Self-modulation of linearly polarized electromagnetic waves in non-Maxwellian plasmas

L. A. Rios[a] and R. M. O. Galvão
Centro Brasileiro de Pesquisas Físicas and Instituto Nacional de Ciência e Tecnologia de Sistemas Complexos, Rua Xavier Sigaud 150, Rio de Janeiro 22290-180, Brazil

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The self-modulation of a linearly polarized electromagnetic wave propagating in a non-Maxwellian plasma is investigated. The plasma electrons (ions) obey a κ distribution function, which has been proved to be appropriate for modeling nonthermal distributions. The fluid model is used to describe the plasma dynamics, and a multiscale perturbation analysis is carried out to obtain the nonlinear Schrödinger equation governing the modulation of the high-frequency field. The effect of superthermal particles on the modulation of the wave and soliton formation is discussed.
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I. INTRODUCTION

Propagation and nonlinear processes associated with linearly polarized electromagnetic (EM) waves in plasmas have been extensively studied, with particular attention to topics such as particle acceleration and self-modulation in laboratory[1–4] and space and astrophysical plasmas.[5–9] The propagation of linearly polarized EM waves in plasmas is a far more complex problem than the propagation of circularly polarized EM waves, since it results in harmonic generation.[10] These harmonics consist of longitudinal oscillations generated by the ponderomotive force of the EM wave. The nonlinear coupling of the EM wave (high frequency) with the electrostatic perturbations causes the amplitude modulation of the high-frequency field, which can be described by a Schrödinger-type equation.[11] Harmonic generation is an important subject by itself, since it may have applications such as the generation of coherent radiation sources.[12,13]

In the present paper we analyze the self-modulation of a linearly polarized EM wave propagating in a non-Maxwellian plasma. A number of experiments indicate the presence of superthermal particles (electrons and ions) in laboratory[14] and space plasmas.[15–17] Particles with velocities exceeding the thermal velocity may arise due to external forces acting on the plasma or due to wave-particle interaction. The presence of superthermal particles results in a distribution function with a high-energy tail, which can be conveniently modeled via a nonthermal distribution function. The family of κ distributions, first discussed by Vasyliunas,[18] is recognized to be highly appropriate for modeling non-Maxwellian plasmas. It has been extensively used to analyze and interpret data on different plasma environments, such as the solar wind,[15,16,19] the Earth's magnetosphere[20–22] and the solar corona.[23,24] The κ distribution is equivalent to the distribution function obtained from the maximization of the Tsallis entropy, the q distribution.[25] The parameters κ and q measure the deviation from the Maxwellian equilibrium ("nonthermality") and are related by the expression $-\kappa=1/(1-q)$ (if the reduced form of the κ distribution is considered[26]). As we will see, the κ function is properly defined only for $\kappa>3/2$, with the Maxwellian distribution recovered for $\kappa\to\infty (q\to 1)$. As the κ function, Tsallis distribution (and statistics) has also been used in the study of many problems related to plasma physics, such as in the analysis of the magnetic field fluctuations in the solar wind[27] and in the experimental investigation of the anomalous diffusion in two-dimensional dusty plasmas.[28]

Although the κ function is commonly used to model non-Maxwellian plasmas, its origin is still not clear.[29] Leubner[30] has suggested the Tsallis nonextensive statistics[25] as the basis to understand the observed nonthermal features in space plasmas. Treumann *et al.*[31] considered the collisionless Vlasov equation and discussed the possibility of a nonthermal distribution arising from a particular collisionless equilibrium state (turbulent but stable states, far from thermal equilibrium). As mentioned earlier, and despite the lack of theoretical justification, the κ distribution has proved to be appropriate for modeling the nonthermal features of different plasma environments. Investigation on the effects of superthermal particles in space plasmas has motivated the development of a plasma dispersion function for κ distributions.[32,33] Recently, Hellberg and Mace[34] introduced the κ-Maxwellian distribution, an anisotropic distribution suitable for magnetized plasmas, where there is a preferred direction in space. This distribution simplifies the problem of finding the generalized plasma dispersion function for wave studies in magnetized plasmas, a hard task when using an isotropic κ function.[35]

Here we investigate the self-modulation of a linearly polarized EM wave propagating in a plasma with particles obeying a κ distribution function. In Sec. II we discuss the model and the basic equations: the fluid model is used to describe the dynamics of the electron-ion plasma and Maxwell's equations describe the behavior of the EM fields. We analyze two cases: the coupling of the EM wave with electron plasma oscillations and ion-acoustic waves. In Sec. III a multiscale perturbation analysis is carried out and the nonlinear Schrödinger (NLS) equation governing the modulation

[a]Electronic mail: rios@cbpf.br.

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of the carrier wave is derived. The effect of nonthermality (via parameter κ) on the modulation of the wave envelope is discussed in Sec. IV, where the influence of the superthermal electrons and ions on the nonlinear frequency shift and soliton formation is investigated.

II. MODEL AND BASIC EQUATIONS

The family of isotropic (three-dimensional) κ distributions has the form

$$f_\kappa(v) = \frac{1}{(\pi\kappa\theta^2)^{3/2}}\frac{\Gamma(\kappa+1)}{\Gamma(\kappa-1/2)}\left(1+\frac{v^2}{\kappa\theta^2}\right)^{-(\kappa+1)}, \quad (1)$$

where v is the velocity, $\Gamma(x)$ is the gamma function, and $\theta=[(\kappa-3/2)/\kappa]^{1/2}v_T$ is a generalized thermal speed ($v_T=\sqrt{2k_BT/m}$). Note that these distributions are properly defined only for $\kappa>3/2$. The correct form of the κ distribution function has been the subject of recent discussion.[36–38] The expansion of the κ function in the limit $\kappa\to\infty$ reveals its similarity with the Maxwellian distribution.[36]

