International Journal of Quality & Reliability Management
Modeling failure rate of a robotic welding station using generalized q-distributions
Edilson M. Assis Ernesto P. Borges Silvio A.B. Vieira de Melo Leizer Schnitman

Received 15 November 2012
Revised 30 October 2013
Accepted 18 September 2014

Modeling failure rate of a robotic welding station using generalized q-distributions

Edilson M. Assis
Department of Production Engineering, Federal University of Bahia, Salvador, Brazil
Ernesto P. Borges
Federal University of Bahia, Salvador, Brazil and National Institute of Science and Technology for Complex Systems, Federal University of Bahia, Salvador, Brazil, and
Silvio A.B. Vieira de Melo and Leizer Schnitman
Department of Production Engineering, Federal University of Bahia, Salvador, Brazil

Abstract

Purpose – The purpose of this paper is to compare four life data models, namely the exponential and the Weibull models, and their corresponding generalized versions, q-exponential and q-Weibull models, by means of one practical application.
Design/methodology/approach – Application of the models to a practical example (a welding station), with estimation of parameters by the use of the least squares method, and the Akaike Information Criterion (AIC).
Findings – The data of the example considered in this paper is divided into three regimes, decreasing, constant and increasing failure rate, and the q-Weibull model describes the bathtub curve displayed by the data with a single set of parameters.
Practical implications – The simplicity and flexibility of the q-Weibull model may be very useful for practitioners of reliability analysis, and its benefits surpasses the inconvenience of the additional parameter, as AIC shows.
Originality/value – The q-Weibull model is compared in detail with other three models, through the analysis of one example that clearly exhibits a bathtub curve, and it is shown that it can describe the whole time range with a single set of parameters.

Keywords Reliability, Bathtub curve, Failure rate, q-distribution
Paper type Research paper

1. Introduction

Reliability modeling is one of the most important steps for Reliability, Availability, Maintainability, and Safety (RAMS) assessment. A growing focus has been placed on RAMS during the design and operation of industrial systems, mainly due to the size and complexity of modern industrial plants. A comprehensive assessment of operational safety requires a systemic approach based on statistical models for the description of failure rates related to equipments and their components. Development, choice or even application of a model to accurately characterize the failure rate is a nontrivial task, and mathematical simulation of reliability performance depends crucially on it. However, life data are still scarce and a predictive capability of the model should be often investigated. It is necessary to have a faithful model to overcome these problems. So, reliability modeling becomes one of the most important steps for RAMS

International Journal of Quality & Reliability Management
Vol. 32 No. 2, 2015
pp. 156-166

0265-671X
DOI 10.1108/IJQRM-11-2012-0151

assessment. In order to improve reliability modeling we take some mathematical functions, originally derived within nonextensive statistical physics, which has been a continuously and increasingly developed along the last two decades. A pair of functions naturally appears from this formalism, namely the q-logarithm, and its inverse, the q-exponential, defined as (see Tsallis, 1994):

$$\ln_q x = \frac{x^{1-q}-1}{1-q}(x>0), \tag{1}$$

$$\exp_q x = [1+(1-q)x]_+^{1/(1-q)}, \tag{2}$$

where the symbol $[a]_+$ means that $[a]_+ = a$ if $a > 0$ and $[a]_+ = 0$ if $a \leqslant 0$. This is an important feature of the q-exponential, once it avoids negative or even complex numbers for $\exp_q x$, and permits its interpretation as probabilities. If the limit $q \to 1$ is taken, these functions recover the usual logarithm and exponential, and thus they are generalizations of these functions. They also satisfy $\ln_q 1 = 0$ and $\exp_q 0 = 1$, $\forall q$. Foundations of nonextensive statistical mechanics and applications for various systems can be found in (Tsallis, 2009). A set of actualized references is maintained in (Tsallis, 2013).

The q-exponential of a negative argument asymptotically becomes a power law for $q > 1$ ($\exp_q(-x) \sim 1/x^n$, with $x > 0$, $n = 1/(q-1)$). One remarkable feature of the q-exponential function is to continuously interpolate between a power law behavior (with $q > 1$) to an exponential behavior (with $q = 1$).

Tsallis *et al.* (1995) were the pioneers in the application of q-exponential and q-logarithm functions in statistical distributions by the generalization of the Gaussian function. The introduction of the parameter q yields the q-Gaussian distribution that recovers the usual Gaussian when $q = 1$. The q-Gaussian generalizes various distributions, e.g., the Dirac delta Lorentzian, and completely flat distribution, according to the value of the parameter q. Prato and Tsallis (1999) modified the q-Gaussians by means of the escort probabilities, which has normalized moment of order zero.

