19. L. Borland, *Closed form option pricing formulas based on a non-Gaussian stock price model with statistical feedback*, Phys. Rev. Lett. **89**, 098701 (2002).
20. L. Borland, *A theory of non-gaussian option pricing*, Quantitative Finance **2**, 415 (2002).
21. R. Osorio, L. Borland and C. Tsallis, *Distributions of high-frequency stock-market observables*, in *Nonextensive Entropy - Interdisciplinary Applications*, eds. M. Gell-Mann and C. Tsallis (Oxford University Press, New York, 2004).
22. S.M.D. Queiros, *On non-Gaussianity and dependence in financial in time series: A nonextensive approach*, Quant. Finance **5**, 475 (2005).
23. F.A. Tamarit, S.A. Cannas and C. Tsallis, *Sensitivity to initial conditions in the Bak-Sneppen model of biological evolution*, Eur. Phys. J. B **1**, 545 (1998).
24. B. Bakar and U. Tirnakli, *Analysis of self-organized criticality in Ehrenfest's dog-flea model*, Phys. Rev. E **79**, 040103(R) (2009).
25. B. Bakar and U. Tirnakli, *Return distributions in dog-flea model revisited*, Physica A **389**, 3382 (2010).
26. A. Celikoglu, U. Tirnakli and S.M.D. Queiros, *Analysis of return distributions in the coherent noise model*, Phys. Rev. E **82**, 021124 (2010).
27. F. Caruso, A. Pluchino, V. Latora, S. Vinciguerra and A. Rapisarda, *Analysis of self-organized criticality in the Olami-Feder-Christensen model and in real earthquakes*, Phys. Rev. E **75**, 055101(R) (2007).

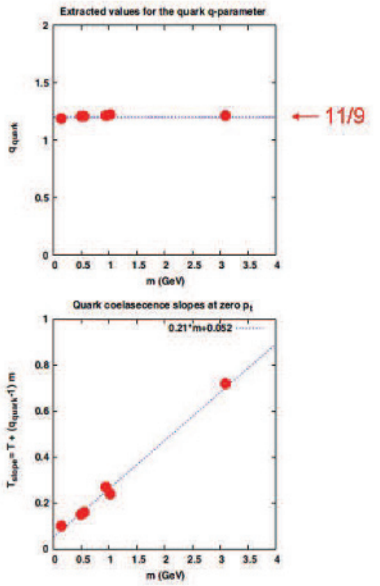


**Fig. 12.** The index $q$ (top) and the temperature (bottom) extracted from hadronic spectra assuming quark coalescence at a sudden hadron formation. See details in [63].

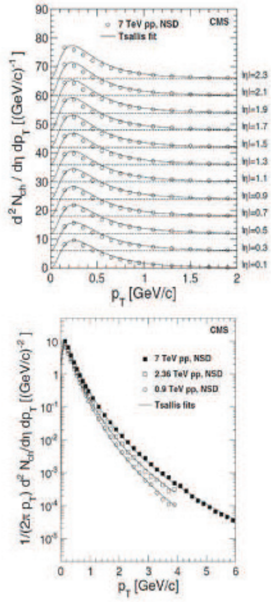


**Fig. 13.** Transverse momenta distributions of charged hadrons in pp collisions, as measured by the CMS Collaboration at LHC, corresponding to 0.9, 2.36 and 7 $TeV$. At these energies it has been obtained $(q, T)$ = $(1.13,\ 0.13\,GeV)$, $(1.15,\ 0.14\,GeV)$, $(1.15,\ o.145\,GeV)$ respectively. See details in [64, 65].

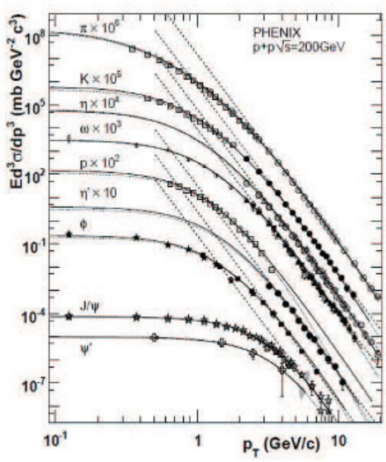


**Fig. 14.** Transverse momenta distributions of various hadrons in pp collisions, as measured by the PHENIX Collaboration, corresponding to 200 $GeV$. At this energy it has been obtained $q \simeq 1.10$. See details in [67].