We consider a linearly polarized EM wave propagating in an unmagnetized electron-ion plasma. The EM wave propagates along the z direction with $\mathbf{E}=(E_x,0,0)$ and $\mathbf{B}=(0,B_y,0)$. As it propagates, electron density perturbations are generated due to the wave ponderomotive force. The electrons are then subject to a total potential $\varphi=\varphi_{sc}+\varphi_p$, where φ_{sc} is the electrostatic potential (produced due to charge separation) and φ_p is the ponderomotive potential. Using the energy conservation relation, we can write the κ distribution function for the electrons in the following form:

$$f_\kappa(v_e) = \frac{N_0}{(\pi\kappa_e\theta_e^2)^{3/2}}\frac{\Gamma(\kappa_e+1)}{\Gamma(\kappa_e-1/2)}\left(1+\frac{v_e^2-2e\varphi/m_e}{\kappa_e\theta_e^2}\right)^{-(\kappa_e+1)}, \quad (2)$$

where N_0 is the electron (ion) equilibrium number density, v_e is the electron velocity, k_B is the Boltzmann constant, e is the fundamental charge, and T_e and m_e are the electron temperature and rest mass, respectively. Integrating the κ distribution over velocity space, one obtains the electron number density

$$N_e(\phi) = N_0\left[1-\frac{\phi}{(\kappa_e-3/2)}\right]^{-(\kappa_e-1/2)}, \quad (3)$$

where $\phi=e\varphi/k_BT_e$ is the normalized total potential. The pressure is given by $P_e=m_e/3\int v_e^2 f_\kappa(v_e)d^3v_e$

$$P_e(\phi) = P_0\left[1-\frac{\phi}{(\kappa_e-3/2)}\right]^{-(\kappa_e-3/2)}, \quad (4)$$

where $P_0=N_0k_BT_e$ is the pressure in the equilibrium state. Assuming that the ions also have a κ velocity distribution function

$$f_\kappa(v_i) = \frac{N_0}{(\pi\kappa_i\theta_i^2)^{3/2}}\frac{\Gamma(\kappa_i+1)}{\Gamma(\kappa_i-1/2)}\left(1+\frac{v_i^2+2e\varphi_{sc}/m_i}{\kappa_i\theta_i^2}\right)^{-(\kappa_i+1)} \quad (5)$$

we have similar expressions for the ion number density and pressure

$$N_i(\phi_{sc}) = N_0\left[1+\frac{\phi_{sc}}{(\kappa_i-3/2)}\right]^{-(\kappa_i-1/2)}, \quad (6)$$

$$P_i(\phi_{sc}) = P_0\gamma_{ie}\left[1+\frac{\phi_{sc}}{\gamma_{ie}(\kappa_i-3/2)}\right]^{-(\kappa_i-3/2)}, \quad (7)$$

where $\gamma_{ie}=T_i/T_e$ and $\phi_{sc}=e\varphi_{sc}/k_BT_e$. Notice that the ponderomotive potential is not included in expression (5), since the ions are too heavy and do not experience the ponderomotive force. Assuming that $e\varphi$ and $e\varphi_{sc}$ are small compared to k_BT_e (ϕ and $\phi_{sc}\ll 1$), we can expand Eqs. (4)–(7) around $\phi(\phi_{sc})=0$ and obtain

$$N_e = N_0(1+\alpha_0\phi+\alpha_1\phi^2+\alpha_2\phi^3\ldots), \quad (8)$$

$$P_e = P_0(1+\phi+\beta_0\phi^2+\beta_1\phi^3\ldots), \quad (9)$$

$$N_i = N_0\left[1-\eta_0\frac{\phi_{sc}}{\gamma_{ie}}+\eta_1\left(\frac{\phi_{sc}}{\gamma_{ie}}\right)^2-\eta_2\left(\frac{\phi_{sc}}{\gamma_{ie}}\right)^3\ldots\right], \quad (10)$$

and

$$P_i = P_0\gamma_{ie}\left[1-\frac{\phi_{sc}}{\gamma_{ie}}+\mu_0\left(\frac{\phi_{sc}}{\gamma_{ie}}\right)^2-\mu_1\left(\frac{\phi_{sc}}{\gamma_{ie}}\right)^3\ldots\right], \quad (11)$$

where $\alpha_0,\alpha_1,\ldots,\beta_0,\beta_1,\ldots,\eta_0,\eta_1,\ldots,\mu_0,\mu_1,\ldots$ are constants depending on κ_e and κ_i. Since we are considering the weak nonlinear regime, only the first nonlinear terms in Eqs. (8)–(11) are kept, and we define

$$\begin{aligned}
\alpha_0 &= (\kappa_e-1/2)/(\kappa_e-3/2),\\
\alpha_1 &= [(\kappa_e-1/2)(\kappa_e+1/2)]/[2(\kappa_e-3/2)^2],\\
\beta_0 &= (\kappa_e-1/2)/[2(\kappa_e-3/2)],\\
\eta_0 &= (\kappa_i-1/2)/(\kappa_i-3/2),\\
\eta_1 &= [(\kappa_i-1/2)(\kappa_i+1/2)]/[2(\kappa_i-3/2)^2],\\
\mu_0 &= (\kappa_i-1/2)/[2(\kappa_i-3/2)].
\end{aligned} \quad (12)$$

Considering the limit $\kappa_{e,i}\to\infty$ (Maxwellian case), we observe that the constants in (12) tend to 1 (α_0 and η_0) and 1/2 (α_1,β_0,η_1 and μ_0). From Eqs. (8)–(11) we notice that, in this limit, the ideal gas law $P_{e,i}=N_{e,i}k_BT_{e,i}$ is recovered. Therefore, Eqs. (8)–(11) together work as an "equation of state" for the non-Maxwellian plasma.

