

Non-extensive statistical analysis of meteor showers and lunar flashes

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ABSTRACT

The distribution of meteor magnitudes is usually supposed to be described by power laws. However, this relationship is not able to model the whole data set, and the parameters are considered to be dependent on the magnitude intervals. We adopt a statistical distribution derived from Tsallis non-extensive statistical mechanics which is able to model the whole magnitude range. We combined meteor data from various sources, ranging from telescopic meteors to lunar impactors. Our analysis shows that the probability distribution of magnitudes of International Meteor Organization (IMO) and Meteor Observation and Recovery Project data are similar. The distribution of IMO visual magnitudes indicates that 2.4 ± 0.5 per cent of the meteors of a shower may be telescopic ($m > 6$). We note that the distribution of duration of lunar flashes follows a power law, and a comparison with the distribution of meteor showers suggests the occurrence of observational bias. The IMO sporadic meteor distribution also seems to be influenced by observational factors.

Key words: meteorites, meteors, meteoroids – methods: statistical.

1 INTRODUCTION

Meteor showers are phenomena resulting from the collision of meteoroid streams with the Earth’s atmosphere. These particles originated from comets and asteroids, generically known as parent bodies. The first dynamic association between a comet and a meteor shower was established in 1867 involving the Leonids and 55P/Tempel-Tuttle (Manson 1995). A similar connection between the asteroid (3200) Phaethon and the Geminids meteor shower was established more than a century later (Whipple 1983). Studies of the physical properties of meteor showers began in the second half of the nineteenth century. These studies were focused on determining the chemical properties and brightness of meteors (Millman 1980). The mass cumulative distribution of a meteor shower is usually assumed to be represented by an exponential function (Zolensky et al. 2006),

$$\log N = A \log M + B, \quad (1)$$

where M is the meteoroid mass, A is the integrated mass index, B is a constant, and N is the meteor flux (number of events per area per time) with mass equal or greater than M . The cumulative distribution of meteor magnitudes m is given by Baggaley (1977)

$$N_m = Cr^m, \quad (2)$$

where N_m is the number of meteors brighter than, or equal to, m , r is the ratio between the number of meteors with magnitudes m and $m + 1$, and C is a constant. Equations (1) and (2) do not properly model all the mass distribution and magnitudes of a meteor shower. This is particularly significant for meteors with $m \leq -3$. Specifically, there is a variation of the mass index for these magnitudes. The use of equation (1) can infer an incorrect estimate of the mass flow incident on the Earth. An empirical solution usually adopted for this problem is to use equation (1) for specific mass intervals, instead of assuming that it is valid for the whole distribution. A fragmentation process can occur during formation, in the transit of meteoroids between the parent body and the Earth, or its interaction with the Earth’s atmosphere (Tóth & Klačka 2004; Ceplecha & Revelle 2005; Jenniskens & Vaubaillon 2007). The frequency of meteoric bombardment on the Moon is presumably similar to that on the Earth. This seems quite reasonable, given the size of a few million kilometres of meteoroid streams associated with annual showers (Nakamura et al. 2000). The visualization of the collision of meteoroids with the Moon is an unusual phenomenon. The only known records, until the 1990s, were associated with more energetic events than those commonly observed on Earth. An example is the photographically recorded flash by Stuart (1956). Based on the obtained image, Buratti & Johnson (2003) estimated that a body of 20 m of diameter crashed on the Moon, creating a 1.5 km diameter crater. The detection of lunar impacts was not possible until the introduction of CCD’s cameras into this type of observation, due to the short duration and low brightness of the events (Ortiz et al. 2002). The first detection of lunar flashes was obtained by

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Bellot Rubio, Ortiz & Sada (2000) and Dunham et al. (2000) for the Leonids meteor shower. The impacts can be associated with bodies with a mass of the order of a few kilograms. The detection of these events is subject to a combination of factors such as moon phase, instruments used, and the nature of the meteoroid. This ensemble of physical and instrumental factors is called observational bias. Observational bias is particularly important in the study of meteors and related phenomena (Hawkes & Jones 1986). As for the parameters associated with lunar flashes, Tost et al. (2006) concluded that most impacts have a typical duration and magnitude from 0.1 to 0.5 s and $m \sim 10$, respectively. Davis (2009) developed a model for the plume of steam due to an impact on the Moon, which suggests that parameters such as brightness and duration of the event are proportional to the mass and kinetic energy of the meteoroid. Bouley et al. (2012) found a direct correlation between the magnitude and duration of the flashes. However, this relationship does not appear to be valid for the Leonids. The difference between Leonids and other meteor showers is likely explained by the higher velocity of this swarm. Models that describe distributions associated with lunar flashes are not yet available in the literature. Such models are important for the identification of the observational bias, and to verify whether the incidence of meteoroid streams on the Moon are consistent with those observed on the Earth. We use non-extensive distributions, that emerge from Tsallis statistics (Tsallis 1988, 2009) to analyse data from meteor surveys and lunar flashes. In particular, we verify that the non-extensive distribution proposed by Sotolongo-Costa et al. (2008) models quite well the cumulative magnitude distribution of meteor showers. For the lunar impacts, the duration of the steam plumes is modelled by a power-law distribution. The correlation between the predictions in both data sets suggests that there is a significant difference in the distribution of sporadic meteors and those associated with a shower. Similarity in the distributions suggests a uniformity of the physical processes that govern the fragmentation of meteoroids and the formation of lunar flashes. The presence of observational bias is suggested in the data of meteors showers and lunar flashes.