One special distribution widely used in RAMS is the Weibull distribution. q-Weibull distribution has been advanced by (Picoli *et al.*, 2003):

$$p(x) = p_0 \frac{\beta x^{\beta-1}}{x_0^{\beta}} \exp_q\left[-\left(\frac{x}{x_0}\right)^{\beta}\right]. \tag{3}$$

Particular cases of Equation (3) are the q-exponential, for $\beta = 1$, and Weibull, for $q = 1$. This function has been applied to a variety of systems, like distributions of dielectric breakdown in oxides, cyclone victims, brand-name drugs by retail sales, highway length (Costa *et al.*, 2006; Picoli *et al.*, 2003). In Sartori *et al.* (2009) we have briefly compared an application of the Weibull and q-Weibull models for a natural gas recovery plant, and in Assis *et al.* (2013) we have explored some mathematical properties of the q-Weibull model, and we have shown that it can exhibit a bathtub curve for particular values of the parameters ($0 < \beta < 1$ and $q < 1$).

The main focus of this work is to present an example that exhibits the bathtub curve, that is properly modeled by the q-Weibull distribution (obviously this model is applicable to other systems). We also use other models, namely the ordinary Weibull, the ordinary exponential, and the q-exponential, for comparison. In Section 2 we review the mathematical expressions adopted, and present the methodology for estimation of parameters. Subsequently, in Section 3, these distributions are applied to life data of a welding station. Finally the last section is dedicated to our conclusions.

2. Life distributions

A statistical distribution of a single continuous variable is fully described by its probability density function (pdf). Most functions commonly used in reliability engineering and life data analysis (reliability function, failure-rate function, mean-time function, median-life function) can be determined directly from the pdf.

Some distributions are more propitious to represent life data and are most commonly called life distributions. A life distribution shows how a population of components fails in time, or how the failures are distributed in time. It is just like any statistical distribution, except that the data involved are time-to-failure or life data. A life distribution is known when the parameters of a properly selected model are estimated.

The use of pdfs in reliability analysis is widespread since long (see Pham and Lai (2007), and also Kotz and Nadarajah (2005) about some quest regarding the originality of the use of pdf in reliability analysis). According to Berberan-Santos *et al.* (2008), the use of stretched exponential (the Weibull distribution can also be called a stretched exponential) has been recorded before the article of Weibull (1951), in a work of Kohlrausch (1854) which describes capacitor discharge.

2.1 q-Weibull and q-exponential distributions

The Weibull distribution is largely used in reliability context. The density of the Weibull distribution, in its three-parameter form, is defined by (Weibull, 1951):

$$f(t) = \frac{\beta(t-t_0)^{\beta-1}}{\theta^{\beta}} \exp\left[-\left(\frac{t-t_0}{\theta}\right)^{\beta}\right], \quad \text{with} t \geqslant t_0, \tag{4}$$

where β is the shape parameter, θ is the scale parameter, and t_0 is the location parameter. The characteristic life is $\eta = \theta + t_0$.

There are many models based on Weibull distribution. Time-depending parameters, exponentiated unreliability function, and nested exponentials are some examples used to extend its applicability. Most of these modifications presents exponential behavior (see Murthy *et al.,* 2004).

Exponential functions are usually found in the description of systems with weak interactions (or, in the limit, no interactions at all). Complex systems usually have long-range spatial interactions or long-term memory or a cooperation/competition, as can be seen in Bak (1997). Statistical distributions of complex systems are usually described by power laws.

Component failures can have multiple causes, that can be recent or not; cooperation and conflict between causes may also happen, so it is not surprising that, in specific situations, complex behavior may appear, and power laws, or power law like distributions are expected.

The description of simple systems is well established, particularly through Boltzman-Gibbs statistical mechanics (Boltzmann weight and Maxwellian distributions, that rely on an exponential basis). These exponential distributions are derived from the Boltzmann-Gibbs-Shannon (BGS) entropy. The theoretical basis of statistical mechanics for complex systems is under intense research recently, and there are many evidences that points toward the nonextensive statistical mechanics, and the Tsallis nonadditive entropy (Tsallis, 2009).