Assuming that all quantities vary only with z, the fluid and Maxwell's equations can be written as

$$\frac{\partial E_x}{\partial z} = -\frac{1}{c}\frac{\partial B_y}{\partial t}, \quad (13)$$

$$\frac{\partial B_y}{\partial z} = \frac{4\pi e}{c}N_e v_{ex} - \frac{1}{c}\frac{\partial E_x}{\partial t} \quad (14)$$

with $E_x=-(1/c)\partial A_x/\partial t$, $B_y=\partial A_x/\partial z$ and

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$$m_e\left(\frac{\partial}{\partial t}+v_{ez}\frac{\partial}{\partial z}\right)v_{ex}=-eE_x+\frac{e}{c}v_{ez}B_y, \tag{15}$$

where Eq. (15) describes the motion of the electrons in the transverse field, and

$$m_eN_e\left(\frac{\partial}{\partial t}+v_{ez}\frac{\partial}{\partial z}\right)v_{ez}=-eN_eE_z-\frac{e}{c}N_ev_{ex}B_y-\frac{\partial P_e}{\partial z}, \tag{16}$$

$$m_iN_i\left(\frac{\partial}{\partial t}+v_{iz}\frac{\partial}{\partial z}\right)v_{iz}=eN_iE_z-\frac{\partial P_i}{\partial z}, \tag{17}$$

$$\frac{\partial N_e}{\partial t}+\frac{\partial (N_ev_{ez})}{\partial z}=0, \tag{18}$$

$$\frac{\partial N_i}{\partial t}+\frac{\partial (N_iv_{iz})}{\partial z}=0, \tag{19}$$

and

$$\frac{\partial E_z}{\partial z}=-4\pi e(N_e-N_i) \tag{20}$$

for the longitudinal plasma motion. E_z stands for the longitudinal electrostatic field, $E_z=-\partial\varphi_{sc}/\partial z$.

III. PERTURBATION ANALYSIS AND NLS EQUATION

We are interested in investigate the low-frequency modulation of the EM wave amplitude. To perform this investigation we carry through a multiscale perturbation analysis based on the Krylov–Bogoliubov–Mitropolsky method for nonlinear wave modulation.[39,40] It is well known that this method is efficient in describing the long time behavior of the solution: It consists in varying the amplitude of the wave so slowly that no secular terms can arise.[40] Following Ref. 40, all the physical quantities can be considered weakly nonlinear waves. Then we can use the expansion

$$f=f_0+\varepsilon f_1(a,a^*,\psi)+\varepsilon^2 f_2(a,a^*,\psi)+\varepsilon^3 f_3(a,a^*,\psi)+\ \dots, \tag{21}$$

where f stands for any physical quantity. Here $\varepsilon \sim O(eA/m_ec^2)\ll 1$, $\psi=kz-\omega t$ is the fast variable and f_0 represents the equilibrium state. The complex amplitude $a=eA/m_ec^2$ of the EM wave is assumed to be a slowly varying function of z and t through the relations

$$\frac{\partial a}{\partial t}=\varepsilon A_1(a,a^*)+\varepsilon^2A_2(a,a^*)+\varepsilon^3A_3(a,a^*)+\dots \tag{22}$$

and

$$\frac{\partial a}{\partial z}=\varepsilon B_1(a,a^*)+\varepsilon^2B_2(a,a^*)+\varepsilon^3B_3(a,a^*)+\ \dots. \tag{23}$$

Equivalent relations can be written for a^*, the complex conjugate to a. We keep all significant terms up to order ε^3; as we will see, this order of approximation is enough for our purpose in this paper.

First we consider the transverse wave. From Eq. (15) and the definitions of E_x and B_y we obtain

$$\left(\frac{\partial}{\partial t}+v_{ez}\frac{\partial}{\partial z}\right)v_{ex}=\left(\frac{\partial}{\partial t}+v_{ez}\frac{\partial}{\partial z}\right)\frac{eA_x}{m_ec} \tag{24}$$

which implies the conservation of the transverse momentum, $m_ev_{ex}-eA_x/c=0$ ($v_{ex}=0$ for $A_x=0$). Using Eqs. (13) and (14) and the result obtained above, we get the wave equation

$$\left(\frac{\partial^2}{\partial z^2}-\frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)a_x=\frac{\omega_{pe}^2}{c^2}n_ea_x. \tag{25}$$

In the above equation a_x and n_e are the normalized vector potential and density, $a_x=eA_x/m_ec^2$ and $n_e=N_e/N_0$, respectively.

Now the perturbation technique is applied to Eq. (25). Writing explicitly the expansion for a_x and n_e around the equilibrium state, we have

$$\begin{bmatrix}a_x\\ n_e\end{bmatrix}=\begin{bmatrix}0\\ 1\end{bmatrix}+\varepsilon\begin{bmatrix}a_{x1}\\ 0\end{bmatrix}+\varepsilon^2\begin{bmatrix}a_{x2}\\ n_{e2}\end{bmatrix}+\varepsilon^3\begin{bmatrix}a_{x3}\\ n_{e3}\end{bmatrix}+\ \dots. \tag{26}$$

As we can notice, $\varepsilon\sim O(eA/m_ec^2)\ll 1$ is also a measure of the amplitude of the EM wave, which is connected to the longitudinal motion through the term v_xB_y in Eq. (16). It is straightforward to see that the longitudinal quantities are harmonics of the transverse wave: they originate from the EM wave through the Lorentz force. Then, it is expected that $v_z,E_z,N,P,\phi\sim O(\varepsilon^2)$ and we can set $v_{z1}=E_{z1}=N_1=P_1=\phi_1=0$.

Substituting (26) into Eq. (25) and separating powers of ε we get, to the first order in ε

$$\frac{\partial^2 a_{x1}}{\partial z^2}-\frac{1}{c^2}\frac{\partial^2 a_{x1}}{\partial t^2}=\frac{\omega_{pe}^2}{c^2}a_{x1}. \tag{27}$$

As a starting solution of the above wave equation, we choose a monochromatic plane wave

$$a_{x1}=ae^{i\psi}+a^*e^{-i\psi}. \tag{28}$$

To obtain a solution different from the trivial one ($a_{x1}=0$), the following relation must be satisfied:

$$D(\omega,k)=\omega^2-\omega_{pe}^2-k^2c^2=0. \tag{29}$$

Expression (29) is the dispersion relation for an EM wave propagating in an unmagnetized plasma, $\omega^2=\omega_{pe}^2+k^2c^2$.