2 NON-EXTENSIVE DISTRIBUTIONS OF METEOR SHOWERS AND LUNAR FLASHES

Boltzmann–Gibbs (BG) statistical mechanics needs the assumption (among others) of short-range interaction between particles, that implies strong chaos at the microscopic level and eventually results in the ergodic hypothesis. Distributions that emerge from BG formalism are based on the exponential function, that has rapidly decaying tails, and thus events far from the expectation values are utterly rare. Long-range gravitational interaction does not satisfy this basic assumption, and application of the statistical mechanics formalism to self-gravitating systems results in difficulties and/or inadequacies, e.g. negative specific heat, non-equivalence of ensembles (Lynden-Bell & Wood 1968; Thirring 1970; Landsberg 1990; Padmanabhan 1990). These inadequacies have been at least partially overcome by the use of non-extensive statistical mechanics, that is a generalization of BG formalism (i.e. BG statistics is a particular case of non-extensive statistics). Non-extensive distributions present tails that are asymptotically power laws, which implies that rare events, though unlikely, do not have vanishingly small probabilities. The generalization is governed by the entropic index q , that controls the power-law regime of the tails; the particular value $q = 1$ recovers the usual BG formalism. Non-Boltzmannian distributions, negative specific heat, and compatibility with non-extensive statistical mechanics have been computationally verified (Latora, Rapis-

arda & Tsallis 2001; Borges & Tsallis 2002; Tsallis et al. 2007). Examples linking non-extensivity to astrophysical systems are galore. We point out three instances that ranges from small bodies in the Solar system, galaxies, and the whole Universe: non-extensive distributions in asteroid rotation periods and diameters have been observed (Betzler & Borges 2012). Models for distribution function for spherical galaxies have been proposed, and density profiles have been shown to be a non-extensive equilibrium configuration, that can be used to describe dark matter haloes. The M33 rotation curve has been successfully fitted within this context (Cardone, Leubner & Del Popolo 2011). Small deviations from Gaussianity of the cosmic microwave background temperature fluctuations have also been verified, and non-extensive distributions seem to properly describe the data by the Wilkinson microwave anisotropy probe (Bernui, Tsallis & Villela 2006). More examples of applications of non-extensive statistical mechanics in astrophysical systems may be found in Tsallis (2009).

Fragmentation models have benefited from developments in material science, combustion technology, and geology, among others. Some attempts to obtain the size distribution function were made using the principle of maximum entropy (Li & Tankin 1987; Sotolongo-Costa, Grau-Crespo & Trallero-Herrero 1998). The expression for the BG entropy, in its continuous representation, is given by

$$S = -k \int p(x) \ln p(x) dx, \quad (3)$$

where $p(x)$ is the probability of finding the system in a specific microscopic state, generically represented by the dimensionless coordinate x , and k is the Boltzmann constant that guarantees dimensional consistency to the expression – we may take here $k \equiv 1$ without loss of generality. BG entropy is valid when short-range interactions are present. A fragmented object may be considered as a collection of parts that have entropy larger than the original (Sotolongo-Costa et al. 2008) ($S(\cup A_i) > \sum_i S(A_i)$). This suggests that it may be necessary to use a non-additive entropy, rather than the additive BG entropy. Tsallis non-additive entropy for the distribution of mass fragments is written as

$$S_q = k \frac{1 - \int_0^\infty p^q(M) dM}{q - 1}, \quad (4)$$

where M is a dimensionless mass, q is the entropic index, and $p(M)$ is the probability density of a fragment to have mass between M and $M + dM$. Maximization of equation (4), constrained to the normalization condition

$$\int_0^\infty p(M) dM = 1, \quad (5)$$

and to the q -expectation value of the mass

$$\int_0^\infty M p^q(M) dM = \langle M \rangle_q, \quad (6)$$

where $\langle M \rangle_q$ is the generalized q -expectation value, according to Curado & Tsallis (1991) (see also Tsallis, Mendes & Plastino 1998 for a broader view of the generalization of the constraints) leads to the mass-probability density of a meteor shower

$$p(M) dM = a(1 + bM)^{-\frac{1}{q-1}} dM. \quad (7)$$

Normalization condition, equation (5), imposes that $a = b(2 - q)/(q - 1)$. The inverse cumulative distribution $P_{\geq}(M) = N_{\geq}(M)/N_t = \int_M^\infty p(M') dM'$, $N_{\geq}(M)$ is the cumulative number of

meteors with mass equal to or greater than M , and N_t is the total number of meteors, leads to

$$N_{\geq}(M) = N_t(1 + bM)^{\frac{2-q}{1-q}} = N_t[\exp_q(-\beta M)]^{2-q} \quad (8)$$

($b = (q - 1)\beta$), that is identified with a q -exponential, defined as

$$\exp_q x \equiv [1 + (1 - q)x]^{\frac{1}{1-q}}, \quad (9)$$

if $[1 + (1 - q)x] > 0$, and $\exp_q x \equiv 0$ if $[1 + (1 - q)x] \leq 0$ (Tsallis 1994). This expression provides, in the asymptotic limit, a power-law regime

$$N(M) \sim M^{-n}, \quad (10)$$

with $n = \frac{2-q}{q-1}$, similar to equation (2). Once the relation between the magnitude and meteor mass can be expressed by an exponential function (Jacchia, Verniani & Briggs 1965), we adopt the expression used by Sotolongo-Costa et al. (2008)

$$M = M_0 e^{-\gamma m}, \quad (11)$$

where $M_0 = M(m = 0)$, and γ is a constant. Following their lines, taking into account that

$$p(m) dm = p(M) dM, \quad (12)$$

and finally integrating between m and $-\infty$, to obtain the inverse cumulative distribution, we get

