

Fingerprint of Tsallis statistics in cosmic ray showers

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Abstract

We investigate the impact of the Tsallis non extensive statistics introduced by intrinsic temperature fluctuations in p-Air ultra high energy interactions on observables of cosmic ray showers, such as the slant depth of the maximum X_{\max} and the muon number on the ground N_{μ} . The results show that these observables are significantly affected by temperature fluctuations and agree qualitatively with the Heitler model predictions.

1 Introduction

The Pierre Auger Observatory [1, 2] has led to great discoveries in the field of ultra-high energy cosmic rays (UHECRs) such as the confirmation of a suppression of the cosmic ray flux at energies above 4×10^{19} eV, limits on photon and neutrino fluxes at ultra-high energies and a hint of large scale anisotropies at energies above 8 EeV. Nevertheless many questions related to these particles are still open. Particularly interesting is the behavior of the slant depth of the shower maximum with energy. Understood in terms of the LHC-tuned shower models, it suggests a gradual shift to a heavier composition, with a large fraction of protons at 10^{18} eV, changing to a heavier composition at $10^{19.5}$ eV [3]. However, we should interpret this result with caution since measurements of shower properties performed by the Auger Collaboration have revealed inconsistencies between data and present shower models. For instance, the Pierre Auger Collaboration has reported the first hybrid measurement of the average muon number in inclined air showers at ultra high energies, suggesting a muon deficit in simulations of about 30% to $80_{-20}^{+17}(\text{sys})\%$ at 10^{19} eV, depending on the hadronic interaction model [4]. Hence the measured behavior of the slant depth of the shower maximum evolution could be understood as a hint of a new hadronic interaction physics at energy scales beyond the reach of LHC.

In this work we will deal with hadronic interactions in a statistical model, as first introduced by Hagedorn [5] ideas in the sixties. Recently a power-law function based on the Tsallis statistics [6] has been widely used on fitting the transverse momentum (p_T) and pseudo-rapidity (η) distributions measured in high-energy collisions [7, 8, 9, 10, 11]

while several studies have been devoted to discuss these results in the literature [12, 13, 14, 15, 16, 17, 18, 19, 20]. The Tsallis statistics, extensively used in different branches of science, is often used to describe systems which display properties like memory effects, long range interactions, intrinsic fluctuations, (multi)fractal phase space and so on. It consists in replacing the classical Boltzmann-Gibbs entropy (S_{BG}) by the form proposed by Tsallis

$$S_q = \frac{(1 - \sum_i p_i^q)}{q - 1} \xrightarrow{q \rightarrow 1} S_{BG} = - \sum_i p_i \ln p_i, \quad (1)$$

where p_i is the probability of a particle occupy the state i and q is the Tsallis index. This definition comprises the Boltzmann-Gibbs entropy as a particular case, where $q = 1$. On the other hand, a straight consequence from this expression is that the generalized entropy is no longer an extensive quantity, once we can verify that

$$S_q(A + B) = S_q(A) + S_q(B) + (1 - q)S_q(A)S_q(B), \quad (2)$$

with the parameter q being a measure of the nonextensivity of the system. As a consequence, we must replace the usual exponential Boltzmann-Gibbs distribution, $\exp(-E/T)$, by the Tsallis power-law distribution

$$f(E) = \frac{(2 - q)}{T} \left[1 - (1 - q) \frac{E}{T} \right]^{\frac{1}{1-q}}, \quad (3)$$

where E is the state energy and T is the temperature of the system.

According to [20], the behaviors presented by the transverse momentum and pseudo-rapidity distributions, in high-energies domain, are best described using a nonexponential distribution, such as the one proposed by Tsallis. In fact, following the ideas discussed in [20], that behavior emerges from fluctuations of the thermal energy within the gas of quarks and gluons before the hadronization process. Using this approach, we can relate the parameter q with those thermal fluctuations,

$$q = 1 + \frac{\sigma_T^2}{\langle T \rangle^2} = 1 + \frac{\langle (\frac{1}{T})^2 \rangle - \langle (\frac{1}{T}) \rangle^2}{\langle (\frac{1}{T}) \rangle^2}. \quad (4)$$

Obviously, when $q = 1$ we recover the expected result obtained in the Boltzmann-Gibbs description, where we get an equilibrium at temperature T .

