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Comb – Model and Extensions

Equation to be considered

$$\frac{\partial}{\partial t}\rho(x,y;t) = \mathcal{D}_y\frac{\partial^2}{\partial y^2}\rho(x,y;t) + \mathcal{D}_x\delta(y)\left(\frac{\partial^2}{\partial x^2} - \overline{v}_x\frac{\partial}{\partial x}\right)\rho(x,y;t) - \nabla\cdot(\vec{v}\rho(x,y;t))\,.$$

It is subjected to the boundary and initial conditions

$$\rho(\pm\infty,y;t) = 0 \text{ and } \rho(x,\pm\infty;t) = 0$$

$$\rho(x,y;0) = \widehat{\rho}(x,y)$$

First case

$$v_x \neq 0,\ v_y \neq 0,\ \text{with } \overline{v}_x = 0$$

For this case, the solution is given by

$$\rho(x,y;t) = -\int_{-\infty}^{\infty} d\overline{y}\,\widehat{\rho}(x,y)\mathcal{G}(x,y,\overline{y};t)$$

$$\mathcal{G}(x,y,\overline{y};t) = -e^{\frac{v_y}{2\mathcal{D}_y}(y-\overline{y})}e^{-\frac{v_y^2}{4\mathcal{D}_y}t}\overline{\mathcal{G}}\,'(x,y,\overline{y};t)$$

$$\overline{\mathcal{G}}\,'(x,y,\overline{y};t) = \frac{1}{\sqrt{4\pi\mathcal{D}_y t}}\delta(x-v_x t)\left(e^{-\frac{(y-\overline{y})^2}{4\mathcal{D}_y t}} - e^{-\frac{(|y|+|\overline{y}|)^2}{4\mathcal{D}_y t}}\right) + \frac{1}{\sqrt{8\mathcal{D}_x\sqrt{\mathcal{D}_y}}}\frac{|y|+|\overline{y}|}{\sqrt{4\pi\mathcal{D}_y}}$$

$$\times\int_0^t d\overline{t}\,\frac{e^{-\frac{(|y|+|\overline{y}|)^2}{4\mathcal{D}_y(t-\overline{t})}}}{[(t-\overline{t})\overline{t}^{\frac{1}{2}}]^{\frac{3}{2}}}\mathrm{H}^{1,0}_{1,1}\left[\sqrt{\frac{2}{\mathcal{D}_x}\sqrt{\frac{\mathcal{D}_y}{\overline{t}}}}\,|x-v_x\overline{t}|\,\middle|^{\left(\frac{1}{4},\frac{1}{4}\right)}_{(0,\,1)}\right]$$

Second Case

$$v_x \neq 0,\ v_y \neq 0,\ \text{with } \overline{v}_x \neq 0$$

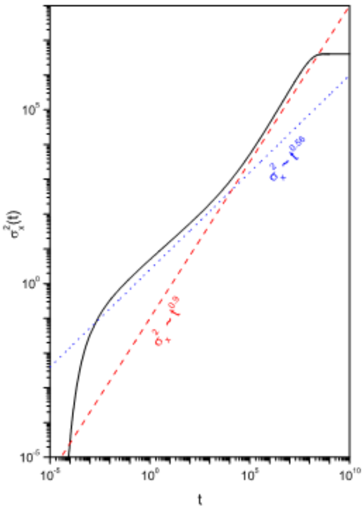
The Green function for this case is

$$\mathcal{G}(x,y,\overline{y};t) = -e^{-\frac{v_y^2}{4\mathcal{D}_y}t}e^{\frac{v_y}{2\mathcal{D}_y}(y-\overline{y})}\tilde{\mathcal{G}}(x,y,\overline{y};t)$$

$$\tilde{\mathcal{G}}(x,y,\overline{y};t) = \frac{1}{\sqrt{4\pi\mathcal{D}_y t}}\delta(x-v_x t)\left(e^{-\frac{(y-\overline{y})^2}{4\mathcal{D}_y t}} - e^{-\frac{(|y|+|\overline{y}|)^2}{4\mathcal{D}_y t}}\right)$$

$$+\frac{1}{t}\int_0^\infty du\,(|y|+|\overline{y}|+2\mathcal{D}_y u)\,\mathcal{G}_y(|y|,|\overline{y}|,2\mathcal{D}_y u;u)\,\mathcal{G}_x(x,-\overline{v}_x u,-v_x t;t)$$

and $\mathcal{G}_\alpha(x,y,z;u) = e^{-\frac{1}{4\mathcal{D}_\alpha u}(x+y+z)^2}/\sqrt{4\pi\mathcal{D}_\alpha u}\,.$



The time behavior of the mean square displacement for $v_y = 5\ 10^{-4}$, $v_x = 0$, $\overline{v}_x = 1$, $\mathcal{D}_y = 5$, $\mathcal{D}_x = 10$ *and* $\tilde{y} = 0.1$