$$N_{\geq}(m) = N_t[\exp_q(-\beta_m e^{-\gamma m})]^{2-q} \quad (13)$$

($\beta_m = \beta M_0$; β_m is taken as a fitting parameter in our procedure). The power of a q -exponential can easily be rewritten as another q' -exponential, with a different index q' :

$$N_{\geq}(m) = N_t \exp_{q'}(-\beta'_m e^{-\gamma m}) \quad (14)$$

with $\frac{1}{1-q'} = \frac{2-q}{q-1}$ and $\beta'_m = (2 - q)\beta_m$. Graphical representation of equation (14) with the ordinate in logarithm scale (log with base 10), and the abscissa (magnitude) in linear scale, as shown in the main panel of Figs 1–3, presents two main regimes. For highly negative magnitudes (i.e. bright objects), equation (14) displays an ascending straight line with slope given by $\gamma/(q' - 1)\log_{10}e$ (the rare events region). For high values of magnitudes, the cumulative distribution tends to its upper value N_t , and the semi-log plot exhibits a quasi-flat region. Elongation of the ascending straight line of the rare events region intercepts the horizontal line of the quasi-flat region at the transition point between regimes, $m^* = \frac{1}{\gamma} \ln[(q' - 1)\beta'_m]$. A similar twofold behaviour occurs regarding the decreasing q -exponential distribution, as explained in Betzler & Borges (2012), and illustrated by its Fig. 1. We have gathered data from meteor showers, and fitted equation (14); it is to be shown in the following sections.

3 OBSERVATIONAL DATA

The cumulative distributions of meteor showers were obtained from the counting of meteors in the range of visual magnitude, provided by the International Meteor Organization (IMO) (available at the VMDB – Visual Meteor Database, <http://www.imo.net/data/visual>). In this analysis, we consider meteor magnitudes ranging from -6 to $+6$ with an interval of one magnitude between the classes. The analysed counting was originated from observers that rated the local sky with limiting magnitude $lm \geq 5.5$. This selection criterion has been adopted in similar studies (Brown & Rendtel 1996; Arlt & Rendtel 2006), and it aims to minimize observational bias. We analysed 10 showers: Geminids (GEM), Orionids (ORI), Quadrantids (QUA),

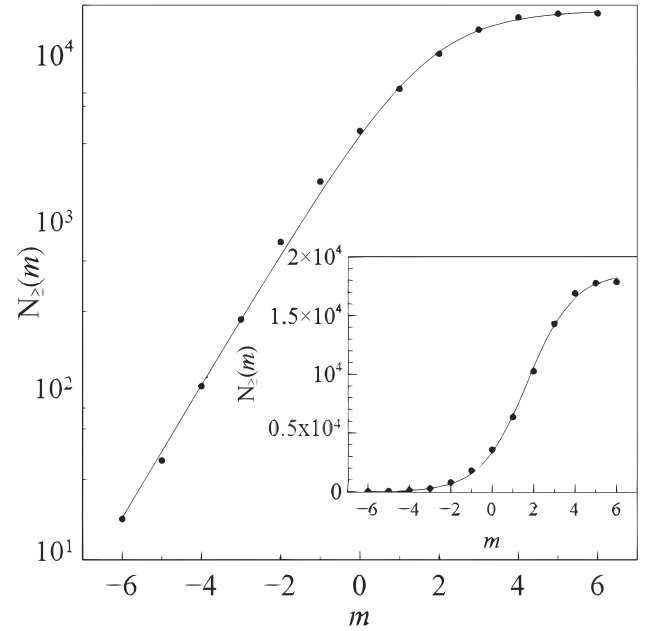


Figure 1. Cumulative distribution $N_{\geq}(m) \times$ magnitude m of PER (2000) (circles), and fitting with equation (14), $N_t = 18660$, $\beta'_m = 4.42$, $\gamma = 0.879$, $q' = 1.97$. Main panel: semi-log scale; inset: linear scale.

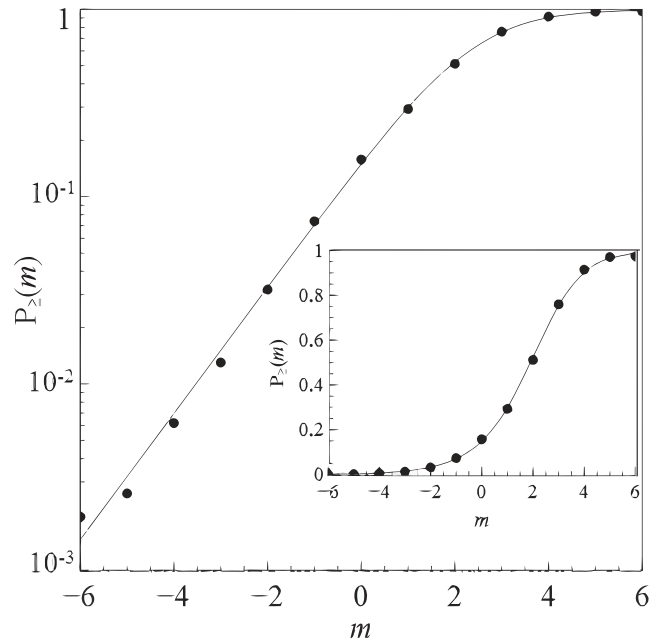


Figure 2. Cumulated probability distributions of CAP (2000–2010) (circles) and fitting with equation (14) with $P_{\geq}(m) = N_{\geq}(m)/N_t$, $N_t = 1536$, $\beta'_m = 10.20$, $\gamma = 1.13$, $q' = 2.42$ (solid line). Main panel: semi-log scale; inset: linear scale.