By assuming such scenario in which the temperature T fluctuates within each collision, the energy distribution of the particles generated in a single high energy interaction follows a power law Tsallis distribution, given by eq. 3. The left panel of Figure 1 presents the Tsallis energy distribution $f(E)$ with a fixed temperature T for different values of q . We can see that higher the q values, the probability for generating particles with larger energy values become greater. As a consequence of the total energy conservation constraint, $\sum_{i=1}^{i=N} E_i = E_{CM}$, where E_{CM} is the total energy of the interaction in the

center of momentum frame, it can be shown that the Tsallis statistics leads to a Negative Binomial multiplicity distribution given by

$$P(N) = \frac{(q-1)^N q-1}{N!} \frac{\Gamma(N+1 + \frac{2-q}{q-1})}{\Gamma(\frac{2-q}{q-1})} \left(\frac{E}{T_L}\right)^N \left[1 - (1-q)\frac{E}{T}\right]^{-N + \frac{1}{1-q}}. \quad (5)$$

Such distribution has a form shown in the right panel of the Figure 1, where it is possible to see how its maximum is affected by q , becoming closer to zero as q grows. Besides, the inset plot of this figure shows that the relationship between the value for the maximum of the multiplicity distribution and q is quite linear, at least in that domain of q values. Therefore, one can see that the introduction of the Tsallis statistics in this context changes the energy, momenta and multiplicity distributions of the particles generated in the hadronic interaction.

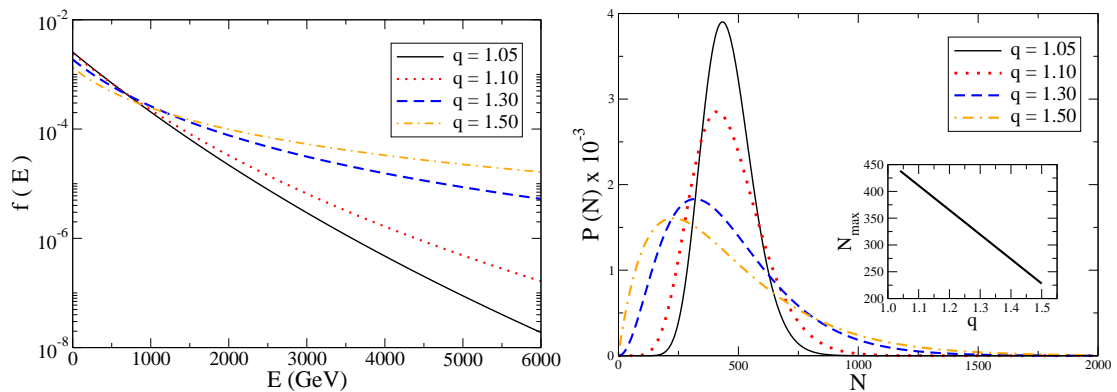


Figure 1: Tsallis energy distributions (left panel) and the corresponding multiplicity distributions (right panel) as a function of q for a fixed value of temperature T .

The transverse momentum and pseudo-rapidity distributions resulting from high energy collisions measured by several experiments show a large discrepancy in the values of the parameter T , reflecting different physics for the transverse and the longitudinal space. The transverse distributions are thermal-like, presenting a parameter T_T almost independent of the energy while those from the longitudinal space have a temperature sensitive to the energy of the collision, understood as the mean energy available per produced particle [20], $T_L = kE_{CM}/\langle N \rangle$, where $\langle N \rangle$ and k are, respectively, the mean multiplicity and inelasticity of the interaction. Moreover, since the measured Tsallis index for the longitudinal space q_L is much larger than measured for the transverse space q_T and $T_L \gg T_T$, the resulting $q \sim q_L$. Also, as verified by simulations, the transverse momentum distribution has a minor contribution on the cosmic ray observables studied in this work. Therefore, from now on, we will assume a statistical equilibrium for the transverse momentum space and we will refer to the entropic index $q = q_L$ and temperature $T = T_L$.