The nonextensive statistical mechanics is a generalization of the Boltzmann-Gibbs statistical mechanics, and it permits to generalize many distributions originally conceived for simple systems. The q-Weibull pdf, expressed by Equation (3), can be generalized as (Picoli *et al.*, 2003):

$$f_q(t) = \frac{\beta(2-q)}{\theta}\left(\frac{t-t_0}{\theta}\right)^{\beta-1}\exp_q\left[-\left(\frac{t-t_0}{\theta}\right)^{\beta}\right], \quad \text{with } t \geqslant t_0, \quad q < 2. \tag{5}$$

The q-Weibull pdf is not normalized for $q \geqslant 2$. The generalized reliability function, $R_q(t) \equiv \int_t^{\infty} f_q(x)dx$, is given by:

$$R_q(t) = \exp_{\frac{1}{2-q}}\left[-(2-q)\left(\frac{t-t_0}{\theta}\right)^{\beta}\right], \quad \text{with} t \geqslant t_0. \tag{6}$$

Note that the index of the q-exponential changes from q to $1/(2-q)$ due to the property of the q-exponential $\int \exp_q x dx = \exp_{\frac{1}{2-q}} x$. The generalized unreliability function is defined as:

$$F_q(t) = 1 - R_q(t). \tag{7}$$

The q-exponential function (Equation (2)) has a cut off whenever $q < 1$ and $x < -1/(1-q)$; when this happens, the q-exponential is defined as zero, avoiding complex values, and this allows its interpretation as a probability. This cut off implies a vertical asymptote in Equation (6), that is a maximum value for lifetime (see Assis *et al.* (2013) for details).

These equations, with general q and $\beta > 0$, are generalizations of three models: the ordinary Weibull (with general $\beta > 0$ and $q = 1$), the ordinary exponential (with $\beta = 1$ and $q = 1$ that is already a particular case of the ordinary Weibull), and the q-exponential (with $\beta = 1$ and general $q < 2$). We compare the four models by means of the example presented in Section 3.

2.2 Failure rate of distributions

The q-Weibull hazard function, $h_q(t)$, is given by Assis *et al.* (2013):

$$h_q(t) = \frac{f_q(t)}{R_q(t)} = \frac{\frac{\beta(2-q)}{\theta}\left(\frac{t-t_0}{\theta}\right)^{\beta-1}\exp_q\left(-\left(\frac{t-t_0}{\theta}\right)^{\beta}\right)}{\exp_{\frac{1}{2-q}}\left(-(2-q)\left(\frac{t-t_0}{\theta}\right)^{\beta}\right)}, \quad q < 2. \tag{8}$$

The behavior of the q-Weibull hazard function may be very different from its particular case h_1 (Weibull hazard function). In total, four different types of failure rate behaviors can be described by Equation (8): (i) monotonically decreasing ($1 < q < 2$ and $0 < \beta < 1$); (ii) monotonically increasing ($q < 1$ and $\beta > 1$); (iii) unimodal ($1 < q < 2$ and $\beta > 1$); (iv) bathtub curve ($q < 1$ and $0 < \beta < 1$). The trivial constant failure rate is obtained for $q = 1$ and $\beta = 1$. The types (iii) and (iv) have a maximum and a minimum, respectively, and this behavior cannot be achieved by the usual ($q = 1$) Weibull hazard function. The q-exponential ($\beta = 1$) and the exponential ($\beta = 1$ and $q = 1$) are particular cases. The constant failure rate of the exponential distribution is given by $\lambda = 1/\theta$.

2.3 Estimation of parameters

Sample data may be conveniently put into a straight line in the form of $y = \beta x + b$ by the change of variables $x = \ln(t - t_0)$, and $y = \ln\{-\ln_{q'}[1 - F_q(t)]\}$, with $q' = 1/(2-q)$ and $b = -\beta \ln[\theta/(2-q)^{\frac{1}{\beta}}]$(note that the q-logarithm is the inverse function of the q-exponential).

Sample data are time-to-failure ranked in ascending order and an estimate of the unreliability can be obtained using an approximation of the median ranks, also known as Bernard's approximation, given by (Johnson, 1951):

$$\hat{F}_i = \frac{i - 0.3}{n + 0.4}, \tag{9}$$

where n is the sample size, i is the order number of failure varying from 1 to n. In this way, for every sample time t_i we obtain:

$$x_i = \ln(t_i - t_0), \tag{10}$$

$$y_i = \ln\left[-\ln_{q'}\left(1 - \hat{F}_i\right)\right]. \tag{11}$$

An alternative to compute $\hat{F}_i$ is to consider it as the sum of relative frequency of occurrence of failure in the previous time intervals. This procedure is suitable for a large number of samples and it is used in Section 3.