Eta Aquariids (ETA), Lyrids (LYR), Capricornids (CAP), Leonids (LEO), Perseids (PER), Alpha Monocerotids (AMO), Southern Taurids (STA), and also sporadic meteors (SPO). The choice of these showers is associated with the variety of physical and dynamic characteristics of each meteoroid stream, which is possibly associated with the differences in their parent bodies. In order to verify whether the parameters of equation (14) have a temporal variation, we have analysed the whole observed meteors in the years 2000,

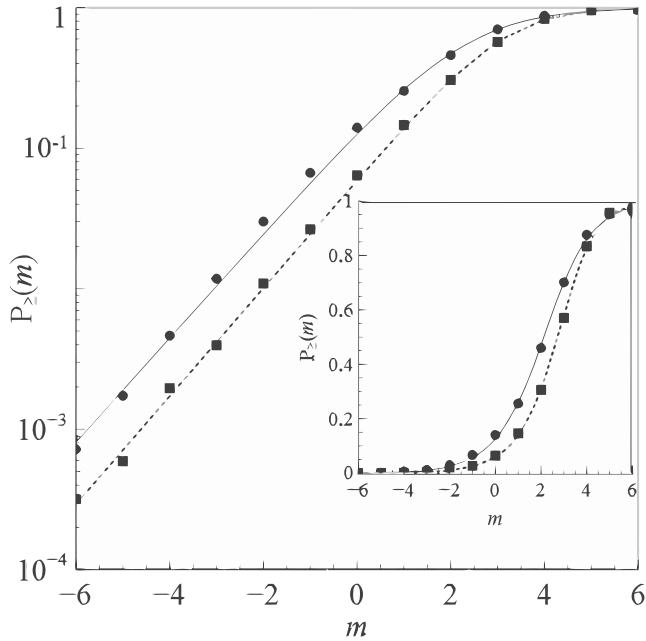


Figure 3. Cumulated probability distributions of LEO (1999) (circles) and SPO (2004) (squares), and their corresponding fittings (equation 14) with $N_t = 60329$, $\beta'_m = 8.72$, $\gamma = 0.99$, $q' = 2.16$ (LEO, solid line) and $N_t = 21942$, $\beta'_m = 48.1$, $\gamma = 1.34$, $q' = 2.52$ (SPO, dashed line).

2002, 2004, 2006, 2008, and 2010 (data from VMDB) according to the limit magnitude criterion mentioned above. Additionally, for the LEO, we have analysed the distribution of meteors in the outburst 1999s. To verify if any trends detected within the VMDB data are valid, we analysed panchromatic magnitudes of fireballs, recorded by the Meteor Observation and Recovery Project (MORP) survey (Halliday, Griffin & Blackwell 1996). From this data set, we studied the SPO, PER, and STA fireballs. Specifically, the PER and STA have presented the largest number of observations in the showers' data base. To demonstrate the possible occurrence of systematic errors between visual, photographic, and TV data, we compared the VMDB and MORP distributions with those detected by a TV camera installed in Salvador (SSA, Bahia, Brazil). The camera uses a CCD 1/3 inch Sony Super HAD EX View. This instrument is pointed towards the zenith, and has a field of 89° . The 49 SPO meteors detected were observed between 2011 July and October. The meteor magnitudes were estimated using the method of Koten (1999). With the same purpose, the SPO MORP magnitude distribution have been compared with the Fireball Database Center (FIDAC) data set (Knöfel & Rendtel 1988) obtained in the years 1993, 1994, 1995, 1996, and 1997, taken from 'Fireball Reports 1993–1996' (available at <http://www.imo.net/fireball/reports>).

We only considered the highest luminosity of detected meteors from the MORP and SSA data. This procedure was adopted to allow a comparison with VMDB and FIDAC visual data. Visual observers generally record only the brightness peak of a meteor (Beech, Nie & Coulson 2007). The duration of lunar flashes were collected by the Automated Lunar and Meteor Observatory (ALaMO) of NASA's Meteoroid Environment Office. The analysed data were obtained between 2005 and 2010 (available at http://www.nasa.gov/sites/default/files/files/ALAMO_lunar_impact_observations319.pdf). Instrumental characteristics of this initiative are presented in Suggs et al. (2008). We obtained the distribution of the duration of lunar flashes of SPO and LEO, GEM, LYR, Taurids (TAU), ORI,

and QUA showers. The duration of the flashes were obtained by multiplying the number of frames in which the events are recorded by 1/30 s, that corresponds to the integration time of the TV cameras used at the ALaMO.

4 PROCESSING AND ANALYSIS

The data from ALaMO, FIDAC, VMDB MORP, and SSA were separately displayed in a crescent cumulative distribution. It was found that ALaMO, FIDAC, MORP, and SSA data are best described by a power law (equation 10). Data from VMDB and SPO MORP, which present a greater magnitude range than the other sources, are better fitted by equation (14) (details are presented in the following Sections 4.1 and 4.2). The parameters were determined with the line search strategy, and the search direction was obtained with the conjugate gradient method. The adequacy of the fittings, as well as the similarity between distributions, was established by the Pearson chi-square test. All fittings present confidence level equal to or greater than 95 per cent.

4.1 Meteor showers

We have verified that equation (14) satisfactorily fits all VMDB meteor showers, and also the SPO (see Fig. 1, with PER, that was chosen as a representative sample because it is one of the most constant and best observed showers; Beech, Illingworth & Brown 2004). We have obtained mean values of $q = 1.57 \pm 0.05$ ($q' = 2.3 \pm 0.3$) and $\gamma = 1.1 \pm 0.2$. The coincidence of the values of q for different showers suggests that the fragmentation process that acts on VMDB meteors are essentially the same for the whole sample. Specifically, the value $q > 1$ implies that there are other effects that influence these distributions besides short-range force.