The goal of the present paper is to study the impact of the temperature T fluctuations, represented by the parameter q , on the shower maximum, X_{\max} , and number of muons

on the ground, N_μ . The simulations performed in this work are described in section 2. In section 3 we present the results of the simulations and discuss them in light of the Heitler Model. Finally, we present the conclusions of this work in section 4.

2 Simulations

For all simulations presented in this work, we have used CORSIKA 7.40 [21] with the interaction models Sibyll 2.1 [22] and GHEISHA 2002d [23], for high and low energy processes, respectively. The muon energy threshold used in the simulations is 0.3 GeV and the array detector position is at 1400 m above sea level, corresponding to the mean altitude of the Pierre Auger Observatory. The air shower simulation chain is as follows: first we simulate the secondaries generated in the collision between a cosmic ray and a nucleus of the upper atmosphere externally by assuming that the hadronization process is described by the Tsallis statistics; the resulting particle list is then inserted back into CORSIKA (using the stacking option and sampling option with thinning = 10^{-6}) to proceed with usual cascade development through the atmosphere. Such a procedure was performed 1000 times for each of several values of q (1.01, 1.025, 1.05, 1.075, 1.10, 1.15, 1.20, 1.25, 1.30, 1.35, 1.40, 1.45 and 1.50) for showers with zenith angle $\theta = 38^\circ$ initiated by a proton of fixed energy $E = 10^{18}$ eV. The reason for limiting the entropic index to $q = 1.5$ in this work is that the mean value of the Tsallis distribution $f(E)$, given by $\langle E \rangle = T/(3 - 2q)$, is well defined only for $1 \leq q \leq 1.5$ [16]. This model assumes that the Sibyll predictions are valid for lower energies since they are tuned by accelerator data while it fails for higher energies. This added to the parametrizations of the LHC transverse momentum and pseudo-rapidity distributions by the Tsallis statistics justifies its use in this work for energies above $E \sim 10^{18}$ eV. The point of the air shower first interaction is determined using the p -Air cross-section predicted by the Epos 1.99 model [24]. The reason for using this value instead of the one predicted by the Sibyll model is that the later presents a large discrepancy in relation to that measured by the Pierre Auger Collaboration [25]. The mean multiplicity $\langle N \rangle$ and inelasticity distribution of the p -Air interaction used in this work were extracted from p -Air interaction simulations using the Sibyll model. Since the Tsallis distribution is non-extensive, generating particle energies E_i according to this distribution subject to the constraint $\sum_{i=1}^{i=N} E_i = E_{CM}$ is not a simple task because the probabilities $f(E_i)$ associated to each particle do not factorize [26]. Therefore, we perform the simulation process according the following procedure: first, we select the number of particles generated in the p -Air interaction using the $P(N)$ expression given by eq. 5 and we assign an energy E_i to each particle i according to the Tsallis distribution. Then two particles i and j are randomly selected and a random fraction ΔE_i of the energy E_i is given to particle j in such a way that the new values of energies are $E_{i,new} = E_i - \Delta E_i$ and $E_{j,new} = E_j + \Delta E_i$. After that, we compute the deviation of this energy distribution in relation to Tsallis distribution $f(E)$ using the D^2

estimator defined by:

$$D^2 = \sum_{i=1}^{N_{bins}} \left(\frac{dN}{dE} - f(E) \right)^2. \quad (6)$$

If the new D^2 value is smaller than the previous one, we accept the changes in energy of the particles i and j , otherwise we cancel them. We keep repeating this procedure for another pair of particles. The whole process continues until the D^2 value is stabilized. Generally, it takes 10^4 iterations to reach such stabilization. To be conservative, the simulations presented in this work were performed using 10^5 iterations for each p-Air interaction. Since we verified through simulations that X_{\max} and N_{μ} are not sensitive to changes in q_T , all simulations corresponding to the transverse space presented in this work were evaluated assuming a statistical equilibrium, i.e, the Hagedorn [5] transverse momentum distribution:

$$\frac{dN}{dp_T} \simeq cp_T \exp\left(-\frac{p_T}{T_T}\right), \quad (7)$$

with $\langle T_T \rangle = 133$ MeV. The type of particles are randomly generated according to Sibyll predictions and once we have generated the particles masses m , the longitudinal momentum is obtained as $p_L = \sqrt{E^2 - m^2 - p_T^2}$. These kinematic variables along with the species of particle complete all the information we need to reintroduce in CORSIKA to proceed with shower propagation through the atmosphere.

3 Results and discussion

In the following we describe the impact of T_L fluctuations, represented by the parameter $q = q_L$, on the shower maximum, X_{\max} , and number of muons on the ground. We will discuss them in terms of the predictions of the Heitler model [27, 28] as well as of the results achieved in reference [29]. Although extremely simple, the predictions of the Heitler model are remarkable. It assumes that the shower maximum is reached when the energies of particles become smaller than a critical energy, in which energy loss processes dominate the production of new particles in the case of electromagnetic component, or the charged pion interaction length becomes larger than the decay length of pions in muons, in the case of the hadronic one. As a consequence, it predicts an increase of the $\langle X_{\max} \rangle$ for smaller mean multiplicities, since larger multiplicities correspond to lower energy per particle. Besides, reference [29] describes a detailed investigation of the impact of the multiplicity, hadronic particle production cross section, elasticity and pion charge-ratio on air shower observables with most of the predictions qualitatively understood within the simple Heitler model and its extension to hadronic component.

The Pearson coefficient ρ was used to assess the degree of correlation between air

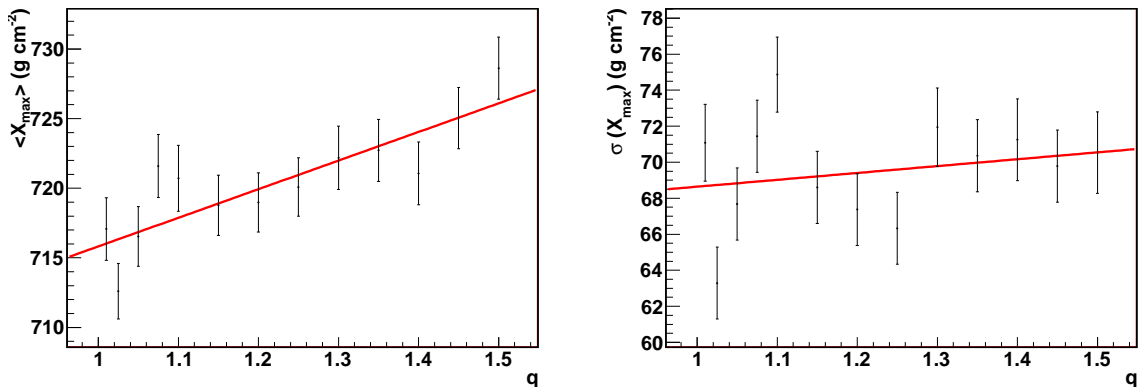


Figure 2: X_{\max} (left) and $\sigma(X_{\max})$ (right) distributions obtained from air shower simulations initiated by the interaction between a proton of $E = 10^{18}$ eV and a nucleus of the upper atmosphere in which the hadronization process is described by the Tsallis statistical.