The parameters of Equation (5) are estimated by the maximization of the coefficient of determination R^2:

$$R^2 = 1 - \frac{\sum_{i=1}^{n}[y_i - \hat{y}_i]^2}{\sum_{i=1}^{n}[y_i - \bar{y}]^2}, \tag{12}$$

with the constraints $\beta > 0$, $\theta > 0$, $t_0 < \eta$, $t_0 < t_{min}$, and $q < 2$, where $\hat{y}_i$ is given by Equation (11), $y = (\sum y_i)/n$ and t_{min} is the lowest sample time. This procedure is obviously also valid for the other three particular models that q-Weibull generalizes, with the additional constraints referred to at the end of Subsection 2.1.

The Akaike Information Criterion (AIC) (Akaike, 1974) is an index that may help the comparison of models with different number of parameters, which is our case. AIC is defined by:

$$AIC = n \ln\left(\frac{RSS}{n}\right) + 2K, \tag{13}$$

where n is the number of data points (x_i, y_i), RSS is the residual sum of squares, and K is the number of parameters of the model. The best model is supposed to be that one with the lowest AIC. AIC requires a bias correction, AIC_c, for small number of points Hurvich and Tsai (1989):

$$AIC_c = n\ln\left(\frac{RSS}{n}\right) + 2K + \frac{2K(K+1)}{n-K-1}. \tag{14}$$

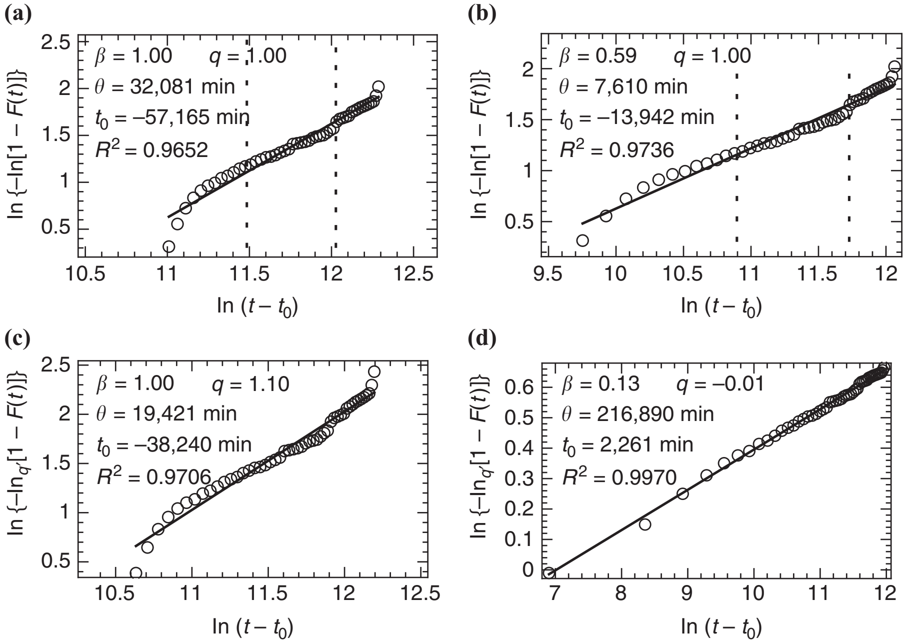
The variable $\Delta_i \equiv IC_i - \min[IC_i]$ may be directly used to compare the models, the best one obviously has $\Delta_i = 0$.

3. Application to a robotic welding station

We considered approximately 1,250 operating times (in minutes) of a robotic welding station used in a manufacturing process. The lifetimes were grouped in 50 intervals and the probability of failure was calculated for each interval by the relative frequency of occurrence. Our analysis is divided in two parts: initially we considered all the operating times as a whole (Figure 1). Then we divided the time-to-failure data in three groups, to evidentiate that there is a mixture of failure models in this example (Figure 3). At the end of this section, we return to all the operating times, without grouping them, to show that the q-Weibull model can fit the whole data set (Figure 4).