28. L.G. Moyano and C. Anteneodo, *Diffusive anomalies in a long-range Hamiltonian system*, Phys. Rev. E **74**, 021118 (2006).
29. C. Anteneodo and C. Tsallis, *Two-dimensional turbulence in pure-electron plasma: A nonextensive thermostatistical description*, J. Molecular Liquids **71**, 255 (1997).
30. R.M. Pickup, R. Cywinski, C. Pappas, B. Farago and P. Fouquet [2009], *Generalized spin glass relaxation*, Phys. Rev. Lett. **102**, 097202.
31. G.L. Ferri, M.F. Reynoso Savio and A. Plastino, *Tsallis? q-triplet and the ozone layer*, Physica A **389**, 1829 (2010).
32. M.L. Lyra and C. Tsallis, *Nonextensivity and multifractality in low-dimensional dissipative systems*, Phys. Rev. Lett. **80**, 53 (1998).
33. U. Tirnakli, C. Tsallis and M.L. Lyra, *Circular-like maps: Sensitivity to the initial conditions, multifractality and nonextensivity*, Eur. Phys. J. B **11**, 309 (1999).
34. E.P. Borges, C. Tsallis, G.F.J. Ananos and P.M.C. Oliveira, *Nonequilibrium probabilistic dynamics at the logistic map edge of chaos*, Phys. Rev. Lett. **89**, 254103 (2002).
35. G.F.J. Ananos and C. Tsallis, *Ensemble averages and nonextensivity at the edge of chaos of one-dimensional maps*, Phys. Rev. Lett. **93**, 020601 (2004).
36. F. Baldovin and A. Robledo, *Nonextensive Pesin identity. Exact renormalizat- ion group analytical results for the dynamics at the edge of chaos of the logistic map*, Phys. Rev. E **69**, 045202(R) (2004).
37. E. Mayoral and A. Robledo, *Tsallis' q index and Mori's q phase transitions at edge of chaos*, Phys. Rev. E **72**, 026209 (2005).
38. A. Pluchino, A. Rapisarda and C. Tsallis, *Nonergodicity and central limit behavior in long-range Hamiltonians*, Europhys. Lett. **80**, 26002 (2007).

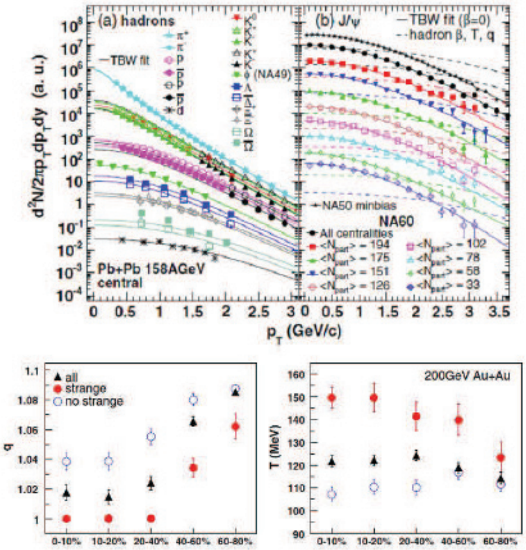


**Fig. 15.** Transverse momenta distributions of charged hadrons in pp and heavy ion collisions, as measured in Brookhaven. See details in [68].

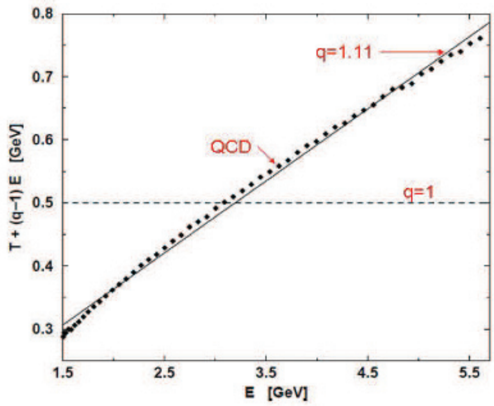


**Fig. 17.** Comparison of QCD diffusion calculation with its corresponding within $q$-statistics: they are consistent for $q = 1.11$. See details in [71].