showers observables and q . It is defined by

$$\rho = \frac{\text{cov}(X, Y)}{\sigma(X) \cdot \sigma(Y)} \quad (8)$$

and measures the linear correlation between two variables X and Y , yielding a value in the interval $[-1, +1]$, with 1 meaning total positive correlation, 0 no correlation, and -1 total negative correlation. $\text{cov}(X, Y)$ is the covariance between X and Y and $\sigma(X)$ and $\sigma(Y)$ are the standard deviations of variables X and Y . The results for the mean depth of shower maximum, $\langle X_{\max} \rangle$, and the fluctuations of X_{\max} are summarized in Figure 2.

The left panel shows the $\langle X_{\max} \rangle$ as a function of q . A strong correlation is observed yielding a Pearson correlation coefficient $\rho = 0.84$. The red line presents the best linear fit corresponding to a $\chi^2_{\nu} = 1.09$. The comparison between the predictions from Heitler model and reference [29] with our results requires caution since we did not change the mean multiplicity in our simulations of the first interaction. However, the changes in distributions of energy and momenta of the particles generated in the first interaction results in a spread of the multiplicity distribution and a shift of its peak to lower values as it is shown in Figure 1. The reason for the strong correlation is that most of the showers generated with larger q values are initiated with smaller multiplicities, or equivalently, with higher energy per particle.

Besides, the right panel of Figure 2 presents the corresponding plot for the $\sigma(X_{\max})$ as a function of q . In this case, the observed correlation is weak, with $\rho = 0.21$ and $\chi^2_{\nu} = 1.84$ corresponding to the best linear fit, shown by the red line. According to the Heitler model, the variance of the X_{\max} distribution depends on the hadronic cross-section interaction, σ_I , and multiplicity, N , via $V(X_{\max}) \propto 1/\sigma_I^2 + \ln(2X_0)^2 V(\ln N)$, where $X_0 \sim 37$ g/cm² is the electromagnetic radiation length. In this work, the hadronic interaction is given by the one corresponding to the interaction between a proton and a nucleus of the atmosphere, $\sigma_I = \sigma_{p-Air}$. Although the $V(\ln N)$ increases with q , the observed correlation between

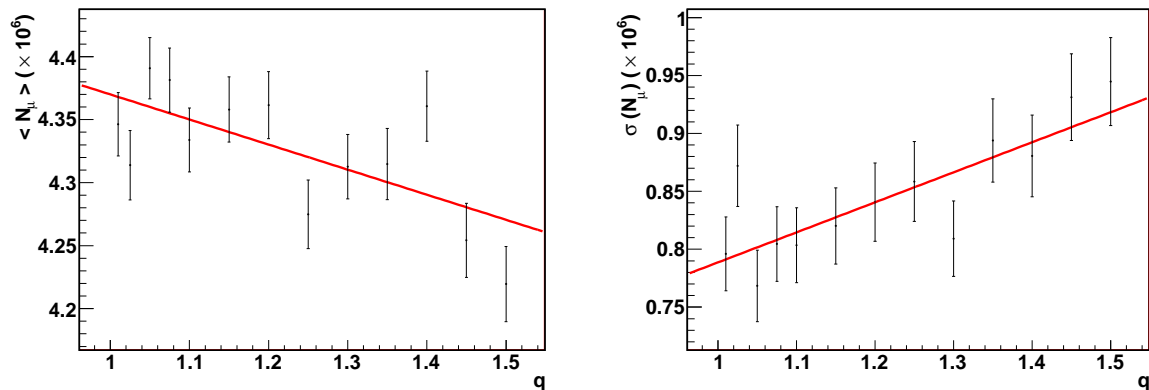


Figure 3: $\langle N_\mu \rangle$ (left) and σ_{N_μ} (right) distributions obtained from air shower simulations initiated by the interaction between a proton of $E = 10^{18}$ eV and a nucleus of the upper atmosphere in which the hadronization process is described by the Tsallis statistical.