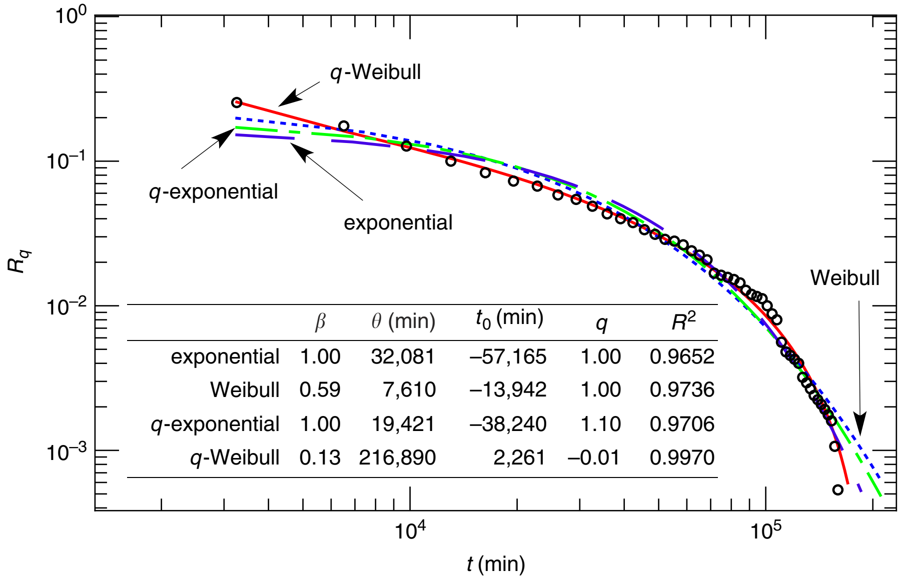
Figure 1 compares the four models by means of the variable y (Equation (11)) as a function of $\ln(t-t_0)$. The main advantage of this representation is that the points should be aligned to a straight line if the data would be perfectly described by the model.

The calculated parameters are indicated in each panel of the Figure 1 and forward in Figures 2 and 4, tabulated for the convenience of the reader. The scales of the abscissas are different for each graph because the models have different values for the parameter t_0.



Notes: (a) Exponential; (b) Weibull; (c) q-exponential; and (d) q-Weibull

Figure 1. Fitting of the time-to-failure data (circles) and fitted curves (solid lines) of the models



	β	θ (min)	t_0 (min)	q	R^2
exponential	1.00	32,081	–57,165	1.00	0.9652
Weibull	0.59	7,610	–13,942	1.00	0.9736
q-exponential	1.00	19,421	–38,240	1.10	0.9706
q-Weibull	0.13	216,890	2,261	–0.01	0.9970

Figure 2. Fitting of the time-to-failure data (circles) and reliability function calculated by the models

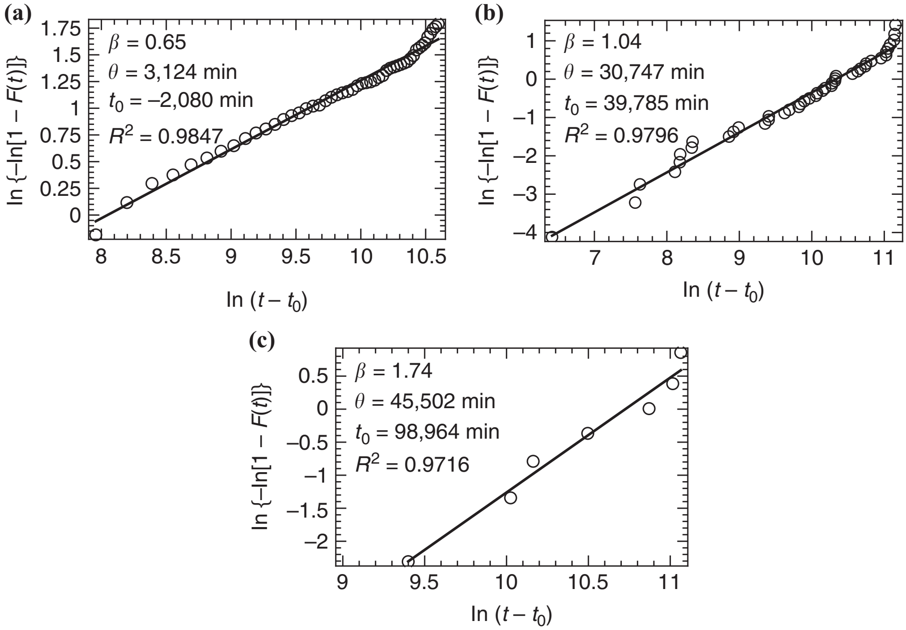
Notes: Exponential (dashed line, violet online); Weibull (dotted line, blue online); q-exponential (dash-dotted line, green online); and q-Weibull (solid line, red online). The inset present the parameters

The vertical lines in Figure 1(a) and (b) will become clear in Figures 3 and 4: they are limiting times (at $t = 40{,}000$ min and $t = 1{,}10{,}000$ min) of three different failure behaviors.

In the following we analyze the fitting of each model, corresponding to each panel of Figure 1.