To order ε^2 we obtain

$$\frac{ie^{i\psi}}{c^2}\left(\frac{\partial D}{\partial\omega}A_1-\frac{\partial D}{\partial k}B_1\right)+\frac{\partial^2 a_{x2}}{\partial z^2}-\frac{1}{c^2}\frac{\partial^2 a_{x2}}{\partial t^2}-\frac{\omega_{pe}^2}{c^2}a_{x2}+c.c.=0, \tag{30}$$

where we have used the relations $\partial D/\partial\omega=2\omega$ and $\partial D/\partial k=-2kc^2$, and $c.c.$ is the complex conjugate to the first term in (30). In finding Eq. (30), we have introduced the operators

$$\partial_t=\varepsilon(A_1\partial_a+A_1^*\partial_{a^*})+\varepsilon^2(A_2\partial_a+A_2^*\partial_{a^*})-\omega\partial_\psi+O(\varepsilon^3),$$

$$\partial_z=\varepsilon(B_1\partial_a+B_1^*\partial_{a^*})+\varepsilon^2(B_2\partial_a+B_2^*\partial_{a^*})+k\partial_\psi+O(\varepsilon^3). \tag{31}$$

In order to make the solution a_{x2} free from secular terms (ψ proportional terms), the condition

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$$A_1+v_gB_1=0 \quad (32)$$

and its complex conjugate relation must be fulfilled. Here

$$v_g=-\frac{\partial D}{\partial k}\bigg/\frac{\partial D}{\partial\omega}=\frac{d\omega}{dk}=\frac{kc^2}{\omega} \quad (33)$$

is the group velocity of the transverse wave $[v_g=v_g(\omega,k)]$. Expression (32) can be written in the form $\partial a/\partial t_1+v_g\partial a/\partial z_1=0$, where $t_1=\varepsilon t$, $z_1=\varepsilon z$ and $O(\varepsilon)$ terms have been neglected. It reveals that, up to order ε^2, the amplitude a is constant in time if we are in the rest frame of the wave packet. Solving now Eq. (30) yields

$$a_{x2}=C_1e^{i\psi}+C_1^*e^{-i\psi}, \quad (34)$$

where C_1 and C_1^* are arbitrary functions of a and a^*.

Proceeding in the perturbation analysis of Eq. (25) [using the expansion (26)] we now collect the terms of order ε^3 and get

$$\left\{\frac{i}{c^2}\left(\frac{\partial D}{\partial\omega}A_2-\frac{\partial D}{\partial k}B_2\right)-\frac{1}{2c^2}\left[\frac{\partial^2D}{\partial\omega^2}\left(A_1\frac{\partial A_1}{\partial a}+A_1^*\frac{\partial A_1}{\partial a^*}\right)+\frac{\partial^2D}{\partial k^2}\left(B_1\frac{\partial B_1}{\partial a}+B_1^*\frac{\partial B_1}{\partial a^*}\right)\right]\right\}e^{i\psi}-\frac{\omega_{pe}^2}{c^2}n_{e2}ae^{i\psi}+\frac{\partial^2a_{x3}}{\partial z^2}-\frac{1}{c^2}\frac{\partial^2a_{x3}}{\partial t^2}-\frac{\omega_{pe}^2}{c^2}a_{x3}+c.c.=0, \quad (35)$$

where we have used the relations $\partial^2D/\partial\omega^2=2$, $\partial^2D/\partial k^2=-2c^2$ and (34). Using the definition

$$\frac{dv_g}{dk}=\frac{\partial v_g}{\partial\omega}\frac{d\omega}{dk}+\frac{\partial v_g}{\partial k}=-\frac{(v_g^2\partial^2D/\partial\omega^2+2v_g\partial^2D/\partial k\,\partial\,\omega+\partial^2D/\partial k^2)}{\partial D/\partial\omega} \quad (36)$$

(in our particular case $\partial^2D/\partial k\partial\omega=0$) and the operator

$$A_1\frac{\partial}{\partial a}+A_1^*\frac{\partial}{\partial a^*}=-v_g\left(B_1\frac{\partial}{\partial a}+B_1^*\frac{\partial}{\partial a^*}\right) \quad (37)$$

besides expressions (31) and (32), we can write

$$\left[\frac{i}{c^2}\left(\frac{\partial D}{\partial\omega}A_2-\frac{\partial D}{\partial k}B_2\right)+\frac{1}{2c^2}\frac{\partial D}{\partial\omega}\frac{dv_g}{dk}\times\left(B_1\frac{\partial B_1}{\partial a}+B_1^*\frac{\partial B_1}{\partial a^*}\right)-\frac{\omega_{pe}^2}{c^2}n_{e2}a\right]e^{i\psi}+\frac{\partial^2a_{x3}}{\partial z^2}-\frac{1}{c^2}\frac{\partial^2a_{x3}}{\partial t^2}-\frac{\omega_{pe}^2}{c^2}a_{x3}+c.c.=0. \quad (38)$$

In order to determine n_{e2} in Eq. (38), we have to consider the equations for the longitudinal motion. First we analyze the coupling of the high-frequency wave with the electron density perturbations. In this case the ions just form a neutralizing background, and Eqs. (17) and (19) can be neglected. We also rewrite Poisson's equation in the form $\partial E_z/\partial z=4\pi e(N_0-N_e)$. Using expressions (8) and (9) we can write

$$\frac{\partial P_e}{\partial z}=\left(\frac{dP_e/d\phi}{dN_e/d\phi}\right)\frac{\partial N_e}{\partial z}\equiv\left(\frac{dP_e}{dN_e}\right)_{\phi=0}\frac{\partial N_e}{\partial z}=\frac{k_BT_e}{\alpha_0}\frac{\partial N_e}{\partial z}. \quad (39)$$

As mentioned before, all the longitudinal quantities are $O(\varepsilon^2)$. Since we carry on the perturbation analysis up to order ε^3, we can neglect terms such as $v_z\partial v_z/\partial z$ and reduce Eqs. (16) and (18), and Poisson's equation to

$$\left(\frac{\partial^2}{\partial t^2}-c_{se}^2\frac{\partial^2}{\partial z^2}\right)n_e+\omega_{pe}^2(n_e-1)=\frac{c^2}{2}\frac{\partial^2a_x^2}{\partial z^2}, \quad (40)$$

where

$$c_{se}=\left[\frac{1}{m_e}\left(\frac{dP_e}{dN_e}\right)_{\phi=0}\right]^{1/2}=\left[\frac{k_BT_e(\kappa_e-3/2)}{m_e(\kappa_e-1/2)}\right]^{1/2}. \quad (41)$$