We have established that 97.6 ± 0.5 per cent of VMDB meteors have magnitudes $m \leq 6$. The rest are telescopic meteors. The counting of telescopic meteors from PER, ORI, and LEO by Porubčan (1973) is much smaller than the present prediction. This suggests that our result may be taken as an upper bound for the number of meteors with $m > 6$.

We did not detect any temporal variation in the probability distribution of meteors. We have compared VMDB data from the year 2000 with the equivalent data obtained up to the year 2010. Specifically for LEO, the counting of meteors belonging to the 1999 storm was compared with those from 2000 to 2010. The temporal independence was also verified. We have concluded that, despite the possibility of a seasonal increase in the total number of meteoroids (N_t), the mass distribution in a meteor shower remains constant. As a consequence of the invariance of the distributions along time, we found that the meteor distribution may be established using sparse data of several years. This hypothesis has been tested using all data from 2000 to 2010 of the CAP shower, and they are well adjusted by equation (14). Probability distribution of the CAP is similar to the distributions of other showers (see Fig. 2). When the showers were separately analysed by their parent bodies, we find that there are no differences between the meteor distributions that originate from comets or from asteroids. This test was done by comparing the distributions of the ORI and ETA (1P/Halley) and GEM (3200 Phaethon) and QUA (2003 EH1; Jenniskens 2004). The probability distributions of showers associated with comets do not differ when they are separated by their dynamic characteristics. To this end, we have compared the data showers associated with the Halley Family (ETA), the Jupiter Family (LEO), and with long period comets (LYR). The only difference that we were able to detect in the VMDB

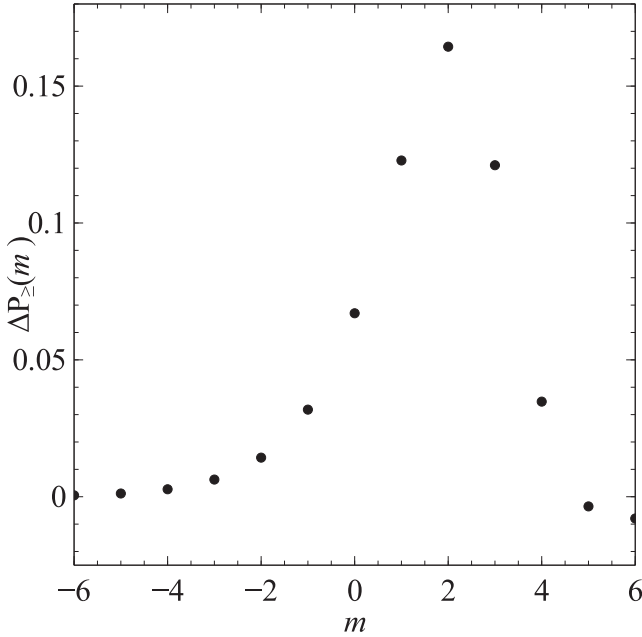


Figure 4. Difference between cumulated probabilities of LEO (1999) and SPO (2004), $\Delta P_{\geq}(m) = P_{\geq}^{\text{LEO}}(m) - P_{\geq}^{\text{SPO}}(m)$, as a function of the magnitudes m , putting in evidence the maximum discrepancy at $m = 2$.

data is between SPO meteors and those of all the other showers (see Fig. 3).

The difference between the SPO and shower probability distributions is maximum for the magnitude 2 (Fig. 4). In this range, there are ~ 20 per cent more meteors in the showers than observed in SPO. The difference is negligible for the magnitudes -6 and $+6$. This difference may be associated with observational bias in the process of data collection. We have found that most observations of annual showers occur 5 ± 2 d before or after the peak of the event. For instance, in the year 2010, VMDB presents 834 entries of SPO ($lm \geq 5.5$), and 434 of those ($\cong 52$ per cent) correspond to the peak period for PER, August 05–19. 1523 PER entries were recorded during this period. This may be evidence that the sky is not systematically monitored except in the annual shower seasons. The occurrence of observational bias in the VMDB SPO data was also suggested when we analysed the data from the MORP survey. The MORP fireballs associated with STA and PER showers were modelled by equations (10) and (11) (Fig. 5).

The comparison between modelled distributions suggested that these showers are similar. SPO data, however, are modelled by equation (14), possibly due to the magnitude range of this sample (Fig. 6). The modelled distributions of showers are not correlated to the observed distribution of the SPO. The number of SPO meteors in the considered magnitude interval is systematically greater than that observed in the showers, and this is a known result (Pawlowski et al. 2001, Rendtel 2006). We obtained $q = 1.51 \pm 0.03$ for the MORP data.

The SSA data are well fitted with equations (10) and (11). The value obtained is $q = 1.46 \pm 0.04$ (see Fig. 7). The FIDAC's data were also adjusted by equations (10) and (11), with $q = 1.8 \pm 0.1$ (Fig. 8). We have found that the FIDAC apparent magnitudes have good adherence to the model. The same is not true for zenithal magnitude distribution. This lack of agreement with the model is associated with the transformation of apparent to zenithal magnitudes. This conversion takes into account an estimate of the fireball