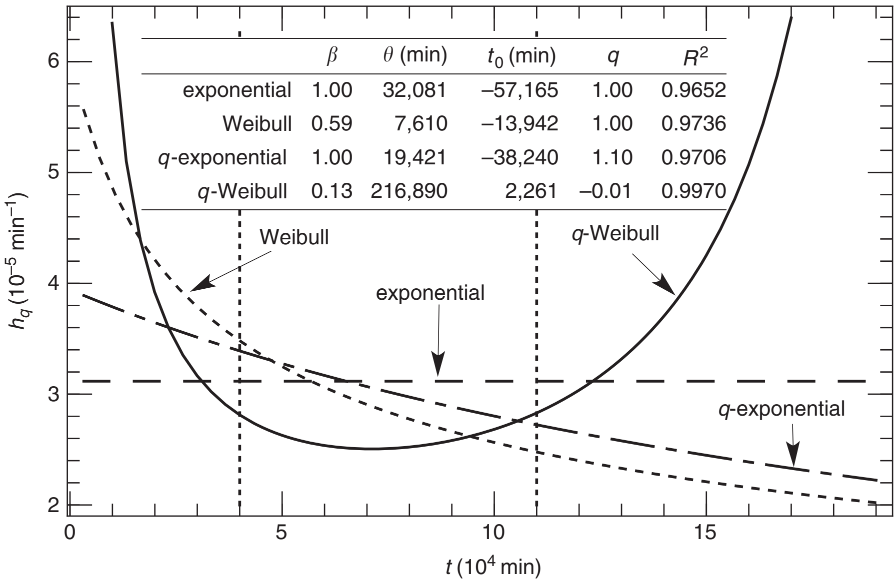
(a) The exponential distribution: the hypotheses compatible for this case are: nonrepairable items, single failure mode, and constant failure rate. Though the coefficient of determination $R^2 = 0.9652$ indicates a good fitness quality, the simplifying assumption of constant failure rate is too strong and does not correspond to the data, as it becomes clear soon. The data include all failure modes and there is no guarantee that the combination of all of them produces constant failure rate. Another way to observe this limitation is to consider a system that presents various failure modes and each of them with constant failure rates. Despite this, the system failure rate is not constant, according to the theory of system reliability (for more details see redundancy in Lewis (1987)).

(b) The Weibull distribution: if the hypothesis of constant failure rate, assumed in the previous model (a), is relaxed to a monotonic behavior, then this situation can be modeled by the Weibull distribution (with $\beta \neq 1$, $q = 1$). This model can describe monotonically decreasing (for $0 < \beta < 1$) or monotonically increasing (for $\beta > 1$) failure rates. Of course the particular case $\beta = 1$ reduces to item (a). The fitting of the Weibull distribution has a coefficient of determination ($R^2 = 0.9736$) slightly larger than that of the exponential distribution. The calculated value of the parameter $\beta < 1$ indicates that the failure rate for this example should be monotonically decreasing, however, it is shown in the next section that this is not true. This result is due to the limitation of the model (b) imposed to the data.



Notes: (a) Decreasing (β<1), time-to-failure 40,000 min; (b) constant ($\beta \approx 1$), 40,000 min <time-to-failure 1,10,000 min; and (c) increasing (β>1), time-to-failure >1,10,000 min

Figure 3. Examples of Weibull fittings for samples grouped by time of failure



	β	θ (min)	t_0 (min)	q	R^2
exponential	1.00	32,081	–57,165	1.00	0.9652
Weibull	0.59	7,610	–13,942	1.00	0.9736
q-exponential	1.00	19,421	–38,240	1.10	0.9706
q-Weibull	0.13	216,890	2,261	–0.01	0.9970

Notes: Vertical lines indicate the separation of the lifetime in three distinct regimes. The parameters are shown in the inset

Figure 4. Failure rate curves calculated by the models exponential (dashed line), Weibull (dotted line), q-exponential (dash-dotted line), and q-Weibull (solid line)

(c) The q-exponential distribution: the generalization of (a) is achieved by the q-exponential distribution ($\beta = 1$, $q \neq 1$). It is worth mention that the q-exponential distribution is not restricted a constant failure rate as it is the case (a). This is monotonic decreasing ($1 < q < 2$) or monotonic increasing ($q < 1$).

(d) The q-Weibull distribution: this is the most general model we are considering, and it presents a nonmonotonic failure rate. Visual inspection of Figure 1(d) indicates that the quality of this fitness is greater than the previous analysis and the coefficient of determination greater than 0.99 confirms it.

Figure 2 presents the fittings for the reliability function $R_q(t)$, as a function of time, for the four models. Note that the q-Weibull distribution is able to describe the whole range of the data, while the others depart from the experimental points for low or high values of time.