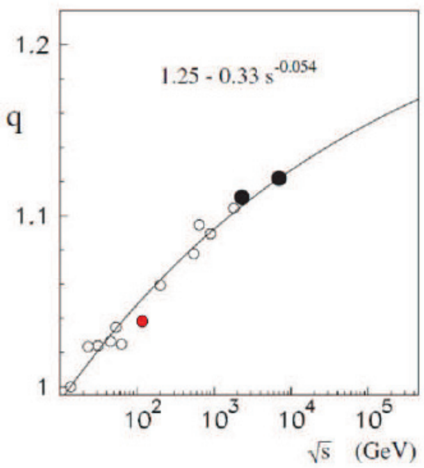


**Fig. 16.** Index $q$ obtained at various energies. See details in [69]. The black dots indicate recent CMS results. The red dot indicates the value obtained in [70].


39. A. Pluchino, A. Rapisarda and C. Tsallis, *A closer look at the indications of q-generalized Central Limit Theorem behavior in quasi-stationary states of the HMF model*, Physica A **387**, 3121 (2008).
40. G. Miritello, A. Pluchino and A. Rapisarda, *Central limit behavior in the Kuramoto model at the 'edge of chaos'*, Physica A **388**, 4818 (2009).
41. M. Leo, R.A. Leo and P. Tempesta, *Thermostatistics in the neighborhood of the π-mode solution for the Fermi-Pasta-Ulam β system: From weak to strong chaos*, J. Stat. Mech. P04021 (2010).
42. D.R. White, N. Kejzar, C. Tsallis,D. Farmer and S. White, *A generative model for feedback networks*, Phys. Rev. E **73**, 016119 (2006).
43. S. Thurner, F. Kyriakopoulos and C. Tsallis [2007], *Unified model for network dynamics exhibiting nonextensive statistics*, Phys. Rev. E **76**, 036111.
44. O. Sotolongo-Grau, D. Rodriguez-Perez, J.C. Antoranz and O. Sotolongo-Costa [2010], *Tissue radiation response with maximum Tsallis entropy*, Phys. Rev. Lett. **105**, 158105 (4 pages).
45. J. S. Andrade Jr., G.F.T. da Silva, A.A. Moreira, F.D. Nobre and E.M.F. Curado [2010], *Thermostatistics of overdamped motion of interacting particles*, Phys. Rev. Lett. **105**, 260601.
46. J.L. Reis Jr., J. Amorim and A. Dal Pino Jr., *Occupancy of rotational population in molecular spectra based on nonextensive statistics*, Phys. Rev. E **83**, 017401 (2011) (4 pages).
47. F.D. Nobre and C. Tsallis, *Infinite-range Ising ferromagnet: thermodynamic limit within generalized statistical mechanics*, Physica A **213**, 337 (1995); Erratum: **216**, 369 (1995).
48. A.O. Caride, C. Tsallis and S. I. Zanette, *Criticality of the anisotropic quantum Heisenberg model on a self-dual hierarchical lattice*, Phys. Rev. Lett. **51**, 145 (1983); **51**, 616 (1983).
49. L.S. Lucena, L.R. da Silva and C. Tsallis, *Departure from Boltzmann-Gibbs statistics makes the hydrogen-atom specific heat a computable quantity*, Phys. Rev. E **51**, 6247 (1995).
50. N.M. Oliveira-Neto, E.M.F. Curado, F.D. Nobre and M.A. Rego-Monteiro, *Approach to equilibrium of the hydrogen atom at low temperature*, Physica A **374**, 251-262 (2007).
51. N.M. Oliveira-Neto, E.M.F. Curado, F.D. Nobre and M.A. Rego-Monteiro, *A simple model to describe the low-temperature behaviour of some atoms and molecules: An application to the hydrogen atom*, J.