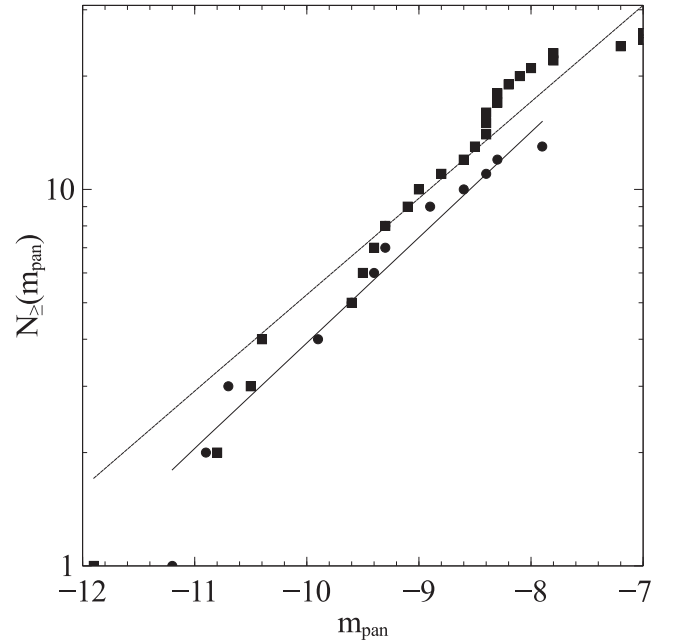


Figure 5. Cumulative distribution of the number of meteors MORP $N_{\geq}(m_{\text{pan}}) \times$ panchromatic magnitude m_{pan} , STA (squares), PER (circles) and their corresponding fittings (equations 10 and 11), STA (solid line, $n = 0.83$), and PER (dashed line, $n = 0.91$). Both fittings using $\gamma = 0.83$ calculated from MORP-SPO data.

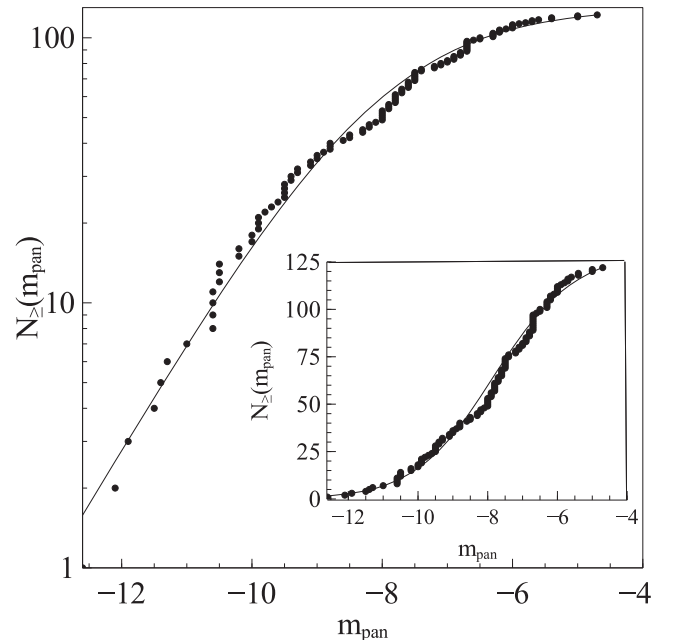


Figure 6. Cumulative distribution of the number of meteors MORP $N_{\geq}(m_{\text{pan}}) \times m_{\text{pan}}$, MORP-SPO (circles), and fitting with equation (14), $N_t = 131$, $\beta'_m = 0.0015$, $\gamma = 0.83$, $q' = 1.86$ (solid line). Main panel: semi-log scale; inset: linear scale.

altitude. The estimate of this parameter can cause the introduction of an additional source of error, as suggested by Bellot Rubio (1995). This problem is particularly important for fireballs brighter than magnitude -6 . We have verified the occurrence of a temporal variation in the distribution of apparent magnitudes comparing the data from 1993 to 1997. The compatibility only occurs when we

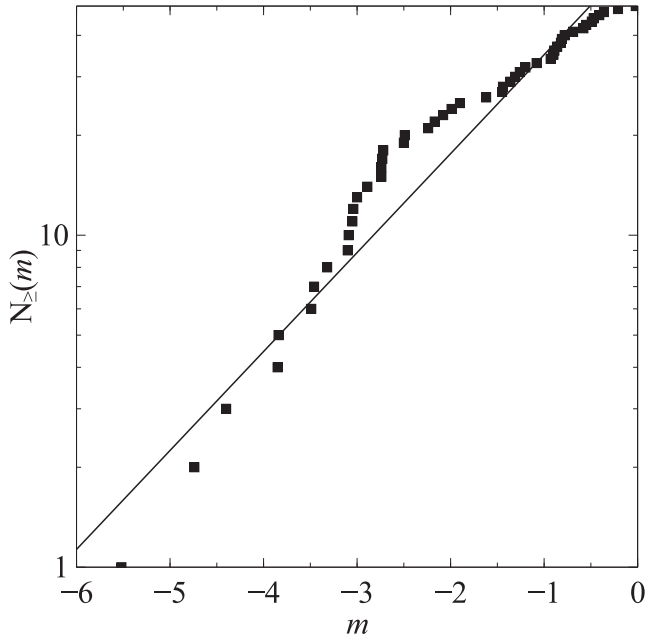


Figure 7. Cumulative distribution $N_{\ge}(m) \times$ magnitude m of meteors detected by an all sky camera at Salvador (SSA, squares). Solid line represents equations (10) and (11) ($n = 1.2$ e $\gamma = 0.59$). The magnitude error is 0.4. Fitting is not good for magnitudes around $m = -2$.