The Figure 1(b) does not present a clear abrupt change of the slope (also known as dog-leg), that is typical when there is a mixture of failure modes, but this is the case for this sample, as can be seen with the following procedure: we divided all 1,250 time-to-failure data in three groups and fitted the usual ($q = 1$) Weibull model separately for each one. For $t \leqslant 40{,}000$ min, we obtained $\beta < 1$ that corresponds to the decreasing failure rate. For the intermediate region $40{,}000 \text{ min} < t \leqslant 1{,}10{,}000$ min, we found $\beta \approx 1$, that predicts a constant failure rate, and the last region, $t > 1{,}10{,}000$ min, produced $\beta > 1$, that corresponds to increasing failure rate. The results of the fittings are shown in Table I. Figure 3 shows these fitting plots. The limiting times 40,000 and 1,10,000 min were found by testing different values, and then we chose those that gave best fits in Figure 3.

The failure rate curves are shown in Figure 4. The curves are made with all the data, without dividing them into the three regions aforementioned. This makes evident that the exponential, the Weibull and the q-exponential models are unable to reproduce the three behaviors detected, separated by vertical lines in the figure (these vertical lines also appear in Figure 1(a) and (b)), with the same parameter set, while the q-Weibull presents a decreasing region, an approximately constant region, and finally an increasing region.

Finally Table II presents AIC_c(bias-adjusted AIC) and Δ_i for the fittings. According to the results, the q-Weibull model is the best fitting model, as indicated by visual inspection on Figures 1 and 2.

Table I.
Parameters and coefficient of determination for each region

Parameters	Decreasing	Constant	Increasing
β	0.65	1.04	1.74
θ (min)	3,124	30,747	45,502
t_0 (min)	−2,080	39,785	98,964
R^2	0.9847	0.9796	0.9716

Table II.
AIC_c and Δ_i

Model	AIC_c	Δ_i
q-Weibull	−472.49	0
Weibull	−270.32	202.17
q-exponential	−246.17	226.32
Exponential	−258.96	213.53

4. Conclusions

We have compared four models for the description of life data of a robotic welding station used in a manufacturing process, a system that exhibits a bathtub behavior in the instantaneous failure rate curve. In total, two of the models are generalized versions of the usual ones (the q-Weibull and the q-exponential). The generalizations are based on the q-exponential function, that has been successfully applied in different systems within nonextensive statistical physics. We have used the approximation of the median ranks, relative frequency of occurrence of failure and the least squares method to estimate the parameters of the models. The results show that the q-Weibull distribution is much more flexible to describe shapes of hazard rate curves than the other analyzed models.

Among the considered models, the q-Weibull distribution was the most appropriate for the considered example. It was able to identify three distinct behaviors of failure rate: decreasing, constant, and increasing, with the same parameter set. These behaviors can be interpreted as three predominant failure modes. In another context the passages could represent infant mortality, lifetime, and aging of an item.

According to this example Weibull and q-exponential models indicate a decreasing failure rate; in fact these models represent only monotonic failure rate shapes. Maintenance policies, risk and costs analyses may be inaccurate if the reliability model cannot recognize nonmonotonic failure rate shape. The q-Weibull model is more indicated for these cases.

We have also taken into account the AIC as an additional source of information to show that the q-Weibull distribution is indeed the best model, among those considered, to represent the data.

The q-distributions can possibly be successfully used in other systems and improve the description of reliability engineering problems.

References

Akaike, H. (1974), "A new look at the statistical model identification", *IEEE Transactions on Automatic Control*, Vol. AC-19 No 6, pp. 716-723.

Assis, E.M., Borges, E.P. and Vieira de Melo, S.A.B. (2013), "Generalized q-Weibull model and the bathtub curve", *International Journal of Quality & Reliability Management*, Vol. 30 No. 7, pp. 720-736.

Bak, P. (1997), *How Nature Works*, Oxford University Press, Oxford.

Berberan-santos, M., Bodunov, E.N. and Valeur, B. (2008), "History of the Kohlrausch (stretched exponential) function: focus on uncited pioneering work in luminescence", *Ann. Phys. (Berlin)*, Vol. 17 No. 7, pp. 460-461.

Costa, U., Freire, V., Malacarne, L., Mendes, R., Picoli, S., de Vasconcelos, E.A. and da Silva, E.F. (2006), "An improved description of the dielectric breakdown in oxides based on a generalized Weibull distribution", *Physica A Statistical Mechanics and its Applications*, Vol. 361 No. 1, pp. 209-215.

Hurvich, C.M. and Tsai, C.L. (1989), "Regression and time series model selection in small samples", *Biometrika*, Vol. 76 No. 2, pp. 297-307.