Phys. B - Atomic, Molecular and Optical Physics **40**, 1975-1989 (2007).
52. G. Kaniadakis, A. Lavagno and P. Quarati, *Generalized statistics and solar neutrinos*, Phys. Lett. B **369**, 308 (1996).
53. I. Bediaga, E.M.F. Curado and J. Miranda, *A nonextensive thermodynamical equilibrium approach in* $e^+e^- \to$ *hadrons*, Physica A **286**, 156 (2000).
54. C. Beck, *Non-extensive statistical mechanics and particle spectra in elementary interactions*, Physica A **286**, 164 (2000).
55. C. Tsallis, J.C. Anjos and E.P. Borges, *Fluxes of cosmic rays: A delicately balanced stationary state*, Phys. Lett. A **310**, 372 (2003).
56. C. Beck, *Generalized statistical mechanics of cosmic rays*, Physica A **331**, 173 (2003).
57. L.F. Burlaga and A.F.-Vinas, *Triangle for the entropic index q of non-extensive statistical mechanics observed by Voyager 1 in the distant heliosphere*, Physica A **356**, 375 (2005).
58. L.F. Burlaga and N.F. Ness, *Compressible "turbulence" observed in the heliosheath by Voyager 2*, Astrophys. J. **703**, 311 (2009).
59. A. Esquivel and A. Lazarian, *Tsallis statistics as a tool for studying interstellar turbulence*, Astrophys. J. **710**, 125-132 (2010).
60. M.A. Moret, V. de Senna, G.F. Zebende and P. Vaveliuk, *X-ray binary systems and nonextensivity*, Physica A **389**, 854-858 (2010).
61. J.C. Carvalho, R. Silva, J.D. do Nascimento and J.R. de Medeiros, *Power law statistics and stellar rotational velocities in the Pleiades*, Europhys. Lett. **84**, 59001 (2008).
62. G. Wilk and Z. Wlodarczyk, *Power laws in elementary and heavy-ion collisions - A story of fluctuations and nonextensivity?*, Eur. Phys. J. A **40**, 299 (2009).
63. T.S. Biro, G. Purcsel and K. Urmossy, *Non-extensive approach to quark matter*, in *Statistical Power-Law Tails in High Energy Phenomena*, Eur. Phys. J. A **40**, 325 (2009).
64. V. Khachatryan et al (CMS Collaboration), *Transverse-momentum and pseudorapidity distributions of charged hadrons in pp collisions at* $\sqrt{s} = 0.9$ *and* $2.36\ TeV$, J. High Energy Phys. **02**, 041 (2010).
65. V. Khachatryan et al (CMS Collaboration), *Transverse-momentum and pseudorapidity distributions of charged hadrons in pp collisions at* $\sqrt{s} = 7\ TeV$, Phys. Rev. Lett. **105**, 022002 (2010).
66. D. d'Enterria, R. Engel, T. Pierog, S. Ostapchenko and K. Werner, *Constraints from the first LHC data on hadronic event generators for ultra-high energy cosmic-ray physics*, 1101.5596 [astro-ph.HE] (2011).
67. Adare et al (PHENIX Collaboration), *Measurement of neutral mesons in* $p + p$ *collisions at* $\sqrt{s} = 200\ GeV$ *and scaling properties of hadron production*, 1005.3674 [hep-ex] (2010).
68. M. Shao, L. Yi, Z.B. Tang, H.F. Chen, C. Li and Z.B. Xu, *Examination of the species and beam energy dependence of particle spectra using Tsallis statistics*, J. Phys. G **37** (8), 085104 (2010).
69. T. Wibig, *The non-extensivity parameter of a thermodynamical model of hadronic interactions at LHC energies*, J. Phys. G: Nucl. Part. Phys. **37**, 115009 (2010) (4 pages).
70. W.M. Alberico, A. Lavagno and P. Quarati, *Non-extensive statistics, fluctuations and correlations in high energy nuclear collisions*, Eur. Phys. J C **12**, 499 (2000).
71. D.B. Walton and J. Rafelski, *Equilibrium distribution of heavy quarks in Fokker-Planck dynamics*, Phys. Rev. Lett. **84**, 31 (2000).