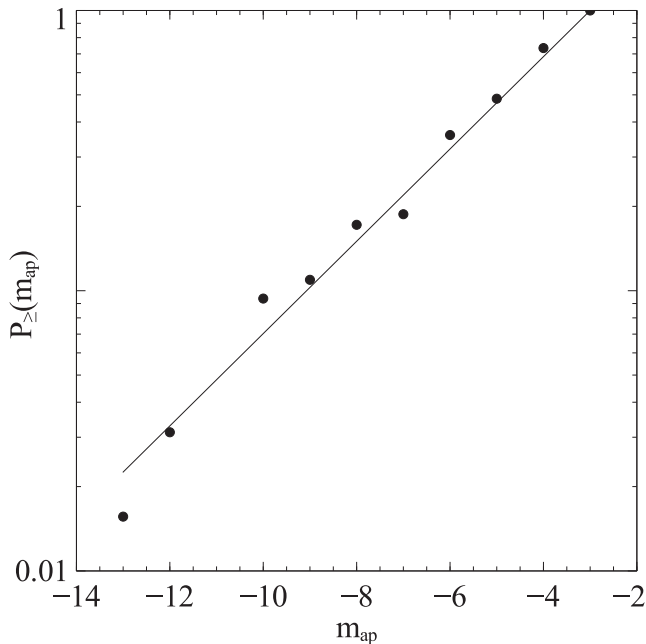


Figure 8. Cumulated probability of FIDAC $P_{\ge}(m_{ap}) \times$ apparent magnitude m_{ap} (data from file FIDAC95.001) Solid line, equations (10) and (11), with $n = 0.33$ and $\gamma = 1.1$.

consider a range of magnitudes between -6 and -3 . This suggests the occurrence of observational bias in the FIDAC sample. This bias may be associated with the difference of the terrestrial surface area covered by the surveys, total duration of the observations and field of view. Considering these factors, a compatibility was obtained by Zotkin & Khotinok (1978) comparing visual data of fireballs observed in the former Soviet Union with meteor photographic net-

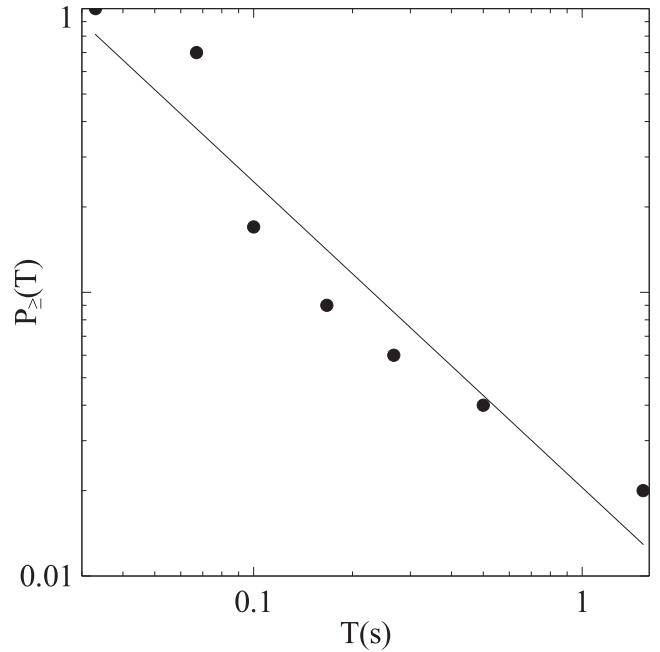


Figure 9. Cumulative probability for the duration $T(s)$ of ALaMO lunar flashes for SPO (circles). The solid line corresponding the fitting of equation (10) with $n = 1.6$.

works. For the same reason, this compatibility also does not occur between the MORP and SSA data.

4.2 Lunar flashes

ALaMO's data are modelled by equation (10) for the impact durations (Fig. 9). Distributions for the LEO, LYR and GEM showers are similar (this corresponds to half of the analysed sample), and this is not observed for the other showers. On the other hand, it would be expected that SPO distribution differs from the shower distributions, but we have observed that SPO distribution present similarities with ORI, GEM, LYR, and LEO. These results disagree with the conclusions obtained with the VMDB and MORP data. This suggests the occurrence of observational bias in the data set, that can be connected to inadequate observation conditions, such as the occurrence of unfavourable Moon phases for the detection of the flashes, and/or bad weather. Due to bias, it is not possible to infer the existence of lunar flashes with duration less than 0.033 s. The mean value for the similar meteor showers is $q = 1.32 \pm 0.02$, and this value is close to the one obtained by photometric data. Since both phenomena are supposedly directly associated to the meteoroid masses, we can conclude that the showers distributions observed on the Earth and on the Moon are compatible. As in the case of meteor showers, we suppose that the mechanisms that govern the duration of lunar flashes are both short- and long-range in nature.

5 CONCLUSIONS

We analysed the distribution of magnitudes of meteors and duration of lunar flashes through non-extensive statistics. We used data from various sources that cover a wide range, from telescopic meteors to lunar impactors. Our main conclusions are as follows.

- (i) The cumulative distribution of the magnitudes of the meteors is well represented by a q -exponential. This distribution is valid

for meteoroids of masses varying from telescopic meteors to lunar impact bodies. A power law, that is the asymptotic limit of the q -exponential, is observed for smaller intervals of masses.

(ii) We estimate that 2.4 ± 0.5 per cent (upper bound limit) of meteoroids in meteor showers are telescopic ($m > 6$).

(iii) The cumulative distributions of all meteor showers registered in VMDB and MORP data sets are similar, independent of time, independent of the type of parent body (whether asteroid or comet), or its dynamic family.

(iv) In the distributions of sporadic (non-shower) meteors the VMDB may present observational bias. For instance, the probability of occurrence of meteors of magnitude 2 in showers is about 20 per cent higher than the equivalent in SPO.

(v) The cumulative distribution of duration of lunar flashes registered by ALAMO is modelled by a power law. The analysis of the distributions suggests that the meteor showers observed on the Earth and on the Moon are compatible.