Johnson, L. (1951), "The median ranks of sample values in their population with an application to certain fatigue studies", *Industry Mathematics*, Vol. 2, pp. 1-9.

Kohlrausch, R. (1854), "Theorie des elektrischen rückstandes in der leidener flasche", *Annalen Der Physik*, Vol. 167 No. 2, pp. 179-214.

Lewis, E.E. (1987), *Introduction to Reliability Engineering*, John Wiley & Sons, New York, NY.

Murthy, D.N.P., Xie, M. and Jiang, R. (2004), *Weibull Models*, John Wiley & Sons Inc., New York, NY.

Nadarajah, S. and Kotz, S. (2005), "On some recent modifications of Weibull distribution", *IEEE Trans. Reliability*, Vol. 54 No. 4, pp. 561-562.

Pham, H. and Lai, C. (2007), "On recent generalizations of the Weibull distribution", *IEEE Trans. Reliability*, Vol. 56 No. 3, pp. 454-458.

Picoli, S., Mendes R. and Malacarne, L. (2003), "*q*-Exponential, Weibull, and *q*-Weibull distributions: an empirical analysis", *Physica A Statistical Mechanics and its Applications*, Vol. 324 Nos 3/4, pp. 678-688.

Prato, D. and Tsallis, C. (1999), "Nonextensive foundation of Lévy distributions", *Physical Review E*, Vol. 60 No. 2, pp. 2398-2401.

Sartori, I., Assis, E.M., da Silva, A.L., Vieira de Melo, R.L.F., Borges, E.P. and Vieira de Melo, S.A.B. (2009), "Reliability modeling of a natural gas recovery plant using q-Weibull distribution", in Alves, R.M.B., Nascimento, C.A.O. and Biscaia, E.C. (Eds), *Computer Aided Chemical Engineering, ISBN 9780444534729*, Elsevier, Vol. 27, pp. 1797-1802.

Tsallis, C. (1994), "What are the numbers that experiments provide?", *Quimica Nova*, Vol. 17 No. 6, pp. 468-471.

Tsallis, C. (2009), *Introduction to Nonextensive Statistical Mechanics, Approaching A Complex World*, Springer, New York, NY.

Tsallis, C. (2013), "Nonextensive statistical mechanics and thermodynamics", Rio de Janeiro, CBPF – Brazilian Center for Research in Physics, available at: http://tsallis.cat.cbpf.br/biblio.htm (accessed October 15, 2013).

Tsallis, C., Levy, S.V.F., Souza, A.M.C. and Maynard, R. (1995), "Statistical-mechanical foundation of the ubiquity of lévy distributions in nature", *Physical Review Letters*, Vol. 75 No. 20, pp. 3589-3593, (erratum Vol. 77, p. 5442, 1996).

Weibull, W. (1951), "A statistical distribution function of wide applicability", *Journal of Applied Mechanics*, Vol. 73, pp. 293-297.

About the authors

Dr Edilson M. Assis is a Professor at the Catholic University of Salvador. He holds MSc Degree in Production Engineering from Federal University of Bahia, Brazil. He is PhD in Industrial Engineering from the Federal University of Bahia (UFBa, 2013). Dr Edilson M. Assis is the corresponding author and can be contacted at: edilsonassis@gmail.com

Dr Ernesto P. Borges is a Professor at the Institute of Physics, Federal University of Bahia, Brazil. He received his BS in Chemical Engineering from the Federal University of Bahia (UFBA, 1986). He obtained his MSc in Chemical Engineering from the Federal University of Rio de Janeiro, Brazil (COPPE/UFRJ, 1993), and his PhD in Physics from the Brazilian Center for Physics Research, Rio de Janeiro, Brazil, (CBPF, 2004). His research interests are mainly concerned in nonextensive statistical mechanics and its applications to complex systems.

Dr Silvio A.B. Vieira de Melo is an Associate Professor at the Industrial Engineering Program, Polytechnic School, Federal University of Bahia, Brazil. He received the BS in Chemical Engineering from the Federal University of Bahia. He obtained the MSc and the PhD both in Chemical Engineering from the Federal University of Rio de Janeiro, Brazil. His research interests are in the areas of reliability data analysis and life testing.

Dr Leizer Schnitman is a Professor at the Industrial Engineering Program, Polytechnic School, Federal University of Bahia, Brazil. He received the BS and also holds his MSc in Electrical Engineering from the Federal University of Bahia. He obtained the PhD in Electronic Engineering from the Aeronautic Institute of Technology, Brazil. His research interests are in the areas of control system and artificial intelligence.