$V(X_{\max})$ and q is weak since the increase of spread of the multiplicity distribution for larger q values is dominated by the first term contribution in $V(X_{\max})$, as a consequence of the relatively small value of the p-Air cross-section.

The impact of q on the mean number of muons in the ground, $\langle N_\mu \rangle$, and the fluctuations of N_μ are summarized in Figure 3. The $\langle N_\mu \rangle$ as a function of q , shown in the left panel, presents a strong anti-correlation with q , with $\rho = -0.91$ and $\chi_\nu^2 = 0.72$ corresponding to the best linear fit, marked in red. A superficial analysis of the left panels of Figures 2 and 3 could indicate wrongly that $\langle N_\mu \rangle$ and $\langle X_{\max} \rangle$ are anti-correlated. The positive correlation between $\langle N_\mu \rangle$ and $\langle X_{\max} \rangle$ position exists but it is weak since muons are hardly attenuated in the atmosphere. Therefore it is not the most important factor for the $\langle N_\mu \rangle$ behavior as a function of q . Indeed muons are mainly produced as a result of pion decay and their abundance in the ground, especially considering the most energetic particles, is strongly correlated with the number of pions in the shower. As a consequence of the reduction of the peak of the multiplicity distribution for larger q values, most showers presents lower production of pions in the first interaction, constituting the main reason for the observed anti-correlation between $\langle N_\mu \rangle$ and q .

On the other hand, the right panel of Figure 3 shows a strong correlation of $\sigma(N_\mu)$ and q , with $\rho = 0.88$, as a natural consequence of the spread of the multiplicity distribution. The red line shows the best linear fit with corresponding $\chi_\nu^2 = 0.52$.

Finally, Table 1 summarizes the correlation coefficients ρ between air shower observables and q obtained in this work.

4 Conclusions

Although the simulations presented in this work are a very simple description of ultra high energy interactions, the results presented here show that intrinsic fluctuations of the sys-

	$\langle X_{\max} \rangle$	$\sigma(X_{\max})$	$\langle N_{\mu} \rangle$	$\sigma(N_{\mu})$
q	$\rho = 0.84$	$\rho = 0.21$	$\rho = -0.91$	$\rho = 0.88$

Table 1: Summary of the correlation coefficients between air shower observables and q obtained in this work.

tem with respect to T_L , given by the parameter $q = q_L$, change the energy, momenta and multiplicity distributions of the particles generated in the interaction between a cosmic ray and a nucleus of the atmosphere, impacting air shower observables such as the slant depth of the maximum X_{\max} and the muon number on the ground N_{μ} . The results show that the higher the temperature fluctuations, the greater the values of the mean slant depth of maximum $\langle X_{\max} \rangle$ and variance of the number of muons on the ground $\sigma(N_{\mu})$, with Pearson correlation coefficients of $\rho = 0.84$ and $\rho = 0.88$, respectively. This results from the spread and shift of the maximum of the multiplicity distribution to lower values for larger temperature fluctuations. Besides, as muons are mainly produced by the decay of charged pions and the shift in the peak of multiplicity distribution reduces the number of such particles generated in the first interaction, the mean number of muons on the ground $\langle N_{\mu} \rangle$ presents a strong negative correlation with q , producing a $\rho = -0.91$. On the other hand, the variance of the slant depth distribution $\sigma(X_{\max})$ presents a very weak correlation with the temperature fluctuations, with $\rho = 0.21$, because the contribution of the hadronic p-Air interaction fluctuations dominates the one originated from the multiplicity distribution. These results agree qualitatively with the Heitler model and reference [29] predictions. Although these results have been obtained for a specific non-extensive hadronic interaction model, we believe that it captures the essential features related to the presence of Tsallis statistics in UHECR showers and can shed light to the understanding of their properties as well as of particle interactions at these energies. Studies regarding different non-extensive particle interaction models are out of the scope of this work and will be addressed in a future work.

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