Impedance, Fractional Diffusion Equation, Experimental Data

Equations

$$\mathcal{A}\frac{\partial^\gamma}{\partial t^\gamma}n_\alpha(z,t) + \mathcal{B}\frac{\partial}{\partial t}n_\alpha(z,t) = -\frac{\partial}{\partial z}j_\alpha(z,t) - \int_{-\infty}^t \varsigma(t-t')n_\alpha(z,t')dt'\,,$$

$$j_\alpha(z,t) = -\mathcal{D}_\alpha\frac{\partial}{\partial z}n_\alpha(z,t) \mp \frac{q\mathcal{D}_\alpha}{k_B T}n_\alpha(z,t)\frac{\partial}{\partial z}V(z,t)$$

$$\frac{\partial^2}{\partial z^2}V(z,t) = -\frac{q}{\varepsilon}\left[n_+(z,t) - n_-(z,t)\right],$$

These equations are subjected to the conditions

$$V\left(\pm\frac{d}{2},t\right) = \pm\frac{V_0}{2}e^{i\omega t}$$

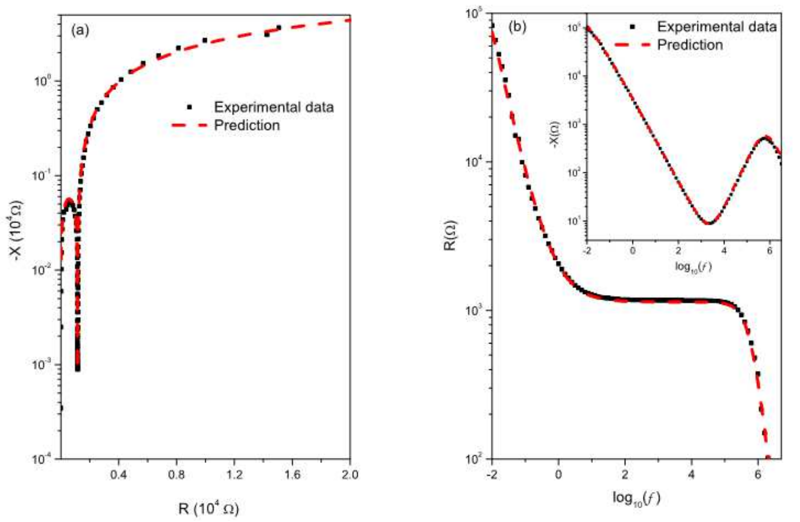
$$j_\alpha(z,t)|_{z=\pm\frac{d}{2}} = \pm k_{\alpha,e}E\,(z,t)|_{z=\pm\frac{d}{2}} \pm \int_0^1 d\overline{\vartheta}\,\widetilde{\tau}(\overline{\vartheta})\int_{-\infty}^t d\overline{t}\mathcal{K}_a(t-\overline{t},\overline{\vartheta})\frac{\partial^{\overline{\vartheta}}}{\partial\overline{t}^{\overline{\vartheta}}}n_\alpha(z,\overline{t})\Big|_{z=\pm\frac{d}{2}}$$

The impedance for this case is given by

$$\mathcal{Z} = \frac{2}{\mathcal{S}\varepsilon\nu_-\Delta(i\omega)}\left(\frac{1}{\lambda^2\nu_-}\tanh(\nu_- d/2) + \frac{d}{2\mathcal{D}}\mathcal{E}(i\omega)\right)$$

$$\Delta(i\omega) = \left(1/\lambda^2 + \chi\left(\Lambda(i\omega)+\omega_e\right)/\mathcal{D}\right)(i\omega+\omega_e)/\nu_- + \left(1/\lambda^2 + \chi\left(i\omega+\omega_e\right)/\mathcal{D}\right)\Upsilon(i\omega)\tanh(\nu_- d/2)$$

with $\mathcal{E}(i\omega) = \chi\left(\Lambda(i\omega)+\omega_e\right) + \chi\nu_-\Upsilon(i\omega)\tanh(\nu_- d/2)$, $\chi = 1/\left(1-\lambda^2\overline{k}_e q/(\mathcal{D}\varepsilon)\right)$, $\omega_e = \overline{k}_e q/\varepsilon$ and $\mathcal{S}$ is the electrode area.



These figure compare the experimental data with the prediction of the model presented here for the real, $R = Re\mathcal{Z}$, *and imaginary,* $X = Im\mathcal{Z}$, *parts of the impedance. We have a good agrement between the experimental data and the predictions for the parameters values:* $\mathcal{S} = 3.14\times10^{-4}\ m^2$, $\varepsilon = 75\varepsilon_0$ $(\varepsilon_0 = 8.85\times10^{-12}\ C^2/(Nm^2))$, $\gamma = 0.98$, $\mathcal{D} = 2.9\times10^{-9}\ m^2/s$, $d = 10^{-3}m$, $\kappa_{a,1} = 8.6\times10^{-5}\ m/s$, $\kappa_{a,2} = 1.03\times10^{-7}\ m/s$, $\overline{k}_e = 0$, $\mathcal{A} = 0.99$, $\lambda = 2.734\times10^{-8}m$, $\tau = 1.5\times10^{-3}s$, $\eta_1 = 0.17$, *and* $\eta_2 = 0.825$.