Defining a generalized temperature $T_{\kappa e}=(\kappa_e-3/2)T_e/(\kappa_e-1/2)$, we can write

$$\lambda_{\kappa e}=\frac{c_{se}}{\omega_{pe}}=\sqrt{\frac{k_BT_{\kappa e}}{4\pi N_0e^2}}=\sqrt{\frac{k_BT_e(\kappa_e-3/2)}{4\pi N_0e^2(\kappa_e-1/2)}}. \quad (42)$$

The parameter $\lambda_{\kappa e}$ can be understood as the Debye length for a κ-plasma.[34] For large values of κ_e, the classical expression for the Debye length is recovered. As $\kappa_e\to3/2$, $\lambda_{\kappa e}$ decreases and tends to zero. Thus one effect of the superthermal electrons is to reduce the shielding length.

Substituting (26) into Eq. (40) we get, to $O(\varepsilon^2)$

$$\left(\frac{\partial^2}{\partial t^2}-c_{se}^2\frac{\partial^2}{\partial z^2}+\omega_{pe}^2\right)n_{e2}=\frac{c^2}{2}\frac{\partial^2a_{x1}^2}{\partial z^2}. \quad (43)$$

The term in the right-hand side originates from the Lorentz force and represents the ponderomotive force. From Eq. (43), we can see that the density perturbations are driven by this term. Using expression (28) for A_{x1} and the operators already defined in (31) we obtain

$$\frac{\partial^2n_{e2}}{\partial\psi^2}-\frac{\omega_{pe}^2}{(k^2c_{se}^2-\omega^2)}n_{e2}=\frac{2c^2k^2}{(k^2c_{se}^2-\omega^2)}a^2e^{2i\psi}+c.c. \quad (44)$$

As discussed before, the longitudinal oscillations are harmonics of the high-frequency field and are generated due to the ponderomotive force of the wave. To avoid a secular behavior of n_{e2}, we exclude the solution $\exp(\omega_{pe}\psi/\sqrt{k^2c_{se}^2-\omega^2})$ when solving Eq. (44) and write

$$n_{e2}=\frac{2c^2k^2}{(4\omega^2-4k^2c_{se}^2-\omega_{pe}^2)}a^2e^{2i\psi}+c.c. \quad (45)$$

Back to Eq. (38), we can write

$$\left[i(A_2+v_gB_2)+\frac{1}{2}\frac{dv_g}{dk}\left(B_1\frac{\partial B_1}{\partial a}+B_1^*\frac{\partial B_1}{\partial a^*}\right)-\frac{\omega_{pe}^2}{2\omega}n_{e2}a\right]e^{i\psi}+\frac{c^2}{2\omega}\left(\frac{\partial^2a_{x3}}{\partial z^2}-\frac{1}{c^2}\frac{\partial^2a_{x3}}{\partial t^2}-\frac{\omega_{pe}^2}{c^2}a_{x3}+\right)+c.c.=0, \quad (46)$$

where the original equation has been divided by $(1/c^2)\partial D/\partial\omega=2\omega/c^2$. Introducing expression (45) into the above equation gives

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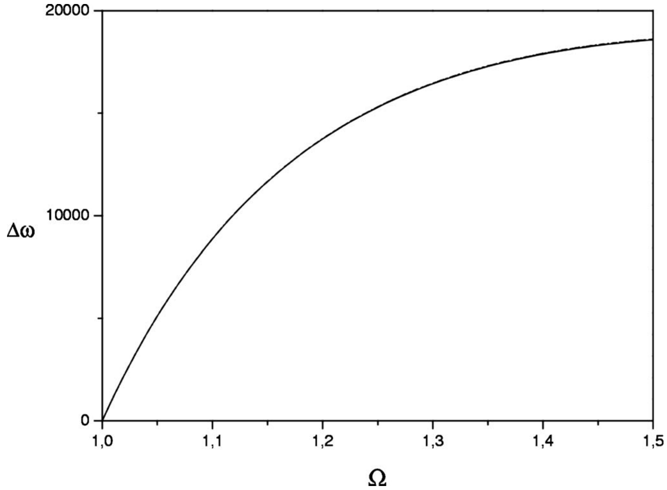

FIG. 1. $\Delta\omega_1$ vs Ω (low-frequency limit) for $n_0=10\ \mathrm{cm}^{-3}$, $k_BT_e=2$ keV and $\kappa_e=1.55$ (solid line), $\kappa_e=3.5$ (dashed line), $\kappa_e=15$ (dotted line), and $\kappa_e=500$ (dot-dashed line, Maxwellian case)—all curves are superimposed, in this case.

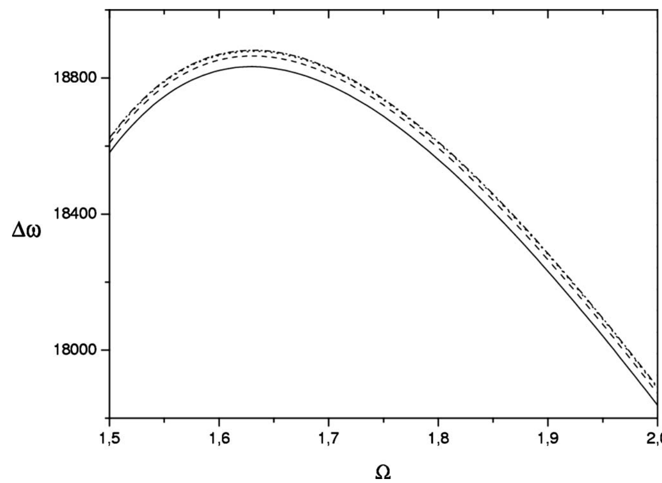

FIG. 2. $\Delta\omega_1$ vs Ω (intermediate frequencies) for $n_0=10\ \mathrm{cm}^{-3}$, $k_BT_e=2$ keV and $\kappa_e=1.55$ (solid line), $\kappa_e=3.5$ (dashed line), $\kappa_e=15$ (dotted line), and $\kappa_e=500$ (dot-dashed line, Maxwellian case)—here the last two curves are superimposed.