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REFERENCES

- Arlt R., Rendtel J., 2006, *MNRAS*, 367, 1721
 Baggaley W. J., 1977, *MNRAS*, 180, 89
 Beech M., Illingworth A., Brown P., 2004, *MNRAS*, 348, 1395
 Beech M., Nie W., Coulson I. M., 2007, *J. R. Astron. Soc. Can.*, 101, 139
 Bellot Rubio L. R., 1995, *A&A*, 301, 602
 Bellot Rubio L. R., Ortiz J. L., Sada P. V., 2000, *Earth Moon Planets*, 82, 575
 Bernui A., Tsallis C., Villela T., 2006, *Phys. Lett. A*, 356, 426
 Betzler A. S., Borges E. P., 2012, *A&A*, 539, A158
 Borges E. P., Tsallis C., 2002, *Physica A*, 305, 148
 Bouley S. et al., 2012, *Icarus*, 218, 115
 Brown P., Rendtel J., 1996, *Icarus*, 124, 414
 Buratti B. J., Johnson L. L., 2003, *Icarus*, 161, 192
 Cardone V. F., Leubner M. P., Del Popolo A., 2011, *MNRAS*, 414, 2265
 Cepelcha Z., Revelle D. O., 2005, *Meteorit. Planet. Sci.*, 40, 35
 Curado E. M. F., Tsallis C., 1991, *J. Phys. A*, 24, L69
 Davis S. S., 2009, *Icarus*, 202, 383
 Dunham D. W., Sterner R., II, Gotwols B., Cudnik B. M., Palmer D. M., Sada P. V., Frankenberger R., 2000, *Occultation Newsl.*, 8, 9
 Halliday I., Griffin A. A., Blackwell A. T., 1996, *Meteorit. Planet. Sci.*, 31, 185
 Hawkes R. L., Jones J., 1986, *QJRAS*, 27, 569
 Jacchia L. G., Verniani F., Briggs R. E., 1965, *SAO Special Report*, 175
 Jenniskens P., 2004, *AJ*, 127, 3018
 Jenniskens P., Vaubaillon J., 2007, *AJ*, 134, 1037
 Knöfel A., Rendtel J., 1988, *J. Int. Meteor. Organ.*, 16, 186
 Koten P., 1999, in Baggaley W. J., Porubčan V., eds, *Proc. Meteoroids '98 Conf., Photometry of TV Meteors*. Astron. Inst., Slovak Acad. Sci., Bratislava, p. 149
 Landsberg P. T., 1990, *Thermodynamics and Statistical Mechanics*. Dover Press, New York
 Latora V., Rapisarda A., Tsallis C., 2001, *Phys. Rev. E*, 64, 056134
 Li X., Tankin R. S., 1987, *Combust. Sci. Tech.*, 56, 65
 Lynden-Bell D., Wood R., 1968, *MNRAS*, 138, 495
 Manson J. W., 1995, *J. Br. Astron. Assoc.*, 105, 219
 Millman P. M., 1980, in Halliday I., McIntosh B. A., eds, *Proc. IAU Symp. 90, One Hundred and Fifteen Years of Meteor Spectroscopy*. Reidel, Dordrecht, p. 121
 Nakamura R., Fujii Y., Ishiguro M., Morishige K., Yokogawa S., Jenniskens P., Mukai T., 2000, *ApJ*, 540, 1172
 Ortiz J. L., Quesada J. A., Aceituno J., Aceituno F. J., Bellot Rubio L. R., 2002, *ApJ*, 576, 567
 Padmanabhan T., 1990, *Phys. Rep.*, 188, 285
 Pawlowski J. F., Hebert T. J., Hawkes R. L., Matney M. J., Stansbery E. G., 2001, *Meteorit. Planet. Sci.*, 36, 1467
 Porubčan V., 1973, *Bull. Astron. Inst. Czech.*, 24, 1
 Rendtel J., 2006, *J. Int. Meteor. Organ.*, 34, 71
 Sotolongo-Costa O., Grau-Crespo R., Trallero-Herrero C., 1998, *Rev. Mex. Fis.*, 44, 441
 Sotolongo-Costa O., Gamez R., Luzon F., Posadas A., Weigandt Beckmann P., 2008, *Apeiron*, 15, 187
 Stuart L. H., 1956, *Strolling Astron.*, 10, 42
 Suggs R. M., Cooke W. J., Suggs R. J., Swift W. R., Hollon N., 2008, *Earth Moon Planets*, 102, 293
 Thirring W., 1970, *Z. Phys.*, 235, 339
 Tost W., Oberst J., Flohrer J., Laufer R., 2006, in *Copernicus Meetings, European Planetary Science Congress 2006, Lunar Impact Flashes: History of observations and recommendations for future campaigns*. Berlin, Germany, p. 546
 Tóth J., Kláčka J., 2004, *Earth Moon Planets*, 95, 181
 Tsallis C., 1988, *J. Stat. Phys.*, 52, 479
 Tsallis C., 1994, *Quim. Nova*, 17, 468
 Tsallis C., 2009, *Introduction to Nonextensive Statistical Mechanics: Approaching a Complex World*. Springer-Verlag, New York
 Tsallis C., Mendes R., Plastino A. R., 1998, *Physica A*, 261, 534
 Tsallis C., Rapisarda A., Pluchino A., Borges E. P., 2007, *Physica A*, 381, 143
 Whipple F. L., 1983, *IAU Circ.*, 3881, 1
 Zolensky M., Bland P., Brown P., Halliday I., 2006, in Lauretta D. S., McSween H. Y., Jr, eds, *Meteorites and the Early Solar System II*. Univ. Arizona Press, Tucson, AZ, p. 869
 Zotkin I. T., Khotinok R. L., 1978, *Meteoritika*, 37, 37

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