$$\left[i(A_2+v_gB_2)+\frac{1}{2}\frac{dv_g}{dk}\left(B_1\frac{\partial B_1}{\partial a}+B_1^*\frac{\partial B_1}{\partial a^*}\right)-\frac{\omega_{pe}^2c^2k^2}{\omega(4\omega^2-4k^2c_{se}^2-\omega_{pe}^2)}|a|^2a\right]e^{i\psi}+\frac{c^2}{2\omega}\left(\frac{\partial^2 a_{x3}}{\partial z^2}-\frac{1}{c^2}\frac{\partial^2 a_{x3}}{\partial t^2}-\frac{\omega_{pe}^2}{c^2}a_{x3}\right)-\frac{\omega_{pe}^2k^2c^2}{\omega(4\omega^2-4k^2c_{se}^2-\omega_{pe}^2)}a^3e^{3i\psi}+c.c.=0. \tag{47}$$

For the solution a_{x3} in Eq. (47) to be secular-free it is necessary that

$$i(A_2+v_gB_2)+\frac{1}{2}\frac{dv_g}{dk}\left(B_1\frac{\partial B_1}{\partial a}+B_1^*\frac{\partial B_1}{\partial a^*}\right)-\frac{\omega_{pe}^2c^2k^2}{\omega(4\omega^2-4k^2c_{se}^2-\omega_{pe}^2)}|a|^2a=0. \tag{48}$$

Noting that

$$A_2=\frac{1}{\varepsilon^2}\frac{\partial a}{\partial t}-\frac{A_1}{\varepsilon}=\frac{\partial a}{\partial t_2}-\frac{A_1}{\varepsilon},$$

$$B_2=\frac{1}{\varepsilon^2}\frac{\partial a}{\partial z}-\frac{B_1}{\varepsilon}=\frac{\partial a}{\partial z_2}-\frac{B_1}{\varepsilon}, \tag{49}$$

$$B_1\frac{\partial B_1}{\partial a}+B_1^*\frac{\partial B_1}{\partial a^*}=\frac{1}{\varepsilon^2}\frac{\partial^2 a}{\partial z^2}=\frac{\partial^2 a}{\partial z_1^2}$$

we obtain, from Eq. (48)

$$i\left(\frac{\partial a}{\partial t_2}+v_g\frac{\partial a}{\partial z_2}\right)+\frac{1}{2}\frac{dv_g}{dk}\frac{\partial^2 a}{\partial z_1^2}-\frac{\omega_{pe}^2c^2k^2}{\omega(4\omega^2-4k^2c_{se}^2-\omega_{pe}^2)}|a|^2a=0. \tag{50}$$

To study the influence of the ions in the modulation of the EM wave, Eqs. (17) and (19) must be taken into account. Manipulating Eqs. (16)–(20) we obtain

$$\frac{\partial^2}{\partial z^2}(\phi_{sc}-p_e)=\frac{m_ec^2}{2k_BT_e}\frac{\partial^2 a_x^2}{\partial z^2} \tag{51}$$

and

$$\frac{\partial^2 n_i}{\partial t^2}=\frac{k_BT_e}{m_i}\frac{\partial^2}{\partial z^2}(\phi_{sc}+p_i), \tag{52}$$

where $n_i=N_i/N_0$, $p_e=P_e/P_0$, and $p_i=P_i/P_0$ and the electron inertia has been neglected in Eq. (16) (terms such as $v_z\partial/\partial z$ have also been neglected).

Now we perform the perturbation analysis (21) in Eqs. (51) and (52). Using Eqs. (8)–(11) and the expansion

$$n_i=1+\varepsilon^2n_{i2}+\varepsilon^3n_{i3}+\ldots \tag{53}$$

we get, to $O(\varepsilon^2)$

$$\left[\frac{\partial^2}{\partial t^2}-(c_{Ti}^2+c_{si}^2)\frac{\partial^2}{\partial z^2}\right]n_{e2}=\frac{m_ec^2}{2m_i}\frac{\partial^2 a_{x1}^2}{\partial z^2}, \tag{54}$$

where the quasineutrality condition $n_{i2}\approx n_{e2}$ has been assumed. Here $c_{Ti}=(k_BT_i/\eta_0m_i)^{1/2}$ and $c_{si}=(m_e/m_i)^{1/2}c_{se}$. Using operators (31) and the solution for a_{x1} we obtain

$$\frac{\partial^2 n_{e2}}{\partial\psi^2}=-\frac{2m_ec^2k^2}{m_i[\omega^2-(c_{Ti}^2+c_{si}^2)k^2]}a^2e^{2i\psi}+c.c. \tag{55}$$

and its solution

$$n_{e2}=\frac{m_ec^2k^2}{2m_i[\omega^2-(c_{Ti}^2+c_{si}^2)k^2]}a^2e^{2i\psi}+c.c. \tag{56}$$

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Therefore, to investigate the wave modulation due to the coupling with the longitudinal ion oscillations we must consider the previous result together with Eq. (46). To avoid secularities in a_{x3} we get

$$i(A_2+v_gB_2)+\frac{1}{2}\frac{dv_g}{dk}\left(B_1\frac{\partial B_1}{\partial a}+B_1^*\frac{\partial B_1}{\partial a^*}\right)-\frac{m_e\omega_{pe}^2c^2k^2}{4m_i\omega[\omega^2-(c_{Ti}^2+c_{si}^2)k^2]}|a|^2a=0. \quad (57)$$

Using the identities already defined in (49) we can write the above equation in the form

$$i\left(\frac{\partial a}{\partial t_2}+v_g\frac{\partial a}{\partial z_2}\right)+\frac{1}{2}\frac{dv_g}{dk}\frac{\partial^2 a}{\partial z_1^2}-\frac{m_e\omega_{pe}^2c^2k^2}{4m_i\omega[\omega^2-(c_{Ti}^2+c_{si}^2)k^2]}|a|^2a=0. \quad (58)$$

As a summary of our results, Eqs. (50) and (58) can be written as follows:

$$i\left(\frac{\partial a}{\partial t_2}+v_g\frac{\partial a}{\partial z_2}\right)+P\frac{\partial^2 a}{\partial z_1^2}+Q|a|^2a=0, \quad (59)$$

where

$$P=\frac{1}{2}\frac{dv_g}{dk}=\frac{c^2\omega_{pe}^2}{2\omega^3} \quad (60)$$

and

$$Q=\begin{cases} Q_1=-\dfrac{\omega_{pe}^2c^2k^2}{\omega(4\omega^2-4k^2c_{se}^2-\omega_{pe}^2)} & \text{(EM wave coupled with electron oscillations)}\\ Q_2=-\dfrac{m_e\omega_{pe}^2c^2k^2}{4m_i\omega[\omega^2-(c_{Ti}^2+c_{si}^2)k^2]} & \text{(EM wave coupled with ion oscillations)}\end{cases} \quad (61)$$

are the dispersion and nonlinear coefficients, respectively. Introducing the coordinate transformation

$$\xi=\frac{1}{\varepsilon}(z_2-v_gt_2)=z_1-v_gt_1=\varepsilon(z-v_gt),$$
$$\tau=t_2=\varepsilon t_1=\varepsilon^2t. \quad (62)$$

Equation (59) can be transformed into the NLS equation

$$i\frac{\partial a}{\partial\tau}+P\frac{\partial^2 a}{\partial\xi^2}+Q|a|^2a=0. \quad (63)$$

The above equation describes the modulation of the EM wave, with the second and last terms accounting for the effects of dispersion and nonlinearity on the dynamics of the envelope.

IV. NONLINEAR FREQUENCY SHIFTS AND ENVELOPE HOLES

As discussed before, the nonlinear interaction between the high-frequency field and the electrostatic perturbations produces an electric field envelope which obeys the NLS equation (63). In this section the effect of nonthermality on the modulation of the envelope is investigated.

A standard stability analysis reveals that the EM wave envelope is unstable only for $Q/P>0$.[11] From Eqs. (29), (60), and (61) we notice that, for our case, Q/P is always negative. This result means that the wave is *modulationally* stable, and phenomena such as wave splitting can occur.[41] The condition $Q/P<0$ means that the group dispersion P and the nonlinear frequency shift $\Delta\omega=-Q$ have the same sign (P and $\Delta\omega>0$). The effect of the superthermal particles appears in the coefficient $\Delta\omega(Q)$, which results from the nonlinear coupling between the EM wave and the electrostatic density perturbations. In Fig. 1 we plot the nonlinear frequency shift $\Delta\omega_1=-Q_1$ for small values of the normalized frequency $\Omega=\omega/\omega_{pe}$ (long-wavelength limit, $v_\phi=\omega/k>c$). For this frequency range, the influence of the superthermal electrons is negligible. However, for high electron temperatures and intermediate values of $\Omega(v_\phi\approx c)$ we observe that the effect of electron nonthermality is to decrease $\Delta\omega$ (Fig. 2). The effect is maximum for the frequency range shown in Fig. 2, but it is still observable for larger values of Ω. Including the ion dynamics we get $\Delta\omega_2=-Q_2\approx m_e\omega_{pe}^2c^2k^2/4m_i\omega^3$, since $(c_{Ti}^2+c_{si}^2)\ll c^2$. Therefore, here the effect of ion nonthermality can be neglected for any frequency range.

It is known that the NLS equation admits, among others, localized solutions in the form of envelope solitons. The condition $Q/P<0$ implies that only dark or gray solitons exist. These solutions are also called envelope holes, since they represent an intensity dip in a continuous-wave background. For linearly polarized EM waves the inclusion of relativistic effects has a profound influence on the dynamics of the wave packet.[8,42] Introducing the real variables ρ and σ, which represent the real and the imaginary parts of a (Ref. 11)

$$a(\xi,\tau)=\sqrt{\rho(\xi,\tau)}e^{i\sigma(\xi,\tau)} \quad (64)$$

we have, for the envelope holes

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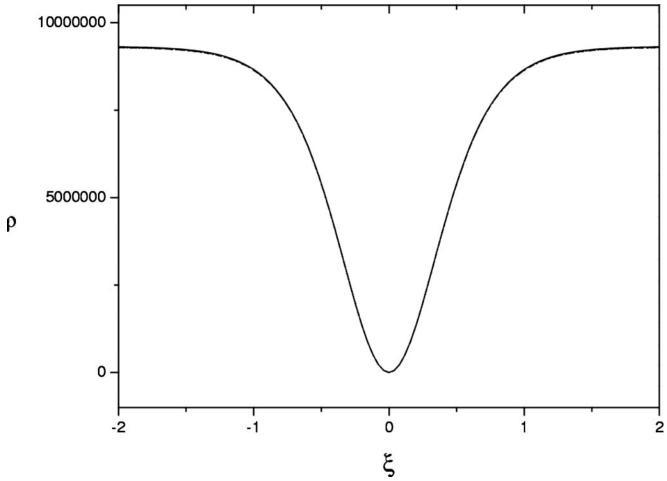

FIG. 3. $|a|^2$ vs ξ for $n_0=10\ \mathrm{cm}^{-3}$, $k_BT_e=2$ keV, $L=1$, $p=1$ (dark soliton), $\Omega=1.8$ and $\kappa_e=1.55$ (solid line), $\kappa_e=3.5$ (dashed line), $\kappa_e=15$ (dotted line), and $\kappa_e=500$ (dot-dashed line, Maxwellian case)—all curves are superimposed.

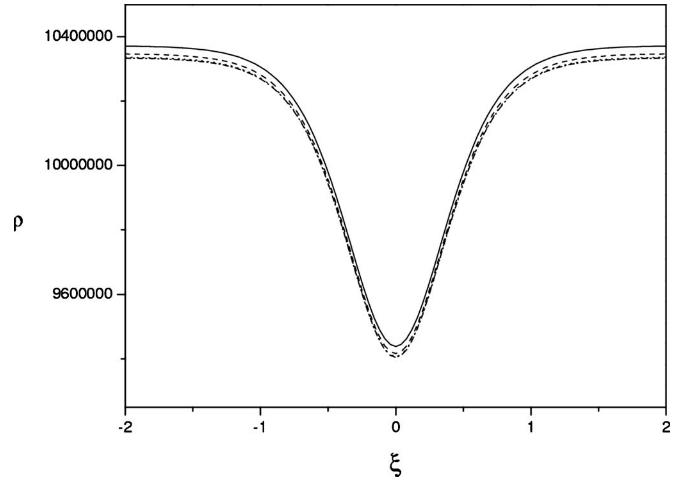

FIG. 5. $|a|^2$ vs ξ for $n_0=10\ \mathrm{cm}^{-3}$, $k_BT_e=2$ keV, $L=1$, $p=0.3$ (gray soliton), $\Omega=5$ and $\kappa_e=1.55$ (solid line), $\kappa_e=3.5$ (dashed line), $\kappa_e=15$ (dotted line), and $\kappa_e=500$ (dot-dashed line, Maxwellian case)—the last two curves are superimposed.

$$\rho=\rho_0[1-p^2\,\mathrm{sech}^2(\sqrt{|\delta|\rho_0}p\xi)] \tag{65}$$

and

$$\sigma=\arcsin\left\{\frac{p\tanh(\sqrt{|\delta|\rho_0}p\xi)}{[1-p^2\,\mathrm{sech}^2(\sqrt{|\delta|\rho_0}p\xi)]^{1/2}}\right\}+\Lambda\tau, \tag{66}$$

where ρ_0 is the soliton amplitude, p is an independent parameter related to the soliton width $L=2/p(\rho_0|\delta|)^{1/2}$ $=$constant, $\delta=Q/P$, and $\Lambda=-\rho_0|\delta|(3-p^2)/2$. In Fig. 3 we plot $\rho(\xi)=|a|^2$ for a dark soliton ($p=1$) for different values of κ_e. As we can see, the effect of electron nonthermality in this case is negligible, even for larger frequencies. Localized stationary solutions for $p=0.3$ (gray solitons, $p<1$) and $\Omega=1.4$ and 5 are shown in Figs. 4 and 5, respectively. As the frequency increases, the amplitude ρ_0 diminishes for all values of κ_e. However, gray solitons with $\kappa_e\to 3/2$ are less affected. Thus, the effect of the superthermal electrons is to slow the decrease of ρ_0. As p increases this effect becomes negligible. Like before, the inclusion of the ion dynamics results in no effect of nonthermality for both dark and gray solitons.

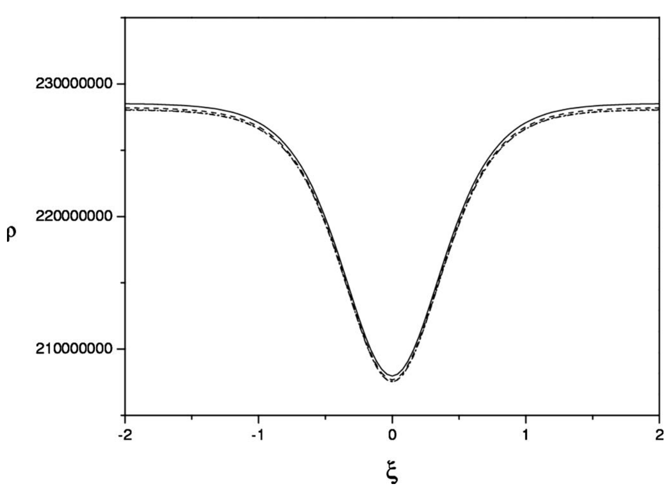

FIG. 4. $|a|^2$ vs ξ for $n_0=10\ \mathrm{cm}^{-3}$, $k_BT_e=2$ keV, $L=1$, $p=0.3$ (gray soliton), $\Omega=1.4$ and $\kappa_e=1.55$ (solid line), $\kappa_e=3.5$ (dashed line), $\kappa_e=15$ (dotted line), and $\kappa_e=500$ (dot-dashed line, Maxwellian case)—the last two curves are superimposed.

As we can see, the electron nonthermality has some influence on the dynamics of the wave packet. Linearly polarized EM waves propagating in plasmas far from thermal equilibrium ($\kappa_e\to 3/2$) generate harmonics that are influenced by the nonthermality of the plasma [Eq. (45)]. The coupling with these harmonics causes the modulation of the EM wave, which is described by the NLS Eq. (63) with a modified nonlinear coefficient Q. Due to electron nonthermality these modulated waves experience smaller frequency shifts, and the resultant gray solitons have larger amplitudes (when compared to EM waves propagating in Maxwellian plasmas). Although these effects are small, they are noticeable at the tail of gray solitons, at least for high temperatures and intermediate to high frequencies. This effect could, in principle, be experimentally detected by an electrostatic probe measuring the shape of the plasma density perturbations associated with the solitons.

V. SUMMARY

In the present paper the self-modulation of a linearly polarized EM wave propagating in a plasma with particles obeying a κ distribution function has been investigated. The fluid model is used to describe the dynamics of the electron-ion plasma, and two cases have been analyzed: first, the coupling of the EM wave with the electron density perturbations, and later the ion dynamics has been included. A multiscale perturbation analysis has been carried out and the NLS equation governing the modulation of the EM wave in both cases has been derived. The effect of nonthermality on the modulation of the wave envelope has been observed only for the EM wave coupled with the electron density perturbations. Superthermal electrons have no influence on the stability of the wave envelope but, for high electron temperatures, we notice that the effect of nonthermality is to reduce

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the nonlinear frequency shift and increase the amplitude of gray solitons (for intermediate to high frequencies of the EM wave). For EM waves coupled with ion-acoustic oscillations no effect of nonthermality has been